9/12/2013 - PF DIN2 - Prof. Thiago Ritto

** Considere todos os pedidos no referencial inercial.

1) (2,0) Calcule as componentes I_{zz} e I_{xy} do sistema mostrado na Fig. 1. Quatro barras soldadas de massa M e comprimento L.

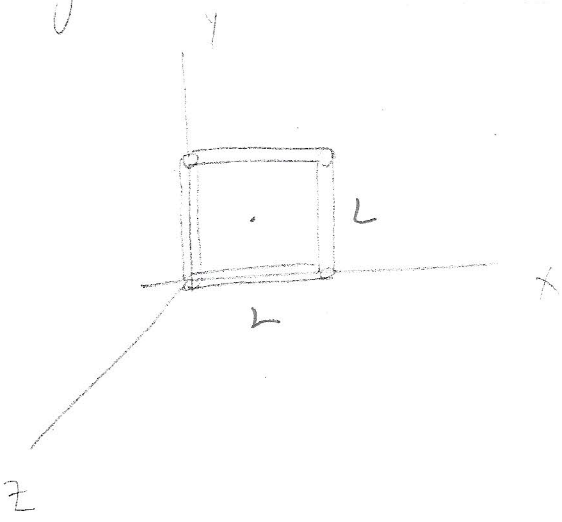
2) (2,0) Considere o sistema mostrado na Fig. 2. Uma barra de comprimento L e massa desprezível está conectada a uma rótula de um lado, e soldada a um disco de diâmetro L e massa M do outro. Suponha que o disco rola sem deslizar. (a) faça o diagrama de corpo livre do sistema (barra + disco), (b) calcule a velocidade do centro de massa do sistema, (c) escreva o vetor velocidade angular do sistema, (d) calcule a energia cinética do sistema, (e) calcule a energia potencial do sistema, (f) obtenha as equações de movimento usando as equações de Lagrange. A base $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ acompanha o movimento da barra, sem fazer o giro do disco.

3) (3,0) Considere o sistema mostrado na Fig. 3. Duas barras soldadas, conectadas a uma terceira barra de massa M e comprimento L por um pino. (a) faça o diagrama de corpo livre da terceira barra, (b) calcule a aceleração do centro de massa da barra, (c) escreva as equações obtidas pela Lei de Newton, (c) calcule a quantidade de movimento angular da barra em relação ao seu cento de massa, (d) escreva as equações obtidas pela Lei de Euler. A base $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ está fixa no referencial inercial, e os outros referenciais estão indicados na figura.

4) (2,0) Um projétil de massa m e velocidade $v_o\mathbf{a}_2$ no instante t_o percorre uma distância h antes de atingir uma barra conforme mostra a Fig. 4. Sendo o impacto perfeitamente plástico, calcule (a) a velocidade do projétil imediatamente antes do impacto, (b) a velocidade do conjunto imediatamente após o impacto, (c) o vetor velocidade angular do conjunto imediatamente após o impacto.

5) (1,0) O satélite esboçado na Fig. 5 tem velocidade spin ω_o e de precessão Ω. Desenhe os cones espacial e do corpo.

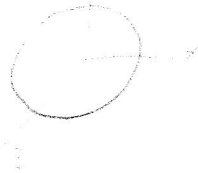
Fig. 1



TABELA

$$I_{yy} = I_{zz} = \frac{1}{12} m l^2$$

$$I_{xx} = I_{xy} = I_{xz} = I_{yz} = 0$$



$$I_{xx} = I_{yy} = \frac{1}{4} m R^2$$

$$I_{zz} = \frac{1}{2} m R^2$$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

Fig 2

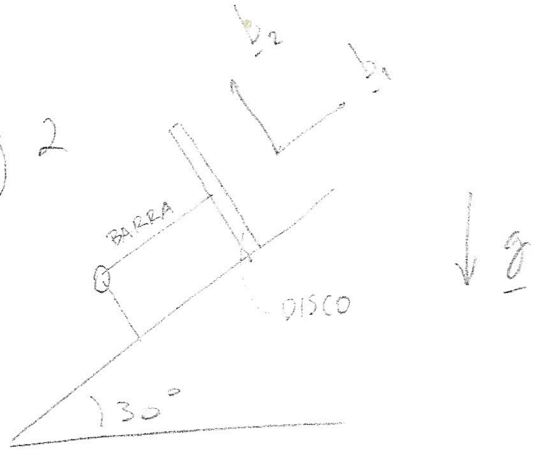
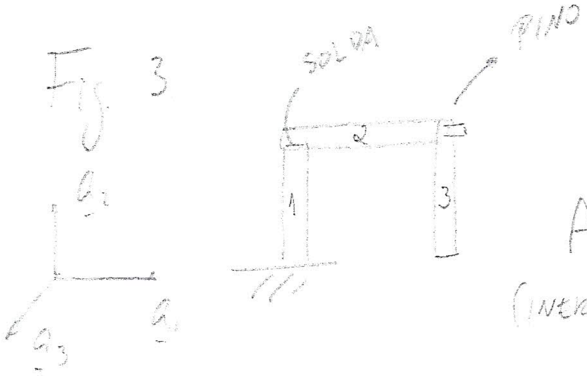


