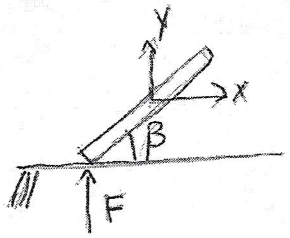
Lista 2.

1)



$$I_{linear} = {}^{R}G_2^{B} - {}^{R}G_1^{B}$$

$$I_{ang} = {}^{R}H_2^{B/*} - {}^{R}H_1^{B/*}$$

$$I_{linear} = \int F\,dt = G_2 - G_1 = m\,{}^{R}V_2^{*} - m\,{}^{R}V_1^{*}$$

$$I_{ang} = \int (F_1\, l\cos\beta\, dt) = I^{*}\omega_2 - I^{*}\cancelto{0}{\omega_1}$$

$$\int F\,dt = -\frac{I^{*}\omega_2}{l c\beta} = m\left({}^{R}V_2^{*} - {}^{R}V_1^{*}\right)$$

$$-\frac{I^{*}\omega_2}{l c\beta} = m\left({}^{R}V_2^{*} - {}^{R}V_1^{*}\right)$$

Considerando o choque elastico: $E_1 = E_2$.

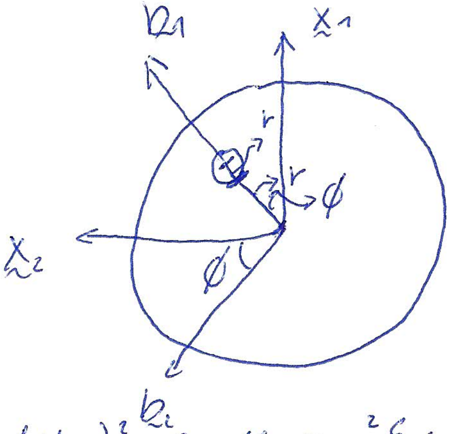
$$\frac{1}{2} m\,{}^{R}V_1^{*2} = \frac{1}{2} I^{*}\omega_2^{2} + \frac{1}{2} m\,{}^{R}V_2^{*2} \Rightarrow I^{*}\omega_2^{2} = m\underbrace{\left({}^{R}V_1^{*2} - {}^{R}V_2^{*2}\right)}_{\left({}^{R}V_1^{*} - {}^{R}V_2^{*}\right)\left({}^{R}V_1^{*} + {}^{R}V_2^{*}\right)}$$

$$\boxed{{}^{R}V_2^{*} - {}^{R}V_1^{*} = \frac{I^{*}\omega_2}{m l c\beta}} \Rightarrow {}^{R}V_1^{*} + {}^{R}V_2^{*} = \omega_2\, l c\beta$$

$$\begin{cases} {}^{R}V_1^{*} - {}^{R}V_2^{*} = \dfrac{I^{*}\omega_2}{m l c\beta} \\ {}^{R}V_1^{*} + {}^{R}V_2^{*} = \omega_2\, l c\beta \end{cases}$$

$$\omega_2 = \frac{6\, c\beta\, {}^{R}V_1^{*}}{l + 3 l c^2\beta} \qquad {}^{R}V_2^{*} = \frac{6\, l\, c^2\beta\, {}^{R}V_1^{*}}{l + 3 l c^2\beta} - {}^{R}V_1^{*}$$

2)



$$p^{*/0} = \frac{M\times 0 + (-m)\times 2r\,\underset{\sim}{b}_1}{M+(-m)} = \frac{2r}{15}\underset{\sim}{b}_1$$

$$^R v^* = -\frac{2}{15} r\dot{\phi}\, b_2$$

$$^R a^* = -\frac{2}{15}\ddot{\phi} r\, \underset{\sim}{b}_2 + \frac{2}{15} r\dot{\phi}^2 b_1 ; \ \ddot{\phi} = \alpha$$

$$M = \rho (4r)^2 \pi \varepsilon = 16\rho\pi r^2\varepsilon ; \ m = \rho\pi r^2 \varepsilon$$

$$M_T = M - m = 15\rho\pi r^2\varepsilon$$

$$\underset{\sim}{F} = m\,\underset{\sim}{a}$$

$$\underbrace{\underset{\sim}{F} = \begin{bmatrix} F_1 - mg \\ F_2 \\ F_3 \end{bmatrix} \quad {}^R a^* = \begin{bmatrix} \frac{2}{15} r\dot{\phi}^2 c\phi + \frac{2}{15} r\ddot{\phi} s\phi \\ \frac{2}{15} r\dot{\phi}^2 s\phi - \frac{2}{15} r\ddot{\phi} c\phi \\ 0 \end{bmatrix}}_{\text{na base } \underset{\sim}{x}}$$

$[I^0]$ na base $\underset{\sim}{b}$, preso no corpo: $[I^0] = \begin{bmatrix} 4Mr^2 & 0 & 0 \\ 0 & 4Mr^2 & 0 \\ 0 & 0 & 8Mr^2 \end{bmatrix} - \begin{bmatrix} \frac{mr^2}{4} & 0 & 0 \\ 0 & \frac{17mr^2}{4} & 0 \\ 0 & 0 & \frac{9}{2}mr^2 \end{bmatrix} =$

$$= \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

Como o movimento é no plano:

