1/12/2014 - PF DIN2 - Prof. Thiago Ritto

1) (2,0) Considere as duas barras soldadas (M, L) mostradas na Fig. 1. Calcule o momento de inércia I_{zz} e o produto de inércia I_{xy} do sistema em relação ao ponto O. Se o vetor velocidade angular desse sistema vale $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z)$, calcule a quantidade de movimento angular em relação à O.

2) (2,0) Considere a tubulação mostrada na Fig. 2. Um líquido de densidade ρ entra com velocidade conhecida $v_1\mathbf{a}_1$ pela esquerda, onde o diâmetro vale D. A tubulação se divide em duas, onde $d = D/2$. Se o regime é permanente, (a) calcule a vazão mássica, (b) calcule a velocidade de saída, (c) calcule as forças resultantes atuando no suporte, (d) calcule os momentos resultantes em relação ao ponto O (centro do suporte).

3) (3,5) Considere o sistema formado por duas barras pendulares mostrado na Fig. 3. O disco (M, R) gira com velocidade $\dot{\phi}$ na direção e sentidos indicados. As duas barras (m, L) estão pinadas na extremidade do disco e podem girar na direção de $\mathbf{b}_1$. Os três graus de liberdade do sistema são os ângulos ϕ (do disco), θ_1 e θ_2 (das barras). (a) calcule a velocidade do centro de massa da barra 1, (b) calcule a energia potencial do sistema, (c) calcule a energia cinética do sistema, (d) obtenha as equações de movimento para os três graus de liberdade (use Lagrange OU Newton/Euler).

4) (2,5) Uma barra presa por uma rótula gira com velocidade $\boldsymbol{\Omega} + \dot{\boldsymbol{\theta}}$ conforme mostra a figura 4. Um disco está pinado na extremidade da barra e gira em torno do seu próprio eixo com velocidade constante $\dot{\psi}$. (a) faça do diagrama de corpo livre do sistema barra/disco e escreva os vetores resultante de força e de momento na base que julgar mais apropriada, e (b) calcule a aceleração do ponto Q do disco, se a aceleração do pino é conhecida $\mathbf{a}^P$.

