22/09/2014 - P1 DIN2 - Prof. Thiago Ritto

1) (2,0) Obtenha o momento de inércia I_{xx} e o produto de inércia I_{xy} em relação ao ponto O da estrutura mostrada na Fig. 1: duas placas quadradas (m_p, L) e uma esfera (m_e, R), soldadas. Calcule também o momento de inércia em relação ao eixo que passa por $\overline{OQ}$.

2) (4,0) Considere o sistema mostrado na Fig. 2. Um pêndulo duplo formado por duas barras (m,L) gira $\Omega \mathbf{a}_1$, com $\Omega = cte$. Os pinos 1 e 2 dão liberdade para as barras girarem em torno da direção de $\mathbf{b}_3$, e $\dot{\theta}_1 = cte$. Pede-se (a) a energia potencial do sistema, (b) a energia cinética do sistema, (c) a aceleração do centro de massa da barra D, (d) a quantidade de movimento angular da barra D em relação ao pino 2, (e) faça o diagrama de corpo livre da barra C e escreva as forças e momentos que nela atuam. A base $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ está fixa no referencial inercial, a base $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ gira $\Omega \mathbf{a}_1$, a base $\{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$ está solidária à barra C e a base $\{\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3\}$ está solidária à barra D.

3) (2,0) Considere o tambor (m, R, I_{zz}) mostrado na Fig. 3. Ele está subindo a rampa graças à força $\mathbf{F}$ (constante) aplicada no fio inextensível. Se o tambor rola deslizando no plano (coeficiente de atrito dinâmico μ_D), (a) faça o diagrama de corpo livre do tambor, (b) escreva as equações de movimento, e (c) indique como calcular o trabalho realizado pelas forças que atuam no tambor.

4) (2,0) Considere a esfera (m, R) mostrada na Fig. 4. Inicialmente ela está com velocidade v_o no sentido positivo de $\mathbf{a}_1$ e ω_o no sentido positivo de $\mathbf{a}_3$. Se no momento do contato da esfera com o solo uma força impulsiva conhecida F (em Δt) aparece na direção negativa de $\mathbf{a}_1$ e uma força $F/10$ aparece na direção negativa de $\mathbf{a}_2$, calcule a velocidade do centro de massa da esfera $\mathbf{v}$ e o vetor velocidade angular da esfera $\boldsymbol{\omega}$ logo após o contato.

