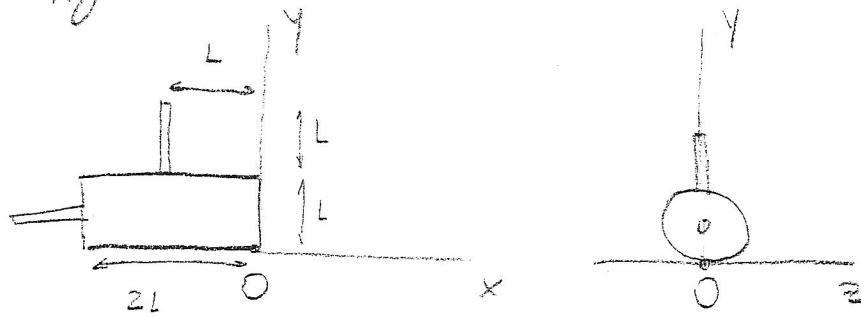


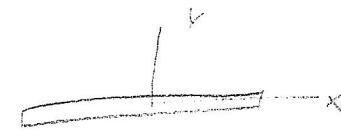
- 1) (2,0) Obtenha o momento de inércia I_{xx} e o produto de inércia I_{xy} em relação ao ponto O da estrutura mostrada na Fig. 1: um cilindro de comprimento $2L$, diâmetro L e massa M com duas barras de comprimento L e massa m soldadas nos locais mostrados na figura.
- 2) (3,0) Considere o sistema mostrado na Fig. 2. Uma barra (massa m e comprimento $2L$) pinada no ponto A numa mesa de diâmetro L que gira com velocidade angular constante $\omega = \omega a_2$. Calcule: (a) a quantidade de movimento angular da barra em relação ao ponto A, (b) a quantidade de movimento angular da barra em relação ao ponto O, e (c) a energia cinética da barra.
- 3) (2,5) Uma partícula de massa m é suspensa por uma mola de rigidez k e comprimento l (em equilíbrio); Fig. 4. As coordenadas generalizadas são o ângulo θ e o deslocamento x a partir do ponto de equilíbrio da mola. Pede-se (a) calcule as energias cinética e potencial do sistema e (b) use as Equações de Lagrange para obter as equações de movimento.
- 4) (2,5) Considere o semi-anel mostrado na Fig. 5. A base $\{\mathbf{b}_1, \mathbf{b}_2\}$ gira junto com o anel. Um torque constante $T\mathbf{b}_3$ é aplicado, e a estrutura gira em torno do ponto O (fixo no referencial inercial). Dada a geometria mostrada na figura e o momento de inércia em relação ao ponto O (I^O), pede-se: (a) o diagrama de corpo livre, (b) a aceleração do centro de massa e (c) as equações de movimento.