FIGURA 1

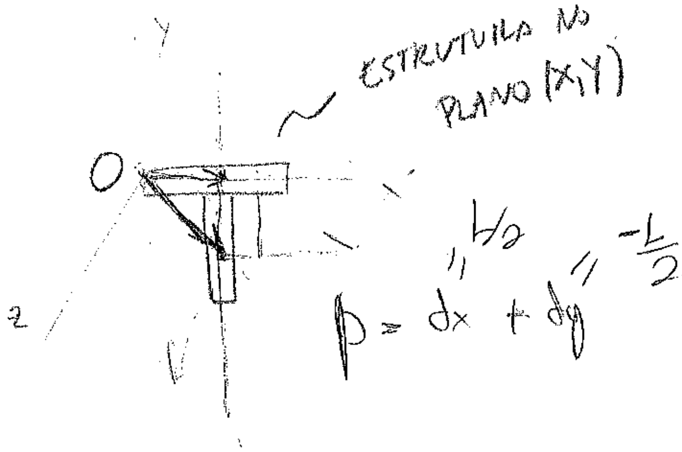


FIGURA 2

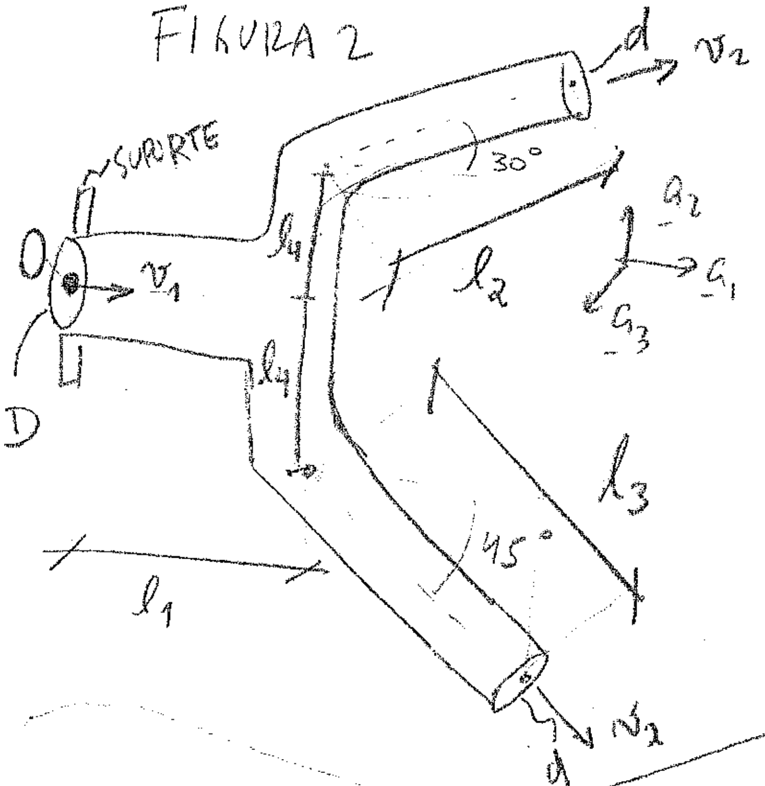
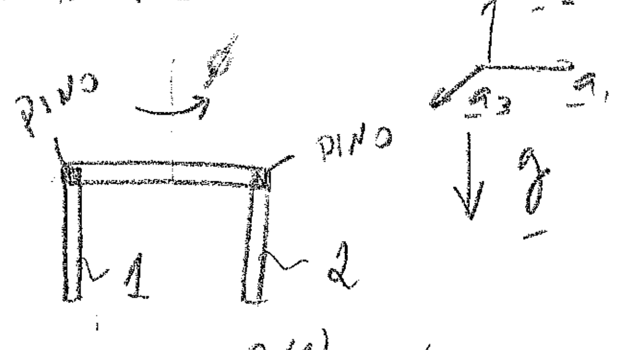


FIGURA 3

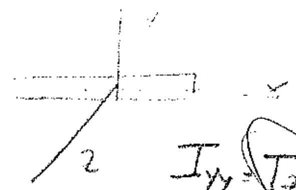


$A \xrightarrow{\Omega(2)} B \xrightarrow{\theta_1(1)} C$ (BARRA 1)

$B \xrightarrow{\theta_2(1)} D$ (BARRA 2)

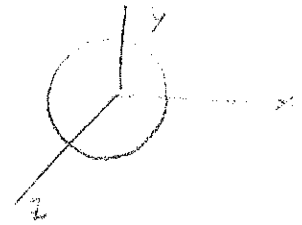
(INERCIAL)

TABELA



$I_{yy} = I_{zz} = \frac{1}{12} m l^2$

$I_{xx} = I_{xy} = I_{xz} = I_{yz} = 0$



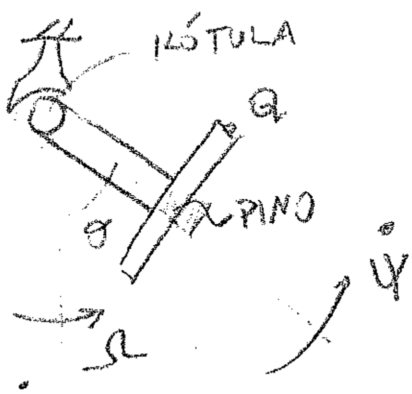
$dx = dz$

$I_{xy} = I_{yz} = I_{xz} = 0$

$I_{xx} = I_{yy} = \frac{1}{4} m r^2$

$I_{zz} = \frac{1}{2} m r^2$

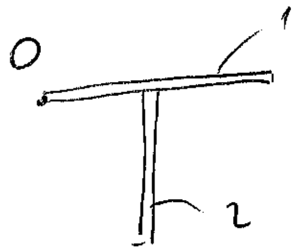
FIGURA 4



$A \xrightarrow{\Omega(2) = \dot{\phi}} B \xrightarrow{\theta(3)} C \xrightarrow{\dot{\psi}(2)} D$

