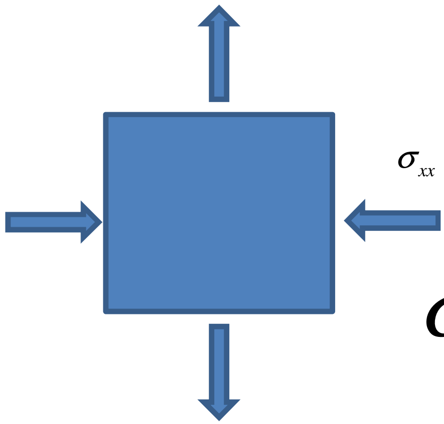
Mecânica dos Sólidos I

Exemplos : Tensão – Deformação

(capítulo IV)

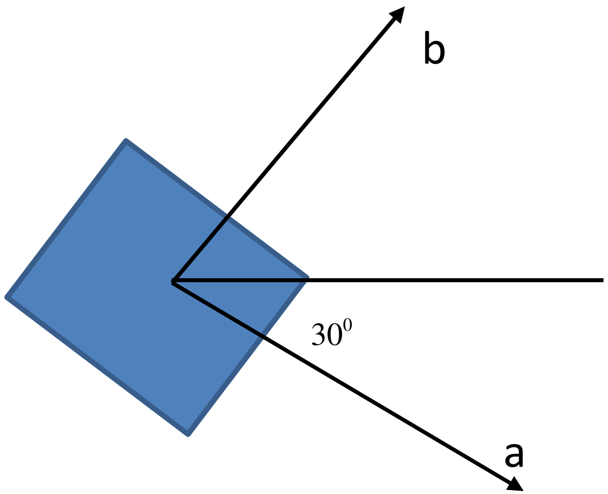
Exemplo 1 (ex. 4.15)

