28/09/2011 - P1 DIN2 - Prof. Thiago Ritto

1) (2,0) Considere o mecanismo mostrado na Fig. 1. A base $\{\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3\}$ está fixa no disco D que está pivotado no ponto O e gira com velocidade $\dot{\theta}_1$ no sentido mostrado na figura. A base $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ está fixa na haste H que está pivotada no ponto Q e forma ângulo θ_2, como mostrado na figura. Os dois corpos estão conectados pelo pino P que está fixo no disco (não há atrito entre o pino e a haste, ou qualquer outra conexão). (a) Faça o diagrama de corpo livre da haste (obs. o ponto C é a localização do centro de massa da haste), (b) escreva o vetor resultante das forças externas que atuam em H usando a base fixa no referencial inercial $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$.

2) (1,5) Considere o tambor mostrado na Fig. 2. A base $\{\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3\}$ está fixa no referencial inercial. A força $\mathbf{F}$ é aplicada na corda que passa pelo tambor fazendo com que ele role sem deslizar durante duas voltas. (a) Calcule a distância percorrida pelo centro de massa do tambor, (b) calcule o trabalho realizado pela força $\mathbf{F}$, pela força de atrito e pela força normal.

3) (4,0) Considere o sistema mostrado na Fig. 3. A base $\{\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3\}$ está fixa no referencial inercial e a base $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ está fixa na barra. Obtenha as equações de movimento para a barra que tem comprimento $2r$. Considere que o atrito é desprezível e que x mede o deslocamento do ponto Q a partir da mola em repouso ($\mathbf{F}_{\text{mola}} = +kx\mathbf{n}_1$). Lembre-se que o número de equações deve ser igual ao número de incógnitas; indique as incógnitas.

4) (2,5) Obtenha o momento de inércia I_{zz} e o produto de inércia I_{xz} da estrutura mostrada na Fig. 4 com relação ao ponto O.