Fig. 1



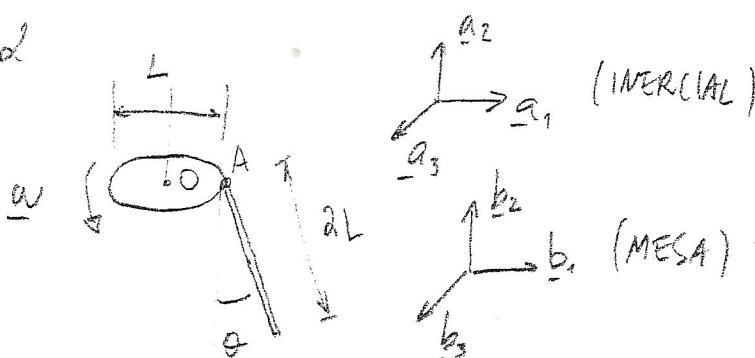
TABELA

BARRA



$$I_{yy} = I_{zz} = \frac{1}{12} m l^2$$

Fig. 2



CILINDRO



$$I_{xx} = I_{yy} = \frac{1}{12} m (3r^2 + h^2)$$

$$I_{zz} = \frac{1}{2} m r^2$$

A $\xrightarrow{\omega(2)}$ B $\xrightarrow{\dot{\theta}(3)}$ C

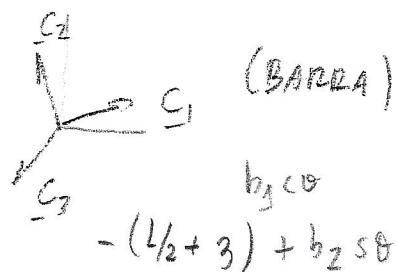
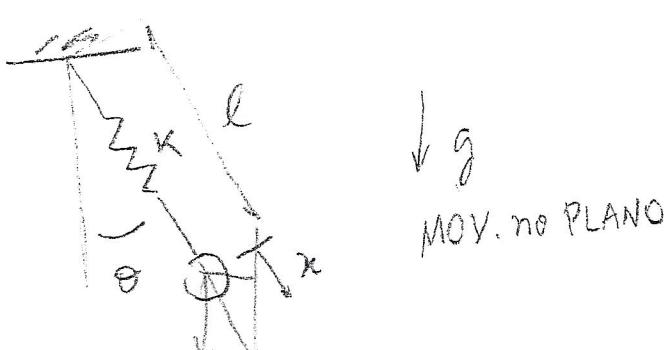
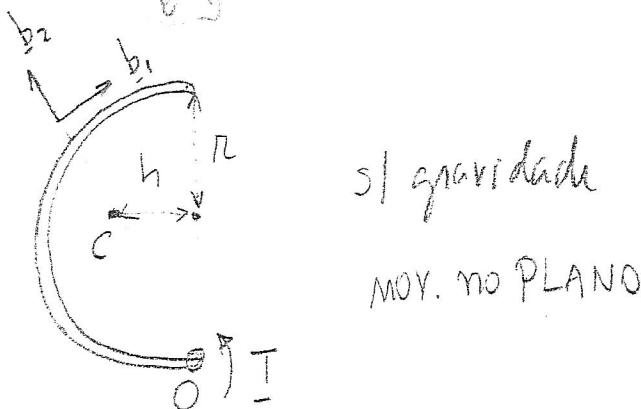


Fig. 3



MOV. no PLANO

Fig. 4

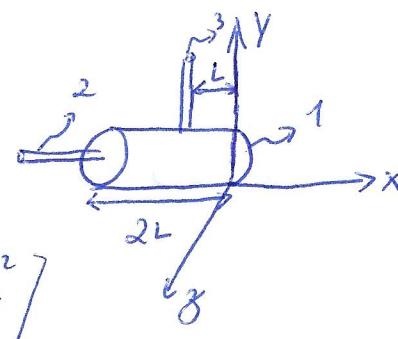


s/ gravidade

MOV. no PLANO

P1. 2013.2

$$1) I_{xx}^{so} = I_{xx}^1 + I_{xx}^2 + I_{xx}^3$$



$$I_{xx}^1 = \frac{1}{2} M \left(\frac{L}{2}\right)^2 + M \left(\frac{L}{2}\right)^2 = \frac{3}{8} M L^2$$

$$I_{xx}^2 = 0 + m \left(\frac{L}{2}\right)^2 = \frac{1}{4} m L^2 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} I_{xx}^{so} = \frac{3}{8} M L^2 + \frac{3}{12} m L^2$$

$$I_{xx}^3 = \frac{1}{12} m L^2 + m \left(\frac{3}{2} L\right)^2 = \frac{7}{3} m L^2$$

$$I_{xy}^s = I_{xy}^1 + I_{xy}^2 + I_{xy}^3$$

$$I_{xy}^1 = 0 - M(-L) \frac{L}{2} = M \frac{L^2}{2}$$

$$I_{xy}^2 = 0 - m \left(-\frac{5}{2} L\right) \frac{L}{2} = \frac{5}{4} m L^2 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} I_{xy}^{so} = M \frac{L^2}{2} + \frac{11}{4} m L^2$$

$$I_{xy}^3 = 0 - m (-L) \frac{3}{2} L = \frac{3}{2} m L^2$$

$$2) a) \underline{H}^A = [\underline{I}^A] \underline{\omega} + \underline{r}^C_A \times m \underline{v}^R_A$$

$$\underline{r}^C_A = -L \underline{c}_2 \quad \underline{v}^R_A = -\omega L \frac{b_3}{2}$$

$$[\underline{I}^A] = \begin{bmatrix} \frac{1}{3} m (2L)^2 & \cancel{I_{xy}}^0 & \cancel{I_{xz}}^0 \\ I_{xy} & 0 & \cancel{I_{yz}}^0 \\ I_{xz} & I_{yz} & \frac{1}{3} m (2L)^2 \end{bmatrix} \text{ na base } \underline{c}$$