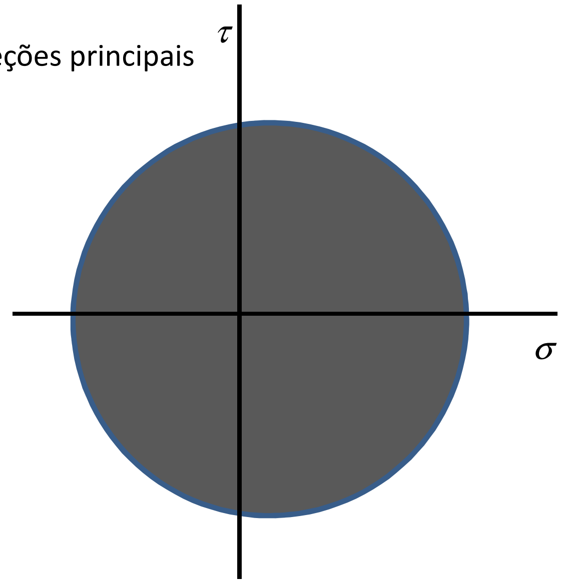
$$\sigma_1 = 75 MN / m^2$$



$\sigma_{xx} = 15 MN / m^2$

$$\sigma_{xy} = 0$$ → Direções principais





$\theta = -30^\circ$

$$\sigma_{aa} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos\theta$$

$$= \frac{-15 + 75}{2} + \frac{-15 - 75}{2} \cos(-60)$$

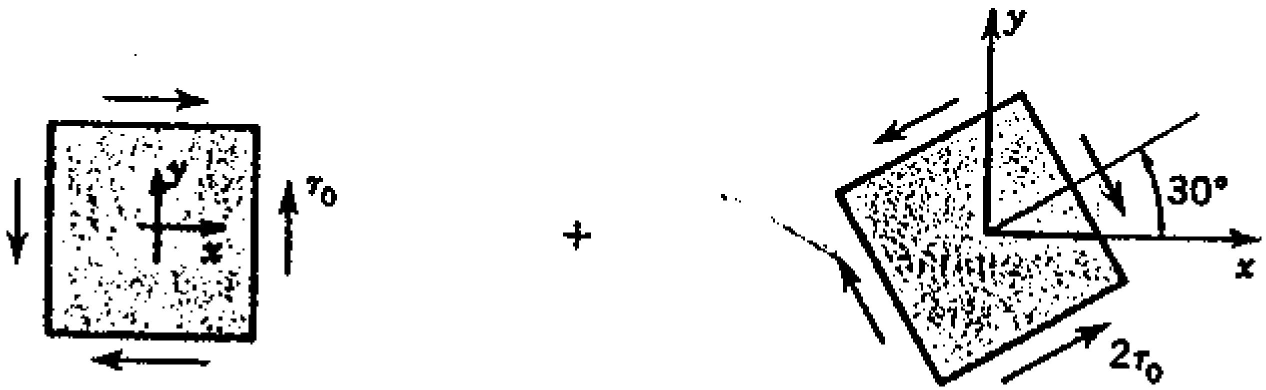
$$\sigma_{aa} = 30 - 22.5 = 7.5 \quad MN/m^2$$

$$\sigma_{ab} = -\frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$$

$$= -\frac{-15 - 75}{2} \sin(-60) = -38.97 \quad MN/m^2$$

E a face b????

Exemplo 2 (ex. 4.26)

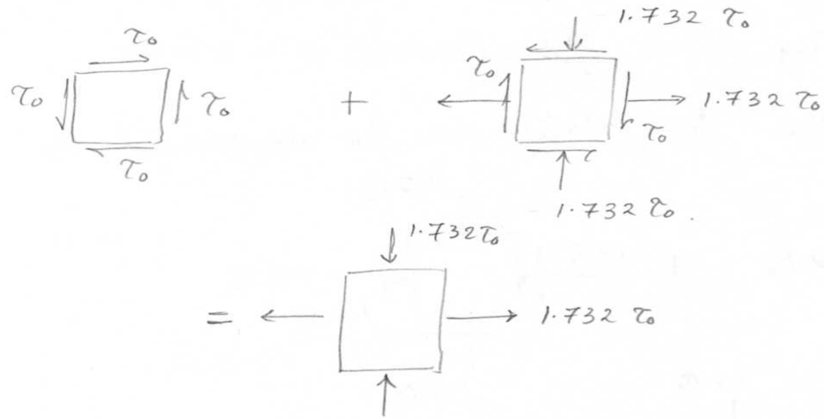


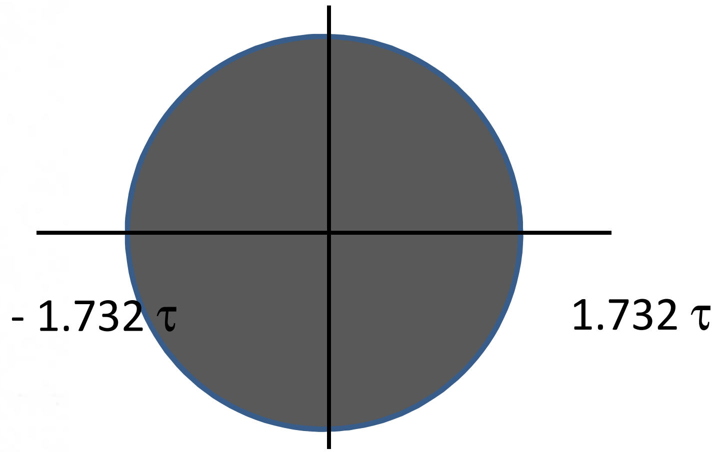
$$\sigma_x = -2\tau_0 \sin(-2\times 30) = 1.732\,\tau_0$$