Fig 3



$$A \xrightarrow[(2)]{\alpha} B \xrightarrow[(1)]{\beta} C$$

(INERCIAL) (BARRA 3)

Fig. 4

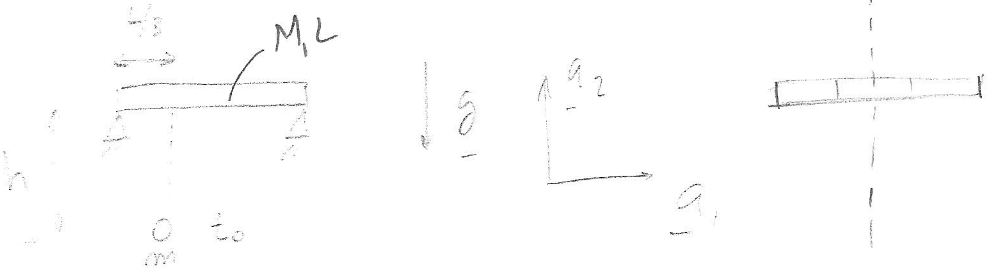
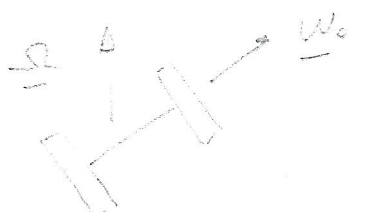
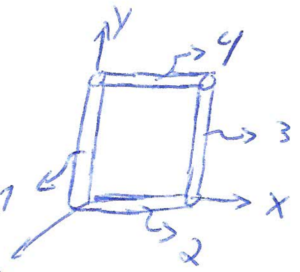


Fig. 5



PF 2013.2



1) $I_{33}^{S} = I_{33}^{1} + I_{33}^{2} + I_{33}^{3} + I_{33}^{4}$

$I_{33}^{1} = \frac{1}{12} m l^2 + m\left(\frac{l^2}{4} + 0\right) = \frac{1}{3} m l^2$

$I_{33}^{2} = \frac{1}{12} m l^2 + m\left(\frac{l^2}{4} + 0\right) = \frac{1}{3} m l^2$

$I_{33}^{3} = \frac{1}{12} m l^2 + m\left(l^2 + \frac{l^2}{4}\right) = \frac{4}{3} m l^2$

$I_{33}^{4} = \frac{1}{12} m l^2 + m\left(\frac{l^2}{4} + l^2\right) = \frac{4}{3} m l^2$

$I_{33}^{S} = \frac{10}{3} m l^2$

$I_{xy}^{S} = I_{xy}^{1} + I_{xy}^{2} + I_{xy}^{3} + I_{xy}^{4}$

$I_{xy}^{1} = 0 - m \times (0) \times \left(\frac{l}{2}\right) = 0$

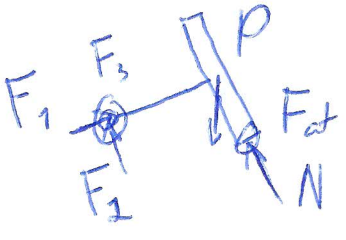
$I_{xy}^{2} = 0 - m \times \left(\frac{l}{2}\right) \times (0) = 0$

$I_{xy}^{3} = 0 - m(l) \times \left(\frac{l}{2}\right) = -m\frac{l^2}{2}$

$I_{xy}^{4} = 0 - m\left(\frac{l}{2}\right)(l) = -m\frac{l^2}{2}$

$I_{xy}^{S} = -m l^2$

2) a)



b) ${}^{R}v^{CM} = L\,\Omega\, b_3$

${}^{R}v^{CM} = \frac{L}{2}\,\omega\, b_3$

$\Omega = \frac{\omega}{2}$

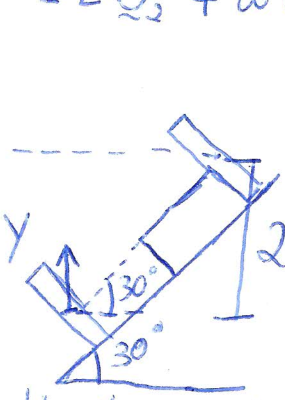
(Considerando velocidade de precessão constante)

c) $\omega = -\Omega b_2 + \omega b_1$

d) $K = \frac{1}{2} m L^2 \Omega^2 + \frac{1}{2} m \frac{L^2}{4} \omega^2$

$= \frac{1}{2} m L^2 \Omega^2 + \frac{1}{2} m L^2 \Omega^2 = m L^2 \Omega^2$

e)