FIG. 1

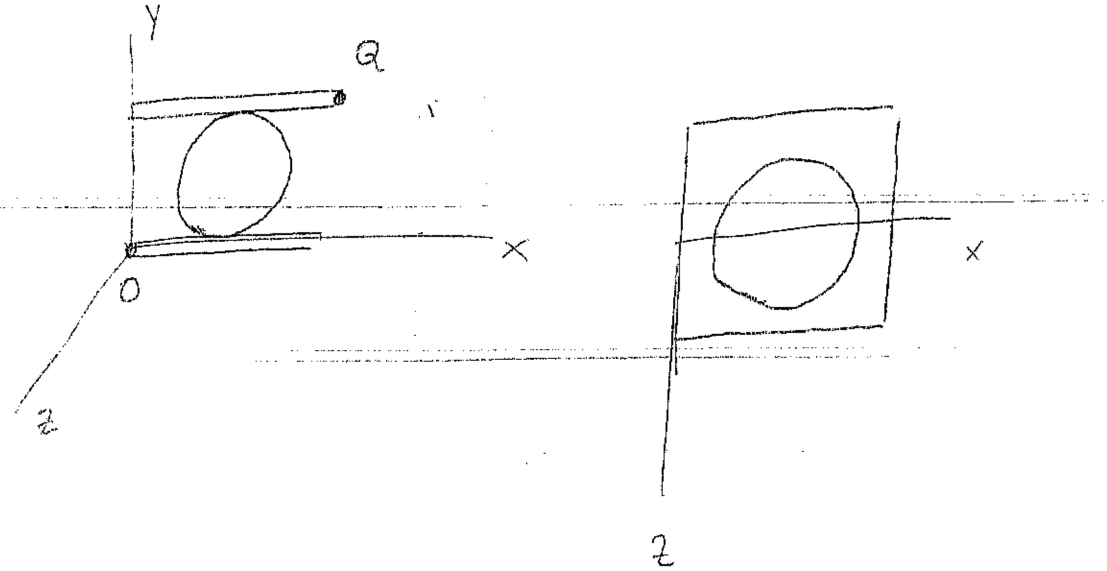


FIG. 2

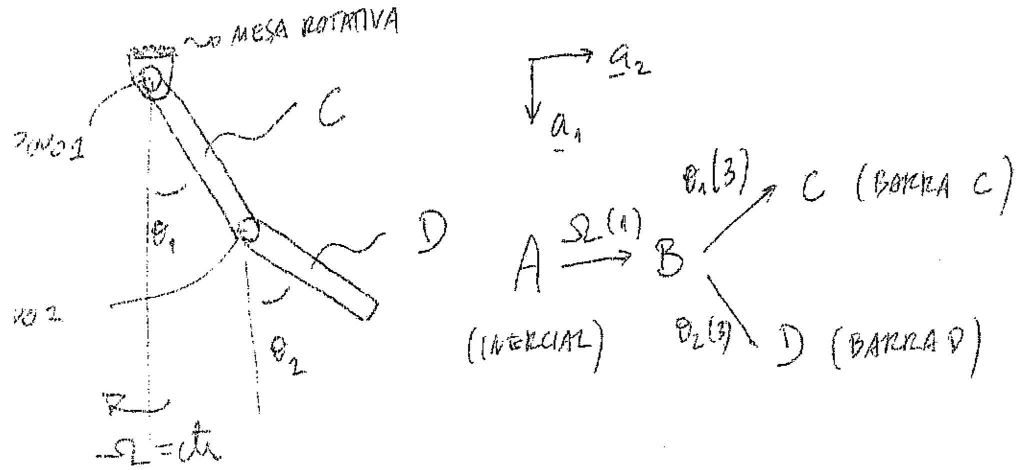


FIG. 3

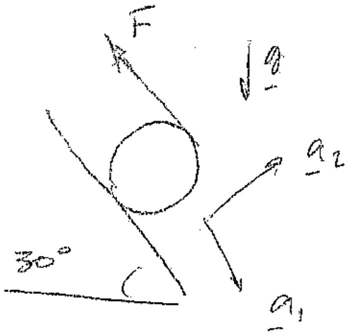
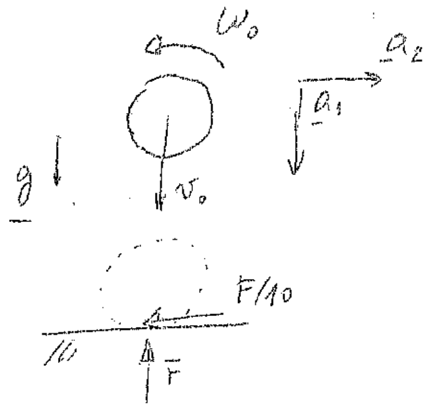
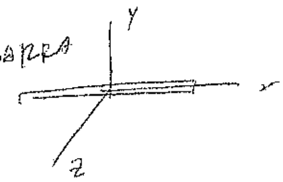


FIG. 4

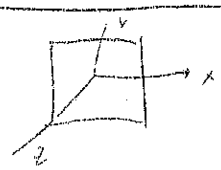


TABELA

BARRA

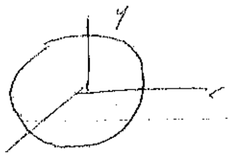


$$I_{xx} = I_{xy} = I_{xz} = I_{yz} = 0$$
$$I_{yy} = I_{zz} = \frac{1}{12} m l^2$$



$$I_{xy} = I_{xz} = I_{yz} = 0$$
$$I_{zz} = \frac{1}{6} m l^2$$

ESFERA

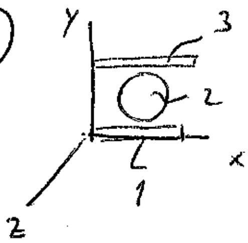


$$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} m r^2$$
$$I_{xy} = I_{xz} = I_{yz} = 0$$

$$I_{..} = I_{xx} = \frac{1}{3} m l^2$$

Gabarito P1 DIN2 22/09/14

(1)



$$I_{xx} = \underbrace{\frac{1}{12} m_p l^2}_{1} + \underbrace{\frac{1}{12} m_p l^2 + m_p (2R)^2}_{2} + \underbrace{\frac{2}{5} m_e R^2 + m_e R^2}_{3}$$

$$I_{xy} = 0 - m_p \frac{L}{\cancel{2}} R \cancel{2} - m_e \frac{L}{2} R$$

$\overline{OQ} = L \underline{b}_1 + 2R \underline{b}_2$ $\quad \|\overline{OP}\| = \sqrt{L^2 + (2R)^2} \longrightarrow \underline{u} = \frac{L}{\sqrt{L^2+(2R)^2}} \underline{b}_1 + \frac{2R}{\sqrt{L^2+(2R)^2}} \underline{b}_2$

$$I_{uu} = I_{xx} u_x^2 + I_{yy} u_y^2 - 2 I_{xy} u_x u_y$$

Faltou $I_{yy} = 2\left(\frac{1}{6} m_p L^2 + m_p \left(\frac{L}{2}\right)^2\right) + \frac{2}{5} m_e R^2 + m_e \left(\frac{L}{2}\right)^2$