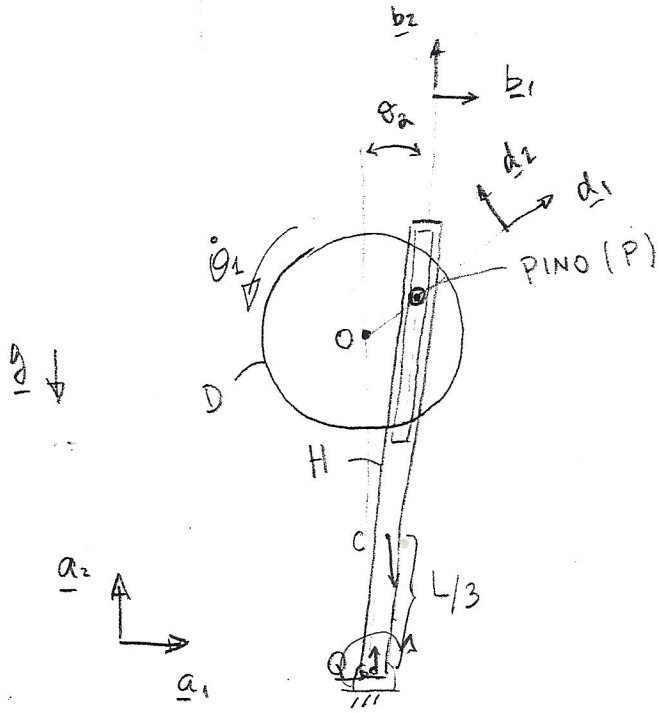

FIGURA 1

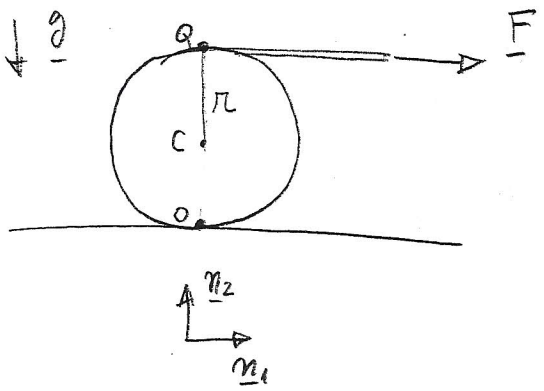

FIGURA 2

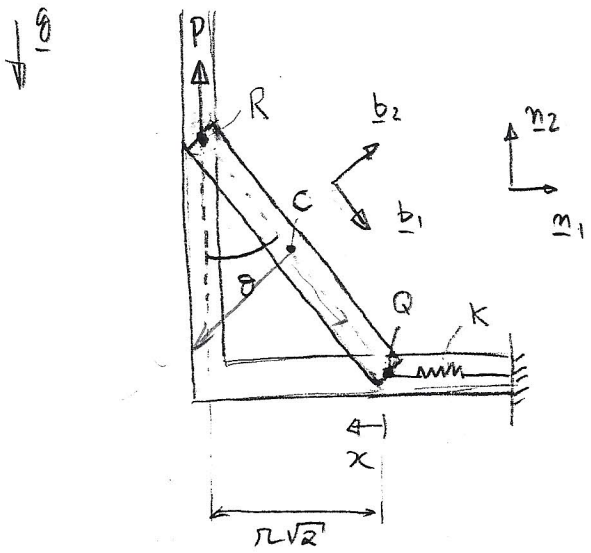

FIGURA 3

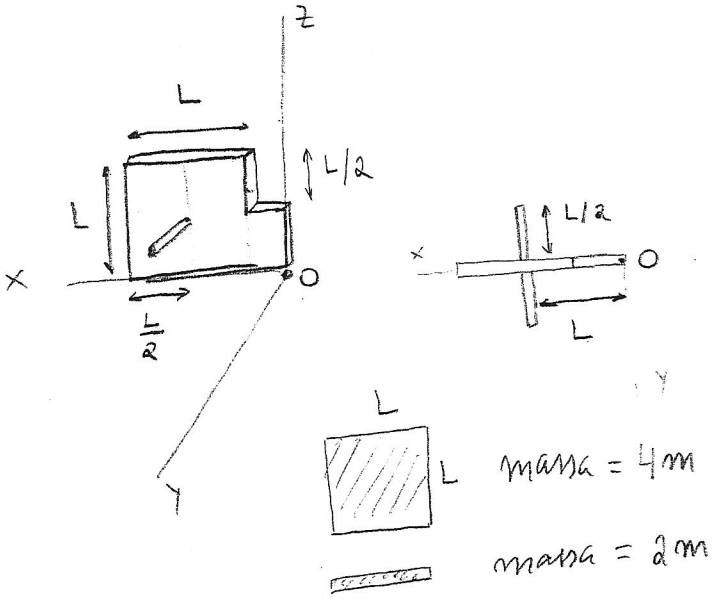

FIGURA 4

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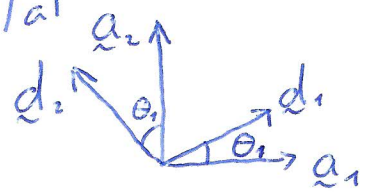
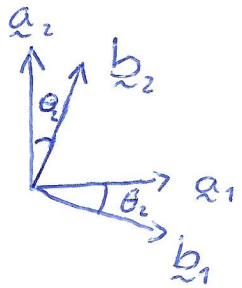
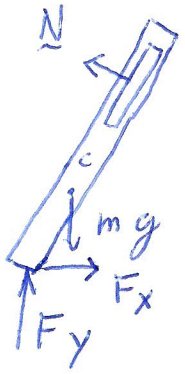
1) a)



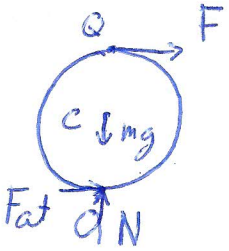




b) $\underset{\sim}{F} = \sum_{i=0}^{n} \underset{\sim}{F}_i = F_x \underset{\sim}{a}_1 + F_y \underset{\sim}{a}_2 - mg\,\underset{\sim}{a}_2 - N\,\underset{\sim}{b}_1$

$= (F_y + N \operatorname{sen}\theta_2 - mg)\,a_2 + (F_x - N\cos\theta_2)\,\underset{\sim}{a}_1$ //

2) a) $\underset{\sim}{x}^c = 2 \times 2\pi r = 4\pi r\ \underset{\sim}{n}_1$

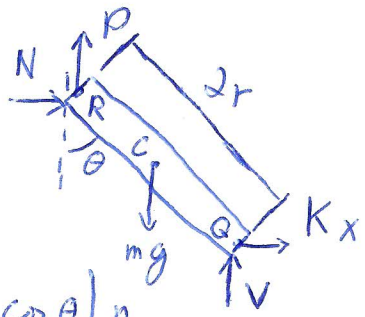


b) $W_{12}^{N} = 0 \Leftrightarrow W_{12}^{N} = \int \underset{\sim}{N}\cdot d\underset{\sim}{r} = \int (0,N)\cdot(dr,0) = 0$

$W_{12}^{Fat} = 0 \Leftrightarrow W_{12}^{Fat} = \int \underset{\sim}{F}_{at}\cdot \cancel{\underset{\sim}{v}}^{\,0}\, dt = 0$

$W_{12}^{F} = \int \underset{\sim}{F}\cdot d\underset{\sim}{r} = F\int dr = F\int_0^{4\pi} 2r\,d\phi = 8F\pi r$

3) $\underset{\sim}{F} = m\,{}^{R}\underset{\sim}{a}^{*}$, $\underset{\sim}{M} = I\,\underset{\sim}{\alpha}$ (movimento no plano)



$\underset{\sim}{F} = (N + K_x)\,\underset{\sim}{n}_1 + (P + V - mg)\,\underset{\sim}{n}_2$