(INERCIAL) (BARRA) (DISCO)

GABARITO PF 2014.2

(1) 

$$I_{zz} = \underbrace{\frac{1}{12} m l^2 + m\left(\frac{l}{2}\right)^2}_{1} + \underbrace{\frac{1}{12} m l^2 + m\left(\left(\frac{l}{2}\right)^2 + \left(\frac{l}{2}\right)^2\right)}_{2}$$

$$= \frac{11}{12} m l^2 \quad 0,8$$

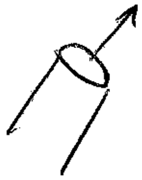
$$I_{xy} = \underbrace{0}_{1} + \underbrace{0 + m\frac{l}{2}\frac{l}{2}}_{2} = \frac{ml^2}{4} \quad 0,8$$

$$\underline{H}^O = \begin{bmatrix} I_{xx} & ml^2/4 & 0 \\ ml^2/4 & I_{yy} & 0 \\ 0 & 0 & \frac{11}{12}ml^2 \end{bmatrix} \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} = \begin{pmatrix} I_{xx} w_x + \frac{ml^2}{4} w_y \\ \frac{ml^2}{4} w_x + I_{yy} w_y \\ \frac{11}{12} m l^2 w_z \end{pmatrix} \quad 0,4$$

(2) a) $m' = \rho A v_1 = \rho \frac{\pi D^2}{4} \left[\frac{kg}{m^3} m^2 \frac{m}{s}\right]$ 0,2

b) $\rho A v_1 = \rho A_2 v_2 + \rho A_2 v_2$ 0,5

$$\rho \frac{\pi D^2}{4} v_1 = 2 \rho \frac{\pi D^2}{16} v_2 \longrightarrow v_2 = 2 v_1$$



$$v_2^{(1)} = v_2 \cos 30\, \underline{a}_1 + v_2 \sin 30\, \underline{a}_2$$

$$v_2^{(2)} = v_2 \cos 45\, \underline{a}_1 - v_2 \sin 45\, \underline{a}_2$$

c) $\underline{F} = m'\left(\underline{v}_2^{(1)} + \underline{v}_2^{(1)} - \underline{v}_1\right)$

$\underline{F} = m'\left(2v_1 \cos 45 + 2v_1 \cos 30 - v_1\right)\underline{a}_1 +$

$m'\left(2v_1 \sin 30 - 2v_2 \sin 45\right)\underline{a}_2$ 0,5

d) $\underline{M}^0 = m'\left(\underline{r}^{2(1)/0} \times \underline{v}_2^{(1)} + \underline{r}^{2(2)/0} \times \underline{v}_2^{(2)} - \cancel{\underline{r}^{1/0}} \times \underline{v}_1\right)$

$\underline{r}^{1/0} \to 0$

$\underline{r}^{2(1)/0} = (l_1 + l_2 \cos 30)\underline{a}_1 + (l_4 + l_2 \sin 30)\underline{a}_2$

$\underline{r}^{2(2)/0} = (l_1 + l_3 \cos 45)\underline{a}_1 - (l_4 + l_3 \sin 45)\underline{a}_2$ 0,8

$\underline{M}^0 = \Big[(l_1 + l_2 \cos 30)2v_1 \sin 30 - (l_4 + l_2 \sin 30)2v_1 \cos 30 +$

$-(l_1 + l_3 \cos 45)2v_1 \sin 45 + (l_4 + l_3 \sin 45)\cos 45\Big]\underline{a}_3$