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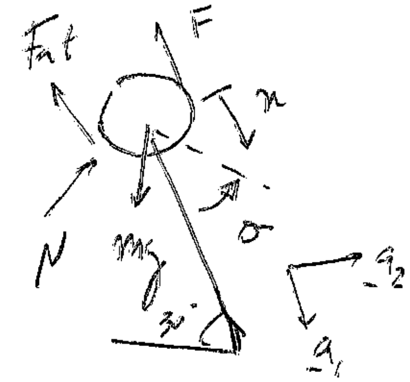
(3)



$$\underline{F} = (-F - F_{at} + mg\sin\theta)\,\underline{a}_1 + (N - mg\cos\theta)\,\underline{a}_2$$

Eqs. de Mov

$$\begin{cases} -F - F_{at} + mg\sin\theta = m\ddot{x} \quad (1) \\ N - mg\cos\theta = 0 \\ I_{zz}\ddot{\theta} = (F - F_{at})\, r \\ F_{at} = \mu_d N \end{cases}$$

INCÓGNITAS $\{x, \theta, N, F_{at}\}$

$W_{peso} = \int_{x_1}^{x_2} mg \sin 30° \, dx = \frac{mg}{2} \Delta x$; $W_F = \int_{x_1}^{x_2} -F\,dx + \int_{\theta_1}^{\theta_2} F r\, d\theta = -F\Delta x + F r \Delta\theta$

$$W_{FAT} = \int_{x_1}^{x_2} -F_{at}\, dx + \int_{\theta_1}^{\theta_2} -F_{at} r\, d\theta = -\mu_d N \Delta x - \mu_d N r \Delta\theta$$

$$W_N = 0$$

(4) $\underline{I} = -F\Delta t\,\underline{a}_1 - \frac{F\Delta t}{10}\,\underline{a}_2 \qquad \underline{I}^{ANG} = -\frac{F\Delta t\,r}{10}\,\underline{a}_3$

$\underline{G}_1 = m v_0\,\underline{a}_1 \;,\; \underline{G}_2 = m\,\underline{v}_2 \;,\; \underline{H}_1 = \frac{2}{5} m k^2 \omega_0\,\underline{a}_3 \;,\; \underline{H}_2 = \frac{2}{5} m r^2\,\underline{\omega}$

Logo $\underline{v}_2 = \left(v_0 - \frac{F\Delta t}{m}\right)\underline{a}_1 - \left(\frac{F\Delta t}{10m}\right)\underline{a}_2$ e $\underline{\omega} = \left(\omega_0 - \frac{F\Delta t}{10}\,\frac{5}{2}\,\frac{1}{mr}\right)\underline{a}_3$

$= \left(\omega_0 - \frac{F\Delta t}{4mr}\right)\underline{a}_3$

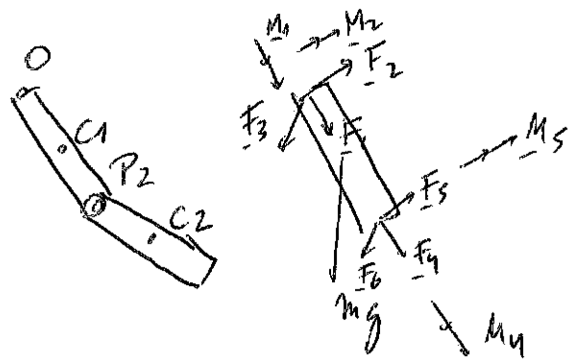
(2) a) $\phi = -\left(mg\frac{L}{2}\cos\theta_1 + mgL\cos\theta_1 + mg\frac{L}{2}\cos\theta_2\right)$

b) $K = \frac{1}{2}\,{}^A\underline{\omega}^C \cdot \underline{\underline{I}}^{C/O} \cdot {}^A\underline{\omega}^C + \frac{1}{2} m\,\underline{v}^{C2} \cdot \underline{v}^{C2} + \frac{1}{2}\,{}^A\underline{\omega}^D \cdot \underline{\underline{I}}^{D/C2} \cdot {}^A\underline{\omega}^D$

c) $\underline{a}^{C2} = \underline{a}^{P2} + {}^A\underline{\omega}^D \times {}^A\underline{\omega}^D \times \underline{r}^{C2/P2} + {}^A\underline{\alpha}^D \times \underline{r}^{C2/P2}$

d) $\underline{H}^{D/O} = \underline{H}^{D/C2} + \underline{r}^{C2/O} \times m\,\underline{v}^{C2}$

e)