$\underset{\sim}{M}^{B/c} = (Vr\operatorname{sen}\theta - Pr\operatorname{sen}\theta - Nr\cos\theta + K_x r\cos\theta)\,\underset{\sim}{n}_3$

${}^{R}\underset{\sim}{a}^{c} = {}^{R}\underset{\sim}{a}^{Q} + \underset{\sim}{\omega}\times(\underset{\sim}{\omega}\times \underset{\sim}{r}^{c/Q}) + \underset{\sim}{\alpha}\times \underset{\sim}{r}^{c/Q}$

${}^{R}\underset{\sim}{a}^{Q} = -\ddot{x}\,\underset{\sim}{n}_1$

$\omega = \dot{\theta}\,\underset{\sim}{n}_3$

$\alpha = \ddot{\theta}\,\underset{\sim}{n}_3$

$\underset{\sim}{r}^{c/Q} = -r\operatorname{sen}\theta\,\underset{\sim}{n}_1 + r\cos\theta\,\underset{\sim}{n}_2$

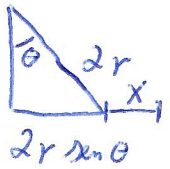
${}^{R}a^{c} = (-\ddot{x} + r\dot{\theta}^2\operatorname{sen}\theta - r\ddot{\theta}\cos\theta)\,n_1 + (-r\ddot{\theta}\operatorname{sen}\theta - r\dot{\theta}^2\cos\theta)\,n_2$

$I^{B/c} = \frac{1}{3}mr$

$$\underset{\sim}{F} = m\underset{\sim}{a} \Rightarrow \begin{cases} N + Kx = m(-\ddot{x} + r\dot{\theta}^2 \operatorname{sen}\theta - r\ddot{\theta}\cos\theta \\ P + V - mg = m(-r\ddot{\theta}\operatorname{sen}\theta - r\dot{\theta}^2\cos\theta) \end{cases}$$

$$\underset{\sim}{M} = I_z \Rightarrow Vr\operatorname{sen}\theta - Pr\cos\theta - Nr\cos\theta + Kxr\cos\theta = \frac{1}{3}mr^2\ddot{\theta}$$

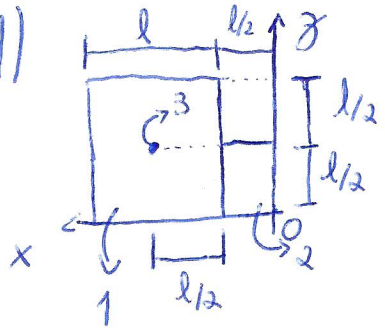
4 incógnitas: $\{V, N, x, \theta\}$ e 3 eqs. No entanto, pode-se relacionar x e θ:

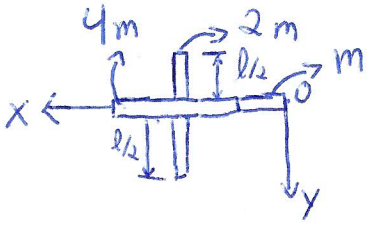


$$\Rightarrow x = r\sqrt{2} - 2r\operatorname{sen}\theta$$

Logo, tem-se 4 equações e 4 incógnitas.

4)





$$I^{*1}_{zz} = \frac{(4m)L^2}{12} \qquad I^{*1}_{xz} = 0$$

$$I^{*2}_{zz} = \frac{m}{12}\left(\frac{L}{2}\right)^2 \qquad I^{*2}_{xz} = 0$$

$$I^{*3}_{zz} = \frac{2mL^2}{12} \qquad I^{3}_{xz} = 0$$

$$I^{01}_{zz} = I^{*1}_{zz} + m(d_x^2 + \cancel{d_y^2}^{\,0}) = \frac{(4m)L^2}{12} + (4m)L^2$$

$$I^{02}_{zz} = I^{*2}_{zz} + m(d_x^2 + \cancel{d_y^2}^{\,0}) = \frac{m}{12}\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{4}\right)^2$$

$$I^{03}_{zz} = I^{*3}_{zz} + m(d_x^2 + d_y^2) = \frac{(2m)L^2}{12} + 2mL^2$$

$$I^{0}_{zz} = I^{01}_{zz} + I^{02}_{zz} + I^{03}_{zz} = \frac{79}{12}mL^2$$

$$I^{01}_{xz} = I^{*1}_{xz} - m d_x d_z = 0 - 4mL\frac{L}{2} = -2mL^2$$

$$I^{02}_{xz} = I^{*1}_{xz} - m d_x d_z = 0 - m\frac{L}{4}\frac{L}{4} = -m\frac{L^2}{16}$$

$$I^{03}_{xz} = I^{*3}_{xz} - m d_x d_z = 0 - (2m)L\frac{L}{2} = -mL^2$$

$$I^{0}_{xz} = I^{01}_{xz} + I^{02}_{xz} + I^{03}_{xz} = -\frac{49}{16}mL^2$$