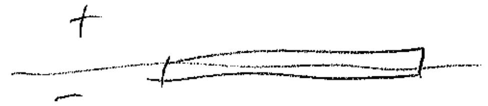
3 $\underline{v}^{*1} = \underline{v}^{P1} + {}^A\underline{\omega}^{B1} \times \underline{r}^{*1/P1}$

$= \dot\phi r \sin\theta_1 \underline{c}_2 + \dot\phi r \cos\theta_1 \underline{c}_3 + \left(\dot\phi \underline{b}_2 + \dot\theta_1 \underline{c}_1\right)\left(-\frac{L}{2}\underline{c}_2\right)$

$\dot\phi \cos\theta_1 \underline{c}_2$
$-\dot\phi \sin\theta_1 \underline{c}_3$ 1,0

$\underline{v}^{*1} = \dot\phi r \sin\theta_1 \underline{c}_2 + \dot\phi r \cos\theta_1 \underline{c}_3 - \dot\theta_1 \frac{L}{2}\underline{c}_3 - \dot\phi \sin\theta_1 \frac{L}{2}\underline{c}_1$

$\underline{v}^{*1} = \begin{pmatrix} -r\dot\phi \sin\theta_1 \frac{L}{2} \\ \dot\phi r \sin\theta_1 \\ \dot\phi r \cos\theta_1 - \dot\theta_1 \frac{L}{2} \end{pmatrix}_C \qquad {}^A\underline{\omega}^{B1} = \begin{pmatrix} \dot\theta_1 \\ \dot\phi r \sin\theta_1 \\ \dot\phi r \cos\theta_1 \end{pmatrix}$

b) 

$\phi_D = 0 \quad \phi_{B1} = -\frac{L}{2}\cos\theta_1\, mg$

0,5 $\quad \phi_{B2} = -\frac{L}{2}\cos\theta_2\, mg$

c)

$K_{DISCO} = \frac{1}{2}\left(\frac{1}{2} m R^2\right)\dot{\phi}^2 \quad ; \quad I_B^* = \begin{bmatrix} \frac{1}{12} m l^2 & 0 \\ & \frac{1}{12} m l^2 \end{bmatrix}$

$K_{B1} = \frac{m}{2}\Big[\dot{\phi}^2 \sin^2\theta_1 \left(\frac{L}{2}\right)^2 + \boxed{\dot{\phi}^2 r^2 \sin^2\theta_1 + \dot{\phi}^2 r^2 \cos^2\theta_1} +$

1,0 $\quad + \dot{\theta}_1^2\left(\frac{L}{2}\right)^2 - \dot{\phi}\, r \cos\theta_1 \dot{\theta}_1 \Big] + \frac{1}{2}\Big[\frac{1}{12} m l^2 \dot{\theta}_1^2 + \frac{1}{12} m l^2 \times$

$\times \dot{\phi}^2 r^2 \cos^2\theta_1\Big]$

K_{B2} = idem B1 c/ θ_2 no lugar de θ_1

d) $K = L - K \qquad$ 1,0

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} = 0$$

$\frac{\partial L}{\partial \phi} = 0 \; ; \; \frac{\partial L}{\partial \dot{\phi}} = \frac{1}{2} m R^2 \dot{\phi} + \frac{m}{2}\Big[2\dot{\phi}\sin^2\theta_1\left(\frac{L}{2}\right)^2 +$

$+ 2\dot{\phi} r^2 - r\cos\theta_1\dot{\theta}_1\Big] + \frac{1}{2}\Big[\frac{1}{12} m l^2\, 2\dot{\phi}\, r^2 \cos^2\theta_1\Big)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = \frac{1}{2} m R^2 \ddot{\phi} + m\Big[\frac{L^2}{4}\Big(\ddot{\phi}\sin^2\theta_1 + \dot{\phi}\frac{d}{dt}(\sin^2\theta_1)\Big) + 2\ddot{\phi} r^2 +$