$$\sigma_y = -(-2\tau_0)\sin(-2\times 30) = -1.732\,\tau_0$$

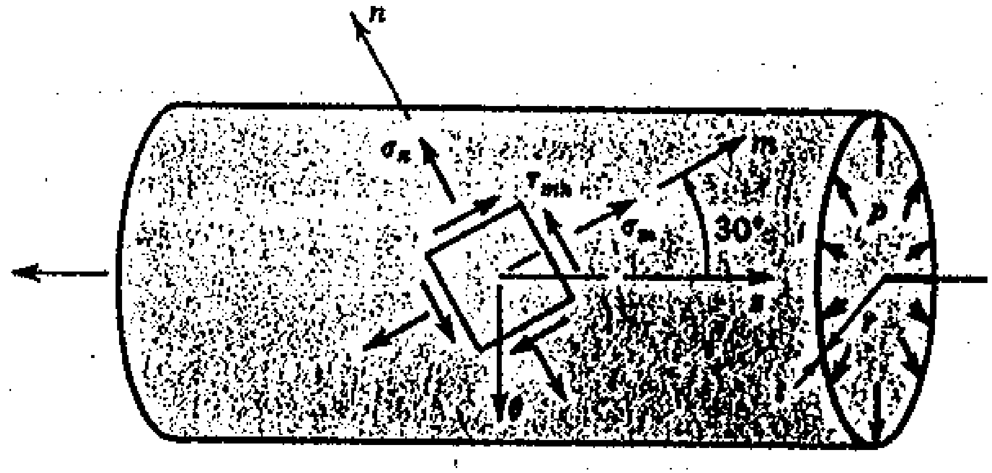
$$\tau_{xy} = -2\tau_0 \cos(-2\times 30) = -\tau_0$$

Therefore the two states can be represented as





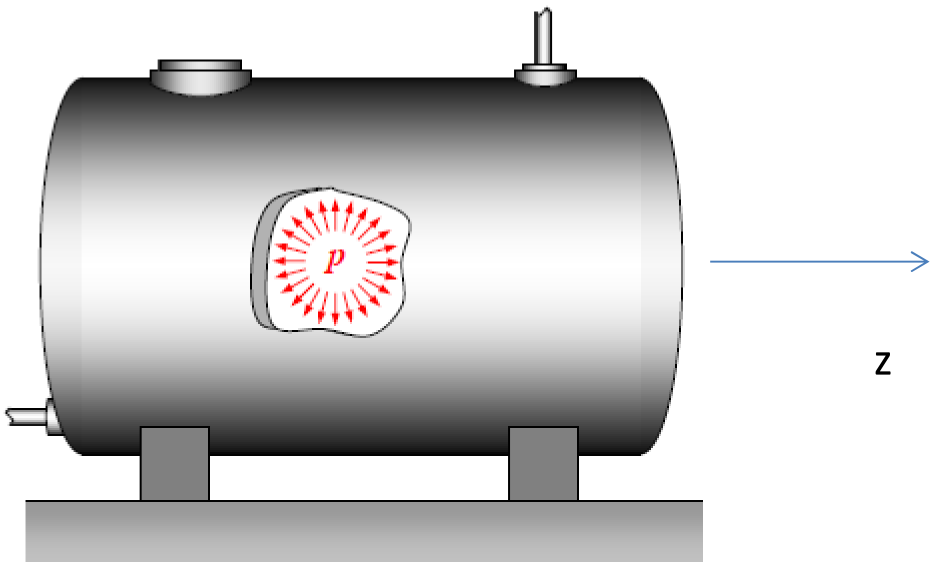
Exemplo 3 (ex. 4.27)



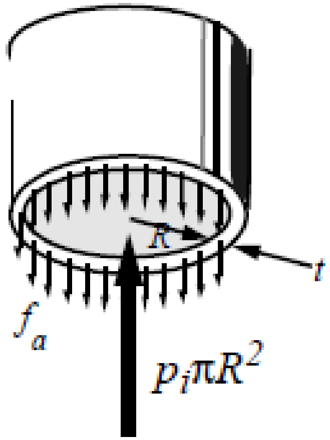
Encontrar os valores da pressão interna p e da força axial F considerando :