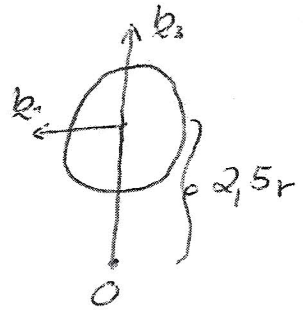
$$M = [I]\alpha$$

$$\left.\begin{aligned} M_1 c\phi - M_2 s\phi &= I_{xx}\cancelto{0}{\alpha_x} = 0 \\ M_1 s\phi + M_2 c\phi &= I_{yy}\cancelto{0}{\alpha_y} = 0 \end{aligned}\right\} M_1 = M_2 = 0$$

$$M_3 - \frac{2}{15} M_T g r s\phi = I_{zz}\alpha$$

3) $K = \frac{1}{2} m v^{*T} \cdot v^{*} + \frac{1}{2} \omega^T [I^*] \omega$

$\underset{\sim}{H}^{D/O} = \underset{\sim}{H}^{D/*} + p^{*/O} \wedge G^D$



$H^{D/*} = [I^*] {}^R\omega^D$

Na base b:

$$[I^*] = \begin{bmatrix} \frac{1}{4} mr^2 & 0 & 0 \\ 0 & \frac{1}{2} mr^2 & 0 \\ 0 & 0 & \frac{1}{4} mr^2 \end{bmatrix} \quad ; \quad {}^R\underset{\sim}{\omega}^D = \begin{bmatrix} \Omega \\ \omega \\ 0 \end{bmatrix}$$

$\underset{\sim}{H}^{D/*} = \frac{1}{4} mr^2 \Omega \, \underset{\sim}{b}_1 + \frac{1}{2} mr^2 \omega \, b_2$

$p^{*/O} = 2{,}5 r \, \underset{\sim}{b}_3$

$G^D = m {}^R V^* = m \, \omega \wedge r = -m \, \Omega \wedge 2{,}5 \wedge r \, b_2$

$H^{D/O} = \frac{26}{4} m r^2 \Omega \, b_1 + \frac{1}{2} m r^2 \omega \, b_2$

$V^* = (0, -2{,}5 \Omega r, 0)$ en b

$K = \frac{25}{8} m \Omega^2 r^2 + \frac{1}{2} \left(\frac{1}{4} mr^2 \Omega + \frac{1}{2} mr^2 \omega \right)$

4) Tomando os dados do exercício anterior como base:

$$K' = \frac{1}{2} m v^{*} \cdot v^{*} + \frac{1}{2} \omega [I^{*}] \omega$$

$$H^{O/o} = H^{O/*} + r^{*/o} \wedge G^{O}$$

$$v^{*} = \begin{bmatrix} v \\ -2,5 r \Omega \\ 0 \end{bmatrix}$$

$$\omega = \begin{bmatrix} \Omega \\ \omega \\ 0 \end{bmatrix}$$

$$r^{*/o} = \begin{bmatrix} vt \\ 0 \\ 2,5r \end{bmatrix}$$

$$H^{O/o} = \begin{bmatrix} \frac{1}{4} m r^2 \Omega \\ \frac{1}{2} m r^2 \omega \\ 0 \end{bmatrix} + m\, r^{*/o} \wedge v^{*} = \begin{bmatrix} \frac{1}{4} m r^2 \Omega + \frac{25}{4} m r^2 \Omega \\ \frac{1}{2} m r^2 \omega + m\, 2,5 r v \\ -2,5 m\, v t\, r \Omega \end{bmatrix}$$

$$K' = \frac{1}{2} m \left(v^2 + \frac{25}{4} r^2 \Omega^2 \right) + \frac{1}{2} \left(\frac{1}{4} m r^2 \Omega + \frac{1}{2} m r^2 \omega \right)$$

5) Por conservação de energia:

$$\Delta\phi = \frac{K}{2}\left[(\Delta x + 2h)^2 - \Delta x^2\right] - (M+m)gh$$

↳ variação de energia potencial

$$\Delta K = \frac{(m+M)}{2}v^2 + \left(\frac{mK^{*2}}{2}\frac{v^2}{r^2}\right) \rightarrow mK^{*2} = I,\ \omega = \frac{v}{r}$$

↳ variação de energia cinética

$$0 = \Delta\phi + \Delta K$$

$$\frac{K}{2}\left[(\Delta x + 2h)^2 - \Delta x^2\right] - (M+m)gh + \frac{(m+M)}{2}v^2 + \frac{mK^*}{2}\frac{v^2}{r^2} = 0$$

$$v = \boxed{\sqrt{\frac{2r^2\left[(M+m)gh - K(2\Delta x h + 2h^2)\right]}{(M+m)r^2 + mK^{*2}}}}$$