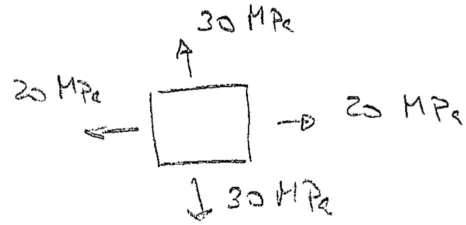
MID. TERM EXAMINATION - SOLUTIONS

QUESTION 1

(1) $$\int_{\Gamma} \underline{s}\; d\Gamma = +16\ \text{KN}\ \underline{i} + 36\ \text{KN}\ \underline{j} - 16\ \text{KN}\ \underline{i} - 36\ \text{KN}\ \underline{j} = \underline{0}$$

(2) $$\sigma_z = \sigma_{zy} = \sigma_{xy} = \sigma_{xz} = 0$$

$$\sigma_x = \frac{16 \times 10^3}{0.2 \times 4 \times 10^{-3}} = 20\ \text{MPa}$$



$$\sigma_y = \frac{36 \times 10^3}{0.3 \times 4 \times 10^{-3}} = 30\ \text{MPa}$$

(3) AS $\sigma_{ij} = 0$ THE PRINCIPAL DIRECTIONS ARE COINCIDENT WITH THE VERTICAL AND HORIZONTAL ONES.

(4) $\underline{n} = -\sin\theta\ \underline{i} + \cos\theta\ \underline{j}$; $\underline{t} = \cos\theta\ \underline{i} + \sin\theta\ \underline{j}$ (TANGENTIAL DIRECTION)

THEN

$$\underline{s} = T\,\underline{n} = -20 \sin\theta\ \text{MPa}\ \underline{i} + 30 \cos\theta\ \text{MPa}\ \