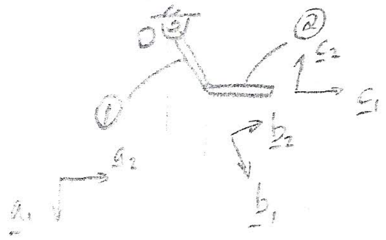
** Considere todos os pedidos no referencial inercial.

1) (2.0) Uma corrente de comprimento L, inicialmente esticada, é solta. Ao mesmo tempo, um suporte atua na parte inferior da corrente mantendo uma aceleração constante $\ddot{y} = a$. Pede-se: (a) a quantidade de movimento linear da corrente e (b) a força R necessária para manter a acelcração constante a (escreva a força em função de g, a e t).

2) (4.5) Considere o problema ilustrado na Fig. 2. Uma barra de massa m e comprimento L está conectada a uma mola de rigidez k na extremidade superior. Esse ponto superior da barra só pode se mover na direção $\mathbf{a}_2$ e não tem momentos resistivos. Considere as velocidades $\Omega\mathbf{a}_2$ e $\omega_o\mathbf{d}_2$ constantes e $\dot{\theta}\mathbf{d}_3$ variável. Pede-se: (a) faça o diagrama de corpo livre da barra e escreva as forças e momento resultantes na base conveniente, (b) escreva as equações relacionadas à lei de Newton, (c) calcule a aceleração angular da barra, (d) escreva a energia cinética da barra, e (e) escreva as equações relacionadas à lei de Euler (ou sua variação). A base $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ está fixa no referencial inercial, a base $\{\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3\}$ está fixa na barra, e os outros referenciais estão indicados na figura.

3) (3.5) Considere o modelo simplificado de uma bicicleta mostrado na Fig. 3, formado por 4 corpos rígidos. Escreva as respostas sem fazer as contas, mas indicando cada termo. Assumindo a hipótese de rolamento sem deslizamento, pede-se: (a) o diagrama de corpo livre do sistema completo e também de cada um dos 4 corpos separadamente, explicitando em que base as forças e momentos estão escritos, (b) a velocidade do centro de massa do corpo 4, (c) escreva as equações relacionadas à lei de Newton para o sistema, (d) escreva as equações relacionadas aos momentos para o sistema. A base $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ está fixa no referencial inercial e os outros referenciais estão indicados na figura. Considere conhecidos os tensores de inércia de cada corpo, em relação ao centro de massa do corpo, na base do corpo.

Ex Calcule $\underline{G}$ da barra 2: $\underline{G}^2 = \int_0^L \underline{v}^P \frac{m}{L} dx$

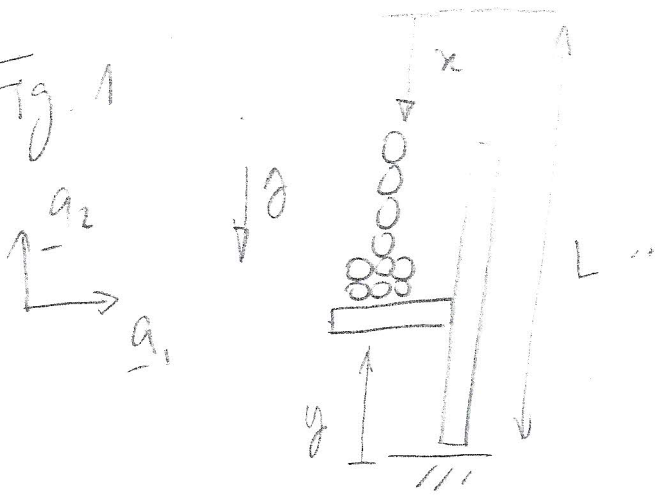


$\underline{v}^P = \frac{d}{dt} \underline{r}^{P/O}$

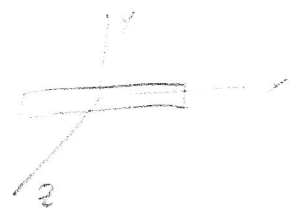
$\underline{r}^{P/O} = L\underline{b}_1 + x\underline{c}_1$

s/ calcular derivadas
s/ fazer transf. de base.
INDICAR TODOS OS TERMOS.