$\underline{F} = (F_2 + F_5)\,\underline{c}_2 + (F_1 + F_4)\,\underline{c}_1 + (F_3 + F_6)\,\underline{c}_3 + mg\,\underline{b}_1$

$\underline{M}^O = (M_2 + M_5)\,\underline{c}_2 + (M_1 + M_4)\,\underline{c}_1 + \left(mg\; mg_1\frac{L}{2} + F_5\ell\right)\underline{c}_3 + -F_6\ell\,\underline{c}_2$

Calculando as componentes...

${}^A\underline{\omega}^C = \Omega\,\underline{b}_1 + \dot{\theta}_1\,\underline{b}_3 \;;\; {}^A\underline{\omega}^D = \Omega\,\underline{b}_1 + \dot{\theta}_2\,\underline{b}_3$

$\underline{\underline{I}}^{C/O} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{3}m\ell^2 & 0 \\ 0 & 0 & \frac{1}{3}m\ell^2 \end{bmatrix} \;;\; \underline{\underline{I}}^{D/C2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12}m\ell^2 & 0 \\ 0 & 0 & \frac{1}{12}m\ell^2 \end{bmatrix} \;;\; \underline{\underline{I}}^{D/P2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{3}m\ell^2 & 0 \\ 0 & 0 & \frac{1}{3}m\ell^2 \end{bmatrix}$

(em C) em (D)

$$\underline{v}^{C2} = \underline{v}^{P2} + {}^{A}\underline{\omega}^{P} \times \underline{r}^{C2/P2}$$

$$\underline{v}^{P2} = {}^{A}\underline{\omega}^{C} \times \underline{r}^{P2/O} \quad ; \quad \underline{r}^{P2/O} = l\,\underline{c}_1$$

$$\underline{a}^{P2} = {}^{A}\underline{\omega}^{C} \times {}^{A}\underline{\omega}^{C} \times \underline{r}^{P2/O}$$

$$\underline{r}^{C2/P2} = \frac{l}{2}\,\underline{d}_1$$

$$\underline{H}^{D/C2} = \left[I^{D/C2} \right] {}^{A}\underline{\omega}^{P}$$

Fazendo as contas...

$${}_{B}T_{C} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad {}_{B}T_{D} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) $${}^{A}\underline{\omega}^{C} = \Omega\cos\theta_1\,\underline{c}_1 - \Omega\sin\theta_1\,\underline{c}_2 + \dot{\theta}_1\,\underline{c}_3$$

Logo $$\frac{1}{2}\,{}^{A}\underline{\omega}^{C} \cdot I^{C/O}\,{}^{A}\underline{\omega}^{C} = \frac{1}{3} m l^2 \Omega^2 \sin^2\theta_1 + \frac{1}{3} m l^2 \dot{\theta}^2$$

$${}_{D}{}^{A}\underline{\omega}^{P} = \begin{pmatrix} \Omega\cos\theta_2 \\ -\Omega\sin\theta_2 \\ \dot{\theta}_2 \end{pmatrix}$$. Logo $$\frac{1}{2}\,{}^{A}\underline{\omega}^{P}\, I^{D/C2}\,{}^{A}\underline{\omega}^{P} = \frac{1}{12} m l^2 \Omega^2 \sin^2\theta_2 + \frac{1}{12} m l^2 \dot{\theta}_2^2$$

$$\underline{v}^{C2} = \underline{v}^{P2} + {}^{A}\underline{\omega}^{P} \times \underline{r}^{C2/P2}$$

$${}_{B}\underline{v}^{C2} = {}_{B}T_{C} \begin{pmatrix} \Omega\cos\theta_1 \\ -\Omega\sin\theta_1 \\ \dot{\theta}_1 \end{pmatrix} \times \begin{pmatrix} l \\ 0 \\ 0 \end{pmatrix} + {}_{B}T_{D} \begin{pmatrix} \Omega\cos\theta_2 \\ -\Omega\sin\theta_2 \\ \dot{\theta}_2 \end{pmatrix} \times \begin{pmatrix} \frac{l}{2} \\ 0 \\ 0 \end{pmatrix}$$

$$ {}_B\underline{v}^{C2} = \begin{pmatrix} -\dot\theta_1\, l \sin\theta_1 - \frac{l}{2}\,\dot\theta_2 \sin\theta_2 \\ \dot\theta_1\, l\cos\theta_1 + \frac{l}{2}\,\dot\theta_2 \sin\theta_2 \\ l\,\Omega\sin\theta_1 + \frac{l}{2}\,\Omega\sin\theta_2 \end{pmatrix} $$