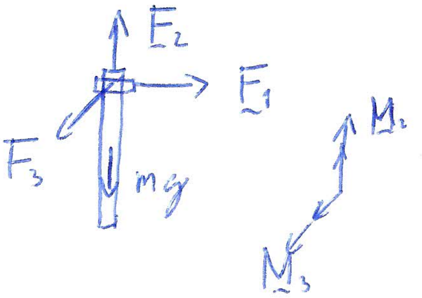
$2L\,\text{sen}(30°) = L$

$y = L\,\text{sen}(\Omega t)$

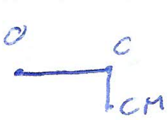
$\phi = m g L\,\text{sen}(\Omega t)$

f) $\mathcal{L} = K - \phi = m L^2 \Omega^2 - m g L\,\text{sen}(\Omega t)$

3) a)



b) $\underset{\sim}{r}^{CM/O} = L\,\underset{\sim}{b}_1 - \frac{L}{2}\underset{\sim}{c}_2$



$$= L\,\underset{\sim}{b}_1 - \left(\frac{L}{2}\cos\beta\,\underset{\sim}{b}_2 - \frac{L}{2}\operatorname{sen}\beta\,\underset{\sim}{b}_1\right)$$

$${}^{R}\underset{\sim}{v}^{CM} = L\dot{\alpha}\,\underset{\sim}{b}_3 + \frac{L}{2}\dot{\beta}\operatorname{sen}\beta\,\underset{\sim}{b}_2 + \frac{L}{2}\dot{\alpha}\operatorname{sen}\beta\,\underset{\sim}{b}_3 + \frac{L}{2}\dot{\beta}\cos\beta\,\underset{\sim}{b}_1$$

$${}^{R}\underset{\sim}{a}^{CM} = L\ddot{\alpha}\,\underset{\sim}{b}_3 - L\dot{\alpha}^2\,\underset{\sim}{b}_1 + \left(\frac{L}{2}\ddot{\beta}\operatorname{sen}\beta + \frac{L}{2}\dot{\beta}^2\cos\beta\right)\underset{\sim}{b}_2 +$$

$$+\left(\frac{L}{2}\ddot{\alpha}\operatorname{sen}\beta + \frac{L}{2}\dot{\alpha}\dot{\beta}\cos\beta\right)\underset{\sim}{b}_3 - \frac{L}{2}\dot{\alpha}^2\operatorname{sen}\beta\,\underset{\sim}{b}_1 + \left(\frac{L}{2}\ddot{\beta}\cos\beta - \frac{L}{2}\dot{\beta}^2\operatorname{sen}\beta\right)\underset{\sim}{b}_1 +$$

$$+\frac{L}{2}\dot{\beta}\dot{\alpha}\cos\beta\,\underset{\sim}{b}_3$$

$${}^{R}\underset{\sim}{a}^{CM} = \begin{bmatrix} \frac{L}{2}\left(\ddot{\beta}\cos\beta - \dot{\beta}^2\operatorname{sen}\beta - \dot{\alpha}^2\operatorname{sen}\beta - 2\dot{\alpha}^2\right) \\ \frac{L}{2}\left(\ddot{\beta}\operatorname{sen}\beta + \dot{\beta}^2\cos\beta\right) \\ \frac{L}{2}\left(2\ddot{\alpha} + \ddot{\alpha}\operatorname{sen}\beta + 2\dot{\alpha}\dot{\beta}\cos\beta\right) \end{bmatrix}_{\text{base } \underset{\sim}{b}}$$

c)

$$\underset{\sim}{F} = m\,{}^{R}\underset{\sim}{a}^{CM} \qquad \underset{\sim}{F} = -mg\,\underset{\sim}{a}_2 + E_2\,\underset{\sim}{c}_2 + E_1\,\underset{\sim}{c}_1 + F_3\,\underset{\sim}{c}_3$$

$$\underset{\sim}{F} = \begin{bmatrix} F_1 \\ F_2\cos\beta - F_3\operatorname{sen}\beta - mg \\ F_3\cos\beta + F_2\operatorname{sen}\beta \end{bmatrix}_{\text{base } \underset{\sim}{b}}$$