Fig. 1



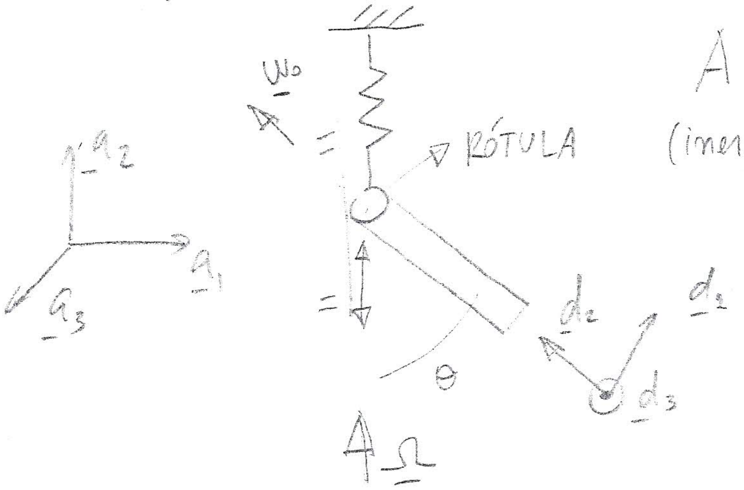
TABELA



$$I_{yy} = I_{zz} = \frac{1}{12} m l^2$$

$$I_{xx} = I_{xy} = I_{xz} = I_{yz} = 0$$

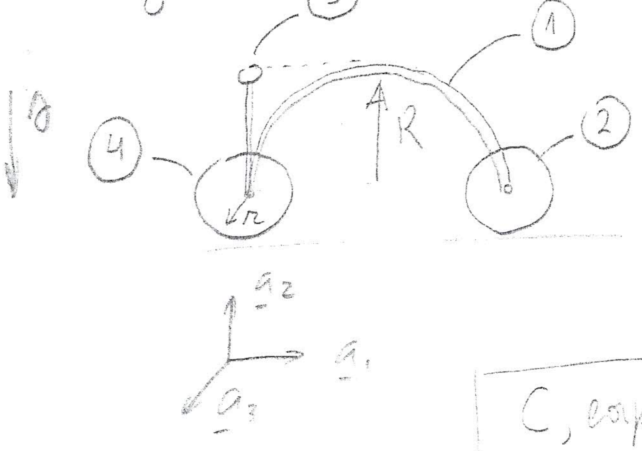
Fig. 2



$$\underset{\text{(inercial)}}{A} \xrightarrow[(2)]{\Omega} B \xrightarrow[(3)]{\dot{\theta}} C \xrightarrow[(2)]{w_0} \underset{\text{(Barra)}}{D}$$

g

Fig. 3



$$\underset{\text{(inercial)}}{A} \xrightarrow[(2)]{\phi} B \xrightarrow[(1)]{\theta} C$$

$C \xrightarrow[(2)]{\alpha} E$, $C \xrightarrow[(3)]{\psi} D$

$E \xrightarrow[(3)]{\beta} F$

C, corpo 1	E, corpo 3
D, corpo 2	F, corpo 4

P2 - 2013.2

1) a) $\underset{\sim}{G} = \left(-\rho(L-x-y)\dot{x} + \rho(x+y)\dot{y}\right)\underset{\sim}{a}_2$

$\ddot{x} = g \qquad \ddot{y} = a$

$\dot{x} = gt \qquad \dot{y} = at$

$x = \frac{gt^2}{2} \qquad y = \frac{at^2}{2}$

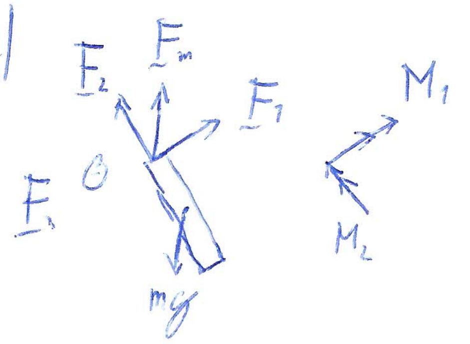
b)

$$R - \rho L g = \dot{G} = \rho(-L\ddot{x} + \dot{x}^2 + x\ddot{x} + \dot{y}\dot{x} + y\ddot{x}) +$$

$$+\rho(x\ddot{y} + \dot{x}\dot{y} + \dot{y}^2 + y\ddot{y}) = \rho(-Lg + g^2t^2 + g^2t^2/2 + agt^2 + agt^2/2 + agt^2/2 +$$

$$+ agt^2 + a^2t^2 + a^2t^2/2) \Rightarrow R = \frac{3}{2}\rho t^2(a+g)^2$$

2) a)