$$\sigma_m = 15.000\,psi \qquad \sigma_n = 5.000\,psi$$

Tensões Planas : Vasos de Pressão de Parede Fina



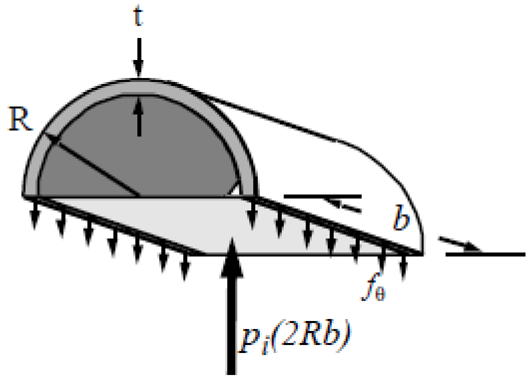
Sistema de Coordenadas Cilíndricas



$$2\pi R \cdot f_a = p_i \cdot \pi R^2$$

$$f_a = p_i \cdot (R/2)$$

$$\sigma_a = p_i \cdot (R/2t)$$



$$p_i \cdot (2Rb) = 2bf_\theta$$

$$\sigma_\theta = p_i \cdot (R/t)$$

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{\theta\theta}}{2} + \frac{\sigma_{xx} - \sigma_{\theta\theta}}{2} \cos(2 \times 30)$$

$$\sigma_m = 0.75\,\sigma_{xx} + 0.25\,\sigma_{\theta\theta} \quad \text{---} \quad (1).$$

$$\sigma_n = \frac{\sigma_{xx} + \sigma_{\theta\theta}}{2} - \frac{\sigma_{xx} - \sigma_{\theta\theta}}{2} \cos(2 \times 30)$$

$$\sigma_n = 0.25\,\sigma_{xx} + 0.75\,\sigma_{\theta\theta} \quad \text{---} \quad (2).$$

From (1) and (2)

$$\sigma_{xx} = \frac{3}{2}\sigma_m - \frac{\sigma_n}{2}$$

$$\sigma_{\theta\theta} = \frac{3\sigma_n - \sigma_m}{2}$$

$$\Rightarrow \quad \sigma_{xx} = 1.5 \times 15000 - 0.5 \times 5000$$

$$= 20{,}000 \text{ psi}$$

$$\sigma_{\theta\theta} = 0$$

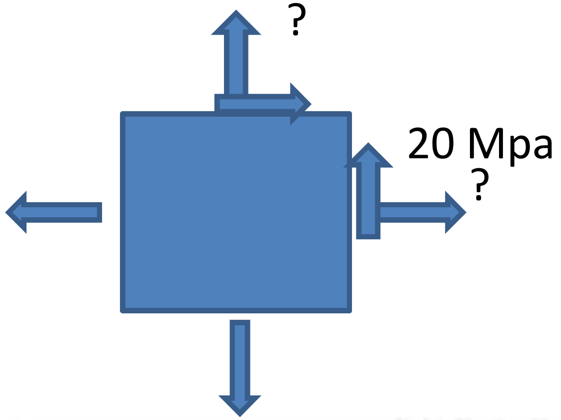
$$\sigma_{xx} = \frac{F}{2\pi r t} \Rightarrow F = 2\pi r t \times 20{,}000$$

$$= 2\pi \times 10 \times 0.1 \times 20{,}000$$

$$= 125{,}663.7 \text{ lb}$$

$$\sigma_{\theta\theta} = 0 \Rightarrow P = 0.$$

Exemplo 4 (ex. 6.28)