$- r\ddot{\theta}_1\cos\theta + r\sin\theta_1\dot{\theta}_1^2\Big] + \frac{1}{2}\Big[\frac{1}{6} m l^2 \ddot{\phi}\, r^2 \cos^2\theta_1 +$

$\frac{1}{6} m l^2 \dot{\phi} r^2 \frac{d}{dt}(\cos^2\theta_1)\Big] -$

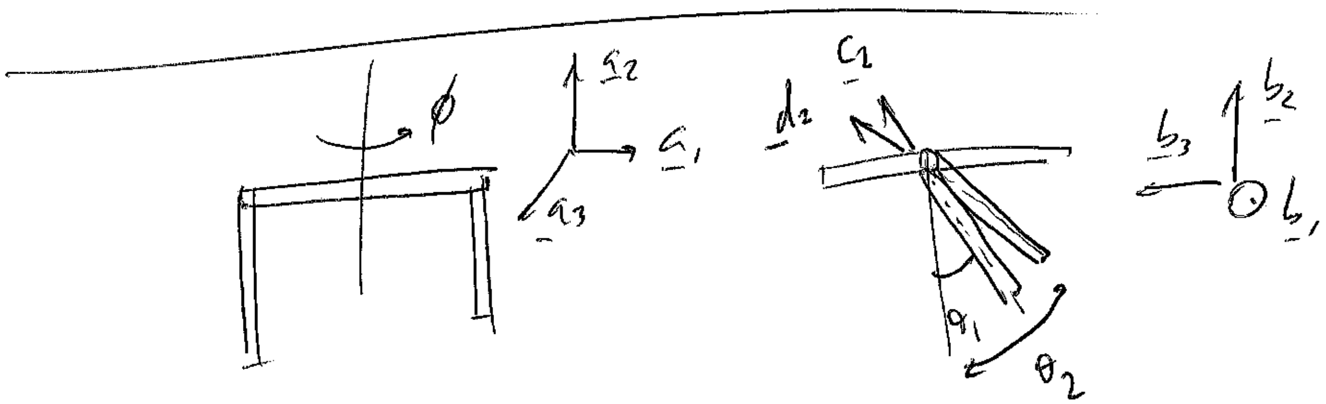
$\therefore$ Vale p/ as duas equações $\begin{cases} \theta = \theta_1 \\ \theta = \theta_2 \end{cases}$

$$\frac{\partial L}{\partial \dot\theta_1} = \frac{m}{2}\left(2\dot\theta_1\left(\frac{L}{2}\right)^2 - \dot\phi r \cos\theta_1\right) + \frac{1}{2}\left[\frac{ml^2}{12} 2\dot\theta_1\right]$$

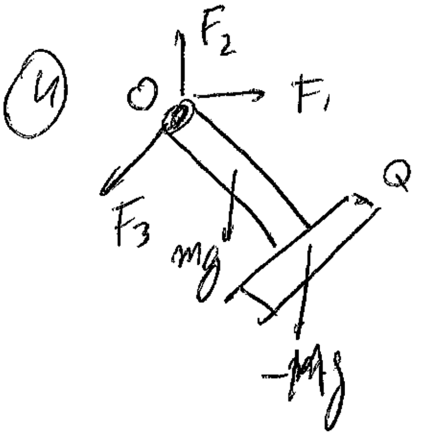
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_1}\right) = \frac{m}{2}\left(2\ddot\theta_1\left(\frac{L}{2}\right)^2\right) - \ddot\phi r \cos\theta_1 + \dot\phi r \cos\theta \dot\theta_1 + \frac{ml^2}{12}\ddot\theta_1$$

$$\frac{\partial L}{\partial \theta_1} = -\frac{L}{2}\sin\theta_1 mg - \frac{L}{2}\sin\theta_2 mg +$$

$$\frac{m}{2}\left[\dot\phi^2 \frac{d}{d\theta_1}(\sin^2\theta_1)\left(\frac{L}{2}\right)^2 + \dot\phi r \sin\theta_1 \dot\theta_1 + \frac{1}{2}\left[\frac{1}{12} ml^2 \dot\phi^2 r^2 \frac{d}{d\theta_1}(\cos^2\theta_1)\right]\right.$$