$\underset{\sim}{F} = -mg\,\underset{\sim}{b}_2 + F_1\underset{\sim}{c}_1 + F_2\underset{\sim}{c}_2 + F_3\underset{\sim}{c}_3 + F_m\underset{\sim}{b}_2$

$\underset{\sim}{M}^O = -mg\,\mathrm{sen}\,\theta\,\underset{\sim}{c}_3 + M_1\underset{\sim}{c}_1 + M_3\underset{\sim}{c}_3$

b) $\underset{\sim}{F} = m\,{}^R\underset{\sim}{a}^C \qquad {}^R\underset{\sim}{a}^C = {}^R\underset{\sim}{a}^O + {}^A\underset{\sim}{\Omega}^C \times ({}^A\underset{\sim}{\Omega}^C \times r^{C/O}) + {}^A\underset{\sim}{\alpha}^C \times r^{C/O}$

${}^R\underset{\sim}{a}^O = \ddot{y}\,\underset{\sim}{b}_2 \qquad {}^A\underset{\sim}{\omega}^D = \Omega\,\underset{\sim}{b}_2 + \omega_0\underset{\sim}{c}_2 + \dot{\theta}\underset{\sim}{c}_3 \qquad {}^A\underset{\sim}{\Omega}^C = \Omega\,\mathrm{sen}\,\theta\,\underset{\sim}{c}_1 + \Omega\cos\theta\,\underset{\sim}{c}_2 + \dot{\theta}\underset{\sim}{c}_3$

$\underset{\sim}{r}^{C/O} = -\frac{L}{2}\underset{\sim}{c}_2 \qquad {}^A\underset{\sim}{\alpha}^C = \frac{{}^R d\Omega}{dt} = \frac{{}^C d\Omega}{dt} + \cancel{\Omega \times \Omega}^{\,0} = \Omega\dot{\theta}\cos\theta\,\underset{\sim}{c}_1 - \Omega\dot{\theta}\,\mathrm{sen}\,\theta\,\underset{\sim}{c}_2 + \ddot{\theta}\underset{\sim}{c}_3$

$$F_1 - (Ky + mg)\,\mathrm{sen}\,\theta = m\left(\ddot{y}\,\mathrm{sen}\,\theta - \frac{L}{2}\Omega^2\,\mathrm{sen}\,\theta\cos\theta + \ddot{\theta}\frac{L}{2}\right)$$

$$F_2 - (Ky + mg)\cos\theta = m\left(\ddot{y}\cos\theta + \frac{L}{2}\left(\Omega^2\,\mathrm{sen}^2\theta + \dot{\theta}^2\right)\right)$$

$$F_3 = m\left(-L\dot{\theta}\Omega\cos\theta - \frac{L}{2}\Omega\dot{\theta}\cos\theta\right) = -3m\frac{L}{2}\dot{\theta}\cos\theta\,\Omega$$

c) ${}^{A}\underset{\sim}{\alpha}^{D} = \frac{{}^{A}d}{dt}(\underset{\sim}{\omega}) = \frac{{}^{C}d}{dt}(\underset{\sim}{\omega}) + \underset{\sim}{\Omega} \times \underset{\sim}{\omega}$

$= (-\omega_0 \dot{\theta} + \Omega \dot{\theta} \cos\theta)\underset{\sim}{c}_1 + (-\Omega \dot{\theta} \operatorname{sen}\theta)\underset{\sim}{c}_2 + (\ddot{\theta} + \omega_0 \Omega \operatorname{sen}\theta)\underset{\sim}{c}_3$

d) $K = \frac{1}{2} m ({}^{R}\underset{\sim}{V}^{C})^T \cdot ({}^{R}\underset{\sim}{V}^{C}) + \frac{1}{2} \underset{\sim}{\omega}^T [I^C] \underset{\sim}{\omega}$

${}^{R}\underset{\sim}{V}^{C} = \dot{y}\, \underset{\sim}{b}_2 + \left(\frac{L}{2} \dot{\theta}\, \underset{\sim}{c}_1 - \frac{L}{2} \Omega \operatorname{sen}\theta\, \underset{\sim}{c}_3\right)$

$[I^C] = \begin{bmatrix} \frac{1}{12} m L^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{12} m L^2 \end{bmatrix}$ $K = \left[\left(\dot{y} \operatorname{sen}\theta + \frac{L}{2}\dot{\theta}\right)^2 + \dot{y}^2 \cos^2\theta + \frac{L^2}{4} \Omega^2 \operatorname{sen}^2\theta\right]\frac{m}{2} +$