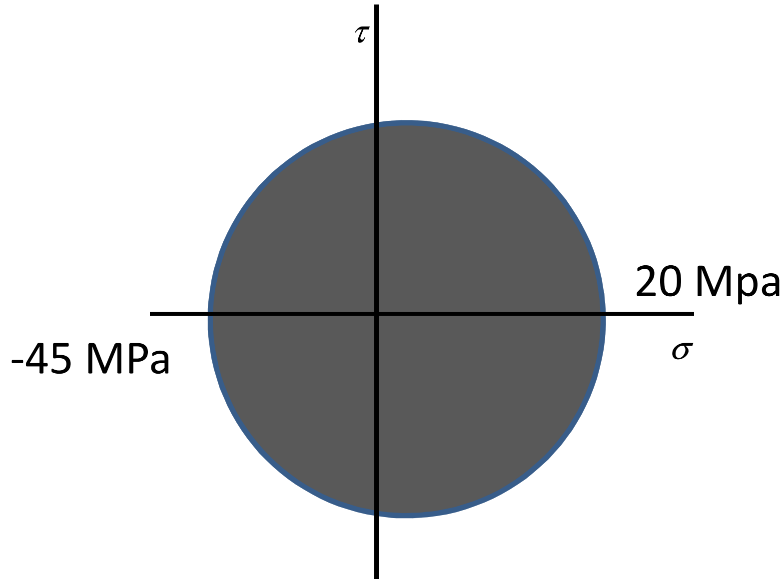


$$\tau_{xy} = -\frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$

$$20 = -\frac{20 + 45}{2}\sin 2\theta$$

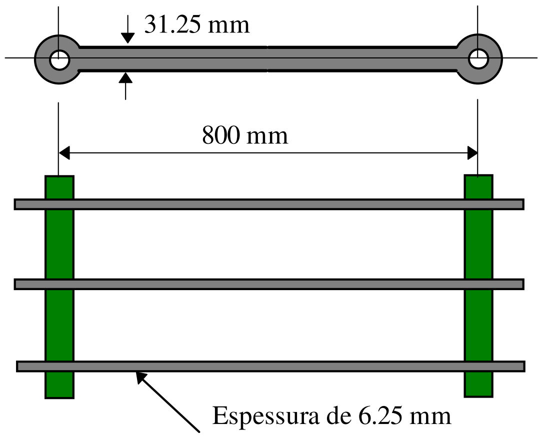
$$\sin 2\theta = -0.615$$

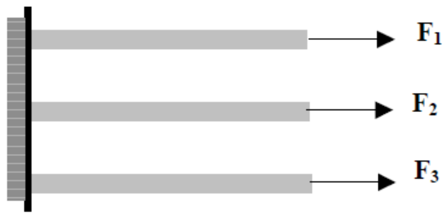
$$\Rightarrow \theta = -18.99^{\circ}.$$



Exemplo 5 (notas de aula prof. Lavínia)

Uma biela consiste em três barras de aço de 6.25 mm de espessura e 31.25mm de largura, conforme esquematizado na figura. Durante a montagem, descobriu-se que uma das barras media 799.925 mm entre os centros do furo e as outras mediam 800 mm. Determinar a tensão em cada barra após a montagem . Utilize para o aço E= 207 GPa.

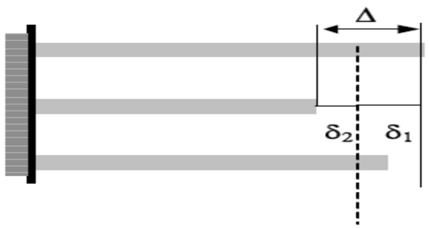




$$F_1 = F_3$$

$$2 \cdot F_1 + F_2 = 0$$

Cinemática



$$\delta_1 + \delta_2 = \Delta$$

Relação Constitutiva Elástica Linear :

$$\delta_1 = -\frac{F_1 \cdot L}{E \cdot A} \qquad \delta_2 = \frac{F_2 \cdot L}{E \cdot A}$$

Solução:

$$-\frac{F_1 \cdot L}{E \cdot A} + \frac{F_2 \cdot L}{E \cdot A} = \Delta \qquad F_1 = -\frac{\Delta \cdot E \cdot A}{L} + F_2$$

Cinemática e Constitutiva

Substituindo no equilíbrio:

$$2 \cdot \left(-\frac{\Delta \cdot E \cdot A}{L} + F_2\right) + F_2 = 0 \qquad F_2 := \frac{2}{3} \cdot \Delta \cdot \frac{E \cdot A}{L} \qquad F_2 = 2.527\,\mathrm{kN}$$

$$F_1 = F_3 \qquad F_1 := -\frac{1}{3} \cdot \Delta \cdot \frac{E \cdot A}{L} \qquad F_1 = -1.263\,\mathrm{kN}$$

$$\sigma_1 := \frac{F_1}{A} \qquad \sigma_2 := \frac{F_2}{A}$$

$$\sigma_1 = -6.5\,\mathrm{MPa} \qquad \sigma_2 = 12.9\,\mathrm{MPa}$$