4



a) $\underline{F} = F_1\,\underline{a}_1 + F_2\,\underline{a}_2 + F_3\,\underline{a}_3 - (mg + Mg)\,\underline{a}_2$

$\underline{M}^O = -mg\frac{l}{2}\sin\theta\,\underline{b}_3 - Mg\,l\sin\theta\,\underline{b}_3$

1,0

b) $\underline{a}^Q = \underbrace{\underline{a}^P}_{\text{conhecida}} + {}^C\underline{\omega}^D \times {}^C\underline{\omega}^D \times \underline{r}^{Q/P} + {}^C\underline{\alpha}^D \times \underline{r}^{Q/P}$

1,5

$\underline{r}^{Q/P} = r\,\underline{d}_1$

$${}^A\underline{\omega}^D = \underline{\omega} = \underbrace{\Omega\,\underline{b}_2}_{\Omega\sin\theta\,\underline{c}_1 + \Omega\cos\theta\,\underline{c}_2} + \underbrace{\dot\theta\,\underline{c}_3}_{-\dot\theta\sin\psi\,d_1 + \dot\theta\cos\psi\,\underline{d}_3} + \dot\psi\,\underline{d}_2$$

$$\underline{\omega} = -\Omega\sin\theta\sin\psi\,\underline{d}_1 - \dot\theta\sin\psi\,\underline{d}_1 + \Omega\cos\theta\,\underline{d}_2 + (\Omega\sin\theta\cos\psi + \dot\theta\cos\psi)\,\underline{d}_3$$

$${}^A\underline{\alpha}^D = \underline{\alpha} = \left(-\Omega\cos\theta\sin\psi\,\dot\theta - \Omega\sin\theta\cos\psi\,\dot\psi - \ddot\theta\sin\psi - \dot\theta\cos\psi\,\dot\psi\right)\underline{d}_1 + \left(-\Omega\sin\theta\,\dot\theta\right)\underline{d}_2 + \left(\Omega\cos\theta\cos\psi\,\dot\theta - \Omega\sin\theta\sin\psi\,\dot\psi + \ddot\theta\cos\psi - \dot\theta\sin\psi\,\dot\psi\right)\underline{d}_3$$

$$\underline{\omega}\times\underline{r}$$

$$\cancel{\underline{r}\times\underline{\omega}} = -r\Omega\cos\theta\,\underline{d}_3 +$$

$$r(\Omega\sin\theta\cos\psi + \dot{\theta}\cos\psi)\,\underline{d}_2$$

$$\underline{\omega}\times\underline{\omega}\times\underline{r} = r\Omega\cos\theta(\Omega\sin\theta\sin\psi - \dot{\theta}\sin\psi)\,\underline{d}_2 +$$

$$-r\Omega\cos(\Omega\cos\theta)\,\underline{d}_1 +$$

$$(-\Omega\sin\theta\sin\psi - \dot{\theta}\sin\psi)\,r(\Omega\sin\theta\cos\psi + \dot{\theta}\cos\psi)\,\underline{d}_3 +$$

$$-(\Omega\sin\theta\cos\psi + \dot{\theta}\cos\psi)\,r(\Omega\sin\theta\cos\psi + \dot{\theta}\cos\psi)\,\underline{d}_1$$

$$\underline{\alpha}\times\underline{r} = \Omega\sin\theta\,\dot{\theta}\,r\,\underline{d}_3 +$$

$$-r(\Omega\cos\theta\cos\psi\,\dot{\theta} - \Omega\sin\theta\sin\psi\,\dot{\psi} + \ddot{\theta}\cos\psi - \dot{\theta}\sin\psi\,\dot{\psi})\,\underline{d}_2$$