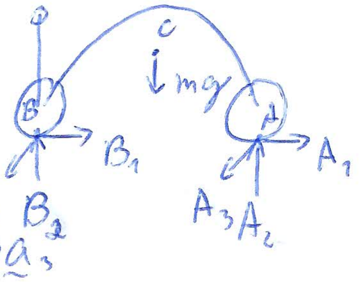
$+ \frac{1}{2} \Omega^2 \operatorname{sen}^2\theta \frac{1}{12} m L^2 + \frac{1}{2} \dot{\theta}^2 \frac{1}{12} m L^2$

e) $M_1 + F_3 \frac{L}{2} = \frac{1}{6} m L^2 (\Omega \dot{\theta} \cos\theta$

$M_2 = 0$

$(K y - F_1)\frac{L}{2} = \frac{1}{12} m L^2 \ddot{\theta} - \frac{1}{12} m L^2 \Omega^2 \operatorname{sen}\theta \cos\theta$

3) a) sistema



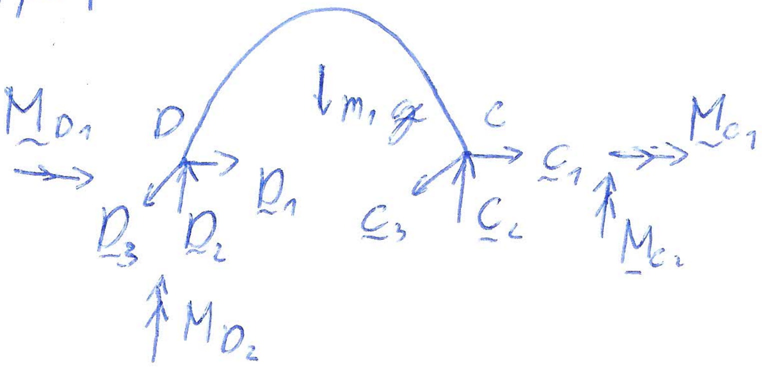
$\underset{\sim}{F} = (A_1 + B_1)\underset{\sim}{a}_1 +$
$+ (A_2 + B_2 - mg)\underset{\sim}{a}_2 +$
$+ (A_3 + B_3)\underset{\sim}{a}_3$

$r^{B/C} = x_B \underset{\sim}{a}_1 + y_B \underset{\sim}{a}_2 + z_B \underset{\sim}{a}_3$

$r^{A/C} = x_A \underset{\sim}{a}_1 + y_A \underset{\sim}{a}_2 + z_A \underset{\sim}{a}_3$

$M^C = \underset{\sim}{r}^{B/C} \times (\underset{\sim}{B}_1 + \underset{\sim}{B}_2 + \underset{\sim}{B}_3) +$
$+ r^{A/C} \times (A_1 + A_2 + A_3)$

Corpo 1

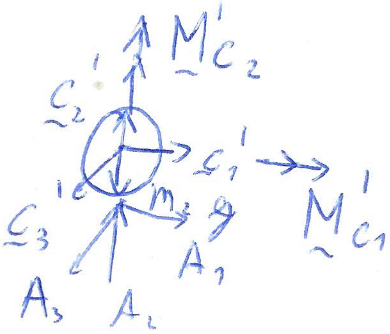


$$\underline{F} = (C_1 + D_1)\underline{a}_1 + (C_2 + D_2 - mg)\underline{a}_2 + (C_3 + D_3)\underline{a}_3$$

$$\underline{M}^{CM} = \underline{r}^{C/CM} \times \underline{F}_C + \underline{r}^{D/CM} \times \underline{F}_D + \underline{M}_{C1} + \underline{M}_{C2} + \underline{M}_{D1} + \underline{M}_{D2}$$

$$r^{D/CM} = x_D \underline{a}_1 + y_D \underline{a}_2 + z_D \underline{a}_3 \; ; \; r^{C/CM} = x_C \underline{a}_1 + \cancel{y_C} \underline{a}_2 + z_C \underline{a}_3$$

Corpo 2

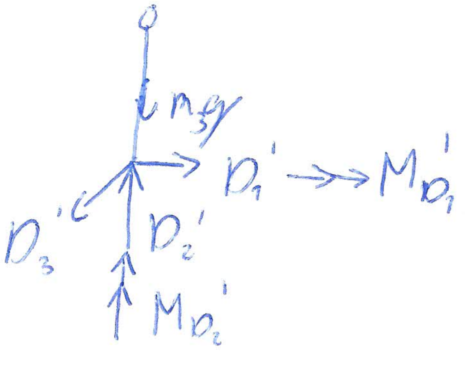


$$\underline{F} = (A_1 + C_1')\underline{a}_1 + (A_2 + C_2' - m_2 g)\underline{a}_2 + (A_3 + C_3')\underline{a}_3$$

$$\underline{M}^C = \underline{r}^{A/C} \times \underline{F}_A + \underline{M}'_{C2} + \underline{M}'_{C1}$$

$$r^{A/C} = -r\,\underline{c}_2$$

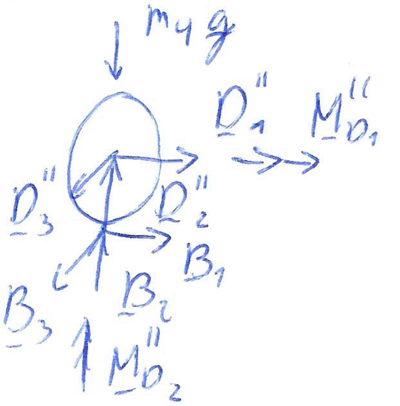
Corpo 3



$$\underline{F} = D_1' \underline{a}_1 + (D_2' - m_3 g)\underline{a}_2 + D_3' a_3$$

$$\underline{M}^{CM} = -\frac{R}{2}\underline{e}_2 \times \underline{F}_D + \underline{M}'_{D1} + \underline{M}_{D2}$$

Corpo 4



$$\underline{F} = D_1'' \underline{a}_1 + (D_2'' + B_2 - m_4 g)\underline{a}_2 + (D_3'' + B_3)\underline{a}_3 + B_1 \underline{a}_1$$

$$\underline{M}^{CM} = -\frac{R}{2} f_2 \times \underline{F}_D + M''_{D1} + M''_{D2}$$

b) $${}^{R}v^{4} = {}^{A}\omega^{F} \times (r\, f_2)$$

$${}^{A}\omega^{F} = \dot{\phi}\, b_2 + \dot{\theta}\, c_1 + \dot{\beta}\, e_3 + \dot{\alpha}\, e_2$$

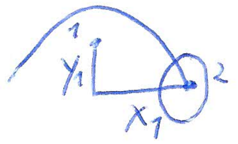
c)

$$F = (A_1 + B_1)\, a_1 + (A_2 + B_2 - mg)\, a_2 + (A_3 + B_3)\, a_3$$

$$F = \dot{G}^{sist} = \dot{G}_1 + \dot{G}_2 + \dot{G}_3 + \dot{G}_4$$

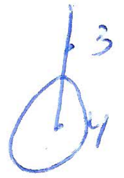
$$G_{sist} = m\,{}^{R}V^{1} + m\,{}^{R}V^{2} + m\,{}^{R}V^{3} + m\,{}^{R}V^{4} \qquad \dot{G} = m\,{}^{R}a^{1} + m\,{}^{R}a^{2} + m\,{}^{R}a^{3} + m\,{}^{R}a^{4}$$

$$a = \frac{{}^{R}d}{dt} V \qquad V^{1} = V^{2} + {}^{A}\omega^{C} \times (x_1\, c_1 + y_1\, c_2)$$



$$V^{2} = V^{4} = {}^{A}\omega^{D} \times (r\, d_2)$$

$$V^{3} = {}^{A}\omega^{C} \times (y_3\, c_2) + V^{4}$$



d)

$$M^{c} = \dot{H}_1^{c} + \dot{H}_2^{c} + \dot{H}_3^{c} + \dot{H}_4^{c}$$

$$H_1^{c} = H^{1} + r^{1/c} \times m V^{1} \quad , \quad H^{1} = [I^{1}]\, {}^{A}\omega^{C}$$

$$H_2^{c} = H^{2} + r^{2/c} \times m V^{2} \quad , \quad H^{2} = [I^{2}]\, {}^{A}\omega^{D}$$

$$H_3^{c} = H^{3} + r^{3/c} \times m V^{3} \quad , \quad H^{3} = [I^{3}]\, {}^{A}\omega^{E}$$

$$H_4^{c} = H^{4} + r^{4/c} \times m V^{4} \quad , \quad H^{4} = [I^{4}]\, {}^{A}\omega^{F}$$