Logo $\frac{1}{2} m\, \underline{v}^{C2}\cdot\underline{v}^{C2} = \frac{1}{2} m\Big[\Big(\dot\theta_1 l\sin\theta_1 - \frac{l}{2}\dot\theta_2\sin\theta_2\Big)^2 +$

$\Big(\dot\theta_1 l\cos\theta_1 + \frac{l}{2}\dot\theta_2\sin\theta_2\Big)^2 +$

$\Big(l\,\Omega\sin\theta_1 + \frac{l}{2}\Omega\sin\theta_2\Big)^2\Big]$

c) $\underline{a}^{C2} = \underline{a}^{P2} + {}^A\underline{\omega}^D \times {}^A\underline{\omega}^D \times \underline{r}^{C2/P2} + {}^A\underline{\alpha}^D \times \underline{r}^{C2/P2}$

$\underline{a}^{P2} = {}^A\underline{\omega}^C \times {}^A\underline{\omega}^C \times l\,\underline{c}_1$

$$ {}_C\underline{a}^{P2} = \begin{pmatrix} -\Omega^2\sin\theta_1\, l - \dot\theta_1^2\, l \\ -l\,\Omega^2\sin\theta_1\cos\theta_1 \\ \Omega\cos\theta_1\,\dot\theta_1\, l \end{pmatrix} $$

$$ {}^A\underline{\omega}^D \times {}^A\underline{\omega}^D \times \frac{l}{2}\underline{d}_1 = \begin{pmatrix} -\Omega^2\sin\theta_2\frac{l}{2} - \dot\theta_2\frac{l}{2} \\ -\frac{l}{2}\Omega^2\sin\theta_2\cos\theta_2 \\ \Omega\cos\theta_2\,\dot\theta_2\,\frac{l}{2} \end{pmatrix}_{BASE\{\underline{d}_1,\underline{d}_2,\underline{d}_3\}} $$

$$ {}^A\underline{\alpha}^D \times \frac{l}{2}\underline{d}_1 = \begin{pmatrix} 0 \\ -\frac{l}{2}\ddot\theta_2 \\ -\Omega\frac{l}{2}\cos\theta_2\,\dot\theta_2 \end{pmatrix}_{BASE\{\underline{d}_1,\underline{d}_2,\underline{d}_3\}} $$

d) $\underline{H}^{P/O} = \underline{H}^{P/C2} + \underline{r}^{C2/O} \times m\,\underline{v}^{C2}$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12}ml^2 & 0 \\ 0 & 0 & \frac{1}{12}ml^2 \end{bmatrix} \begin{pmatrix} \Omega\cos\theta_2 \\ -\Omega\sin\theta_2 \\ \dot{\theta}_2 \end{pmatrix} + \underbrace{\left(l\,\underline{c}_1 + \frac{l}{2}\underline{d}_1 \right)}_{} \times m\,\underline{v}^{C2}$$

$$\left(l\cos\theta_1 + \frac{l}{2}\cos\theta_2 \right)\underline{b}_1 + \left(l\sin\theta_1 + \frac{l}{2}\sin\theta_2 \right)\underline{b}_2 \times m\,\underline{v}^{C2}$$

Logo $\underline{H}^{P/O}$ na base $\{\underline{b}_1, \underline{b}_2, \underline{b}_3\}$

$$= \begin{pmatrix} \left(l\sin\theta_1 + \frac{l}{2}\sin\theta_2 \right)\left(l\Omega\sin\theta_1 + \frac{l}{2}\Omega\sin\theta_2 \right) \\ \left(l\cos\theta_1 + \frac{l}{2}\cos\theta_2 \right)\left(l\Omega\sin\theta_1 + \frac{l}{2}\Omega\sin\theta_2 \right) \\ \left(l\cos\theta_1 + \frac{l}{2}\cos\theta_2 \right)\left(\dot{\theta}_1 l\cos\theta_1 + \frac{l}{2}\dot{\theta}_2 l\cos\theta_2 \right) + \\ \left(l\sin\theta_1 + \frac{l}{2}\sin\theta_2 \right)\left(\dot{\theta}_1 l\sin\theta_1 + \frac{l}{2}\dot{\theta}_2 l\sin\theta_2 \right) \end{pmatrix}$$