$$F_1 = m\frac{L}{2}\left(\ddot{\beta}\cos\beta - \dot{\beta}^2\operatorname{sen}\beta - \dot{\alpha}^2\operatorname{sen}\beta - 2\dot{\alpha}^2\right)$$

$$F_2\cos\beta - F_3\operatorname{sen}\beta - mg = m\frac{L}{2}\left(\ddot{\beta}\operatorname{sen}\beta + \dot{\beta}^2\cos\beta\right)$$

$$F_3\cos\beta + F_2\operatorname{sen}\beta = m\frac{L}{2}\left(2\ddot{\alpha} + \ddot{\alpha}\operatorname{sen}\beta + 2\dot{\alpha}\dot{\beta}\cos\beta\right)$$

d) $\underset{\sim}{M}^{F/CM} = F_3 \frac{L}{2} \underset{\sim}{c}_1 - F_1 \frac{L}{2} \underset{\sim}{c}_3 + M_2 \underset{\sim}{a}_2 + M_3 \underset{\sim}{a}_3$

$$\underset{\sim}{M}^{F/CM} = \begin{bmatrix} F_3 \frac{L}{2} - M_3 \operatorname{sen}\alpha \\ M_2 \cos\beta + M_3 \cos\alpha \operatorname{sen}\beta \\ M_3 \cos\alpha \cos\beta - M_2 \operatorname{sen}\beta - F_1 \frac{L}{2} \end{bmatrix}$$

$$[I^{CM}] = \begin{bmatrix} \frac{1}{12} mL^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{12} mL^2 \end{bmatrix} \qquad \omega = \dot{\alpha} \underset{\sim}{b}_2 + \dot{\beta} \underset{\sim}{c}_1 = \dot{\alpha} \cos\beta \underset{\sim}{c}_2 - \dot{\alpha} \operatorname{sen}\beta \underset{\sim}{c}_3 + \dot{\beta} \underset{\sim}{c}_1$$

$$F_3 \frac{L}{2} - M_3 \operatorname{sen}\alpha = \frac{1}{12} mL^2 \ddot{\beta} + \frac{1}{12} mL^2 (\dot{\alpha} \cos\beta)(-\dot{\alpha} \operatorname{sen}\beta)$$

$$M_2 \cos\beta + M_3 \cos\alpha \operatorname{sen}\beta = 0 \times \frac{d}{dt}(\dot{\alpha} \cos\beta) + \left(\frac{1}{12} mL^2 - \frac{1}{12} mL^2\right) \omega_x \omega_z = 0$$

$$M_3 \cos\alpha \cos\beta - M_2 \operatorname{sen}\beta - F_1 \frac{L}{2} = \frac{1}{12} mL^2 (-\ddot{\alpha} \operatorname{sen}\beta - \dot{\alpha} \dot{\beta} \cos\beta) + - \frac{1}{12} mL^2 (\dot{\beta})(\dot{\alpha} \cos\beta)$$

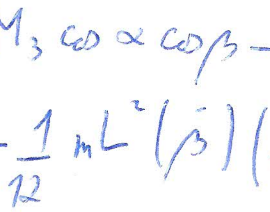
4) a) $\frac{1}{2} \cancel{m} V_0^2 = \frac{1}{2} \cancel{m} V^2 + \cancel{m} gh \Longleftrightarrow$ Conservação de energia.

$$V = \sqrt{V_0^2 - 2gh}$$

b) Conservação da quantidade de movimento linear

$$mV = (m + M) V' \Rightarrow V' = \frac{m}{m+M} \sqrt{V_0^2 - 2gh}$$

$$\underset{\sim}{\omega} = - \frac{d m V}{\left(\frac{1}{12} ML^2 + md\right)} \hat{z}$$

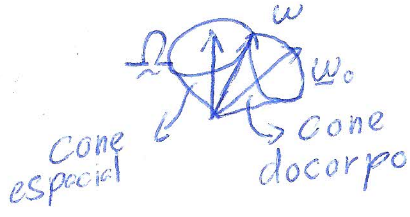
c) 

$$X_{CM} = \frac{m \frac{L}{3} + M \frac{L}{2}}{m + M}$$

Conservação da quantidade de movimento angular

$$H^{S/CM}_{antes} = H^{S/CM}_{depois} \Rightarrow \overbrace{\left(x_{CM} - \frac{L}{3}\right)}^{d} mV = \left(\frac{1}{12} ML^2 + m\left(x_{CM} - \frac{L}{3}\right)^2\right) \omega$$

5)



$\underset{\sim}{\Omega}$
ω
$\underline{\omega}_o$
cone
espacial
cone
docorpo