Lassando tudo para a base \underline{c} :

$$\underline{H}^A = [\underline{I}^A] \begin{pmatrix} \omega \sin \theta \\ \omega \cos \theta \\ \dot{\theta} \end{pmatrix} + m(0 - L \dot{\theta})_x (0 \cdot 0 - \omega \frac{L}{2}) = \left(\frac{4}{3} m L^2 \omega \sin \theta + \omega \frac{L^2}{2} m \right) \underline{c}_1 + \frac{4}{3} m L^2 \dot{\theta} \underline{c}_3$$

$$b) H^0 = H^A + r^A \times G^B$$

$$r^A \times G^B = \left(\frac{L}{2} \cos \theta, -\frac{L}{2} \sin \theta, 0 \right) \times \left(L \ddot{\theta}, 0, -\omega \frac{L}{2} - \omega \sin \theta L \right) m$$

$$\overset{R}{v}^{CM} = v^A + \overset{A}{\omega} \times r^A = -\omega \frac{L}{2} b_3 + (\omega b_2 + \dot{\theta} b_3) \times (-L c_2) = -\omega \frac{L}{2} c_3 + L \ddot{\theta} c_1 - \omega \omega b_3$$

$$H^0 = \left[\frac{4}{3} mL^2 \omega \sin \theta + \omega \frac{L^2}{2} m + m \left(-\frac{\omega L^2}{4} \sin \theta - \omega \frac{L^2}{2} \sin^2 \theta \right) \right] c_1 +$$

$$+ m \left(\frac{L^2}{4} \omega \cos \theta + \frac{L^2}{2} \omega \sin \theta \cos \theta \right) c_2 + m \left(\frac{4}{3} L^2 \ddot{\theta} - \frac{L^2}{2} \dot{\theta} \sin \theta \right) c_3$$

$$c) K = \frac{1}{2} m \overset{R}{v}^{CM T} \cdot \overset{R}{v}^{CM} + \frac{1}{2} \omega^T [I^{CM}] \omega$$

$$= \frac{1}{2} m \left(L^2 \ddot{\theta}^2 + \omega^2 L^2 \left(\sin \theta - \frac{1}{2} \right)^2 \right) + \frac{1}{2} (\omega \cos \theta - \omega \sin \theta \dot{\theta}) [I^{\theta}] \begin{pmatrix} \omega \cos \theta \\ -\omega \sin \theta \\ \dot{\theta} \end{pmatrix}$$

$$= \frac{1}{2} m \left(L^2 \ddot{\theta}^2 + \omega^2 L^2 \left(\sin \theta - \frac{1}{2} \right)^2 \right) + \frac{1}{12} m (2L)^2 \left(\frac{1}{2} \omega^2 \cos^2 \theta + \frac{1}{2} \dot{\theta}^2 \right)$$

$$3) K = \frac{1}{2} m \overset{R}{v}^* T \overset{R}{v}^* ; \quad \phi = -m(L+x) \cos \theta g + \frac{1}{2} K_x^2$$

$$\overset{R}{v}^* = \ddot{x} b_1 + \dot{\theta} (L+x) b_2$$

$$L = \frac{1}{2} m \ddot{x}^2 + \frac{1}{2} m \dot{\theta}^2 L^2 + \frac{1}{2} m \dot{\theta} 2Lx + \frac{1}{2} m \dot{\theta} x^2 + mL \cos \theta g + mx \cos \theta g - \frac{K_x^2}{2}$$

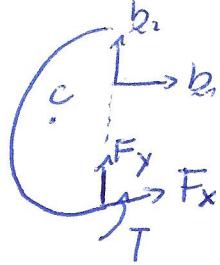
$$\frac{\partial L}{\partial x} = m \ddot{\theta} L + m \dot{\theta} x + m \cos \theta g - K_x; \frac{d}{dt} \left(\frac{\partial L}{\partial \ddot{x}} \right) - \frac{\partial L}{\partial x} = 0 \Rightarrow$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \ddot{x}} \right) = m \ddot{x} ; \Rightarrow m \ddot{x} + K_x - m \ddot{\theta} L - m \dot{\theta} x - m \cos \theta g = 0$$

$$\frac{\partial L}{\partial \theta} = -mgL \sin \theta - mgx \sin \theta$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) = m \ddot{\theta} L^2 + mL \ddot{x} + mx \ddot{x} \quad \left. \right\} m(\ddot{\theta} L^2 + \ddot{x} L + x \ddot{x} + gL \sin \theta + gx \cdot \sin \theta) = 0$$

91 a)



$$b) \frac{d}{dt} \dot{a}^c = \frac{d^2}{dt^2} p^c$$

$$p^c = r \dot{b}_1 - h \ddot{b}_1$$

$$\frac{dp^c}{dt} = r \ddot{\theta} b_1 - h \ddot{\theta} b_2$$

$$\frac{d^2}{dt^2} p^c = -r \ddot{\theta} b_1 - r \ddot{\theta}^2 b_1 + h \ddot{\theta}^2 b_1 - h \ddot{\theta} b_2 //$$

c)

$$F = m \ddot{a}^c$$

$$F_x = h \ddot{\theta}^2 - r \ddot{\theta}$$

$$F_y = -r \ddot{\theta}^2 - h \ddot{\theta}$$

$$M^c = I^c \alpha \Rightarrow T = I_{yy} \ddot{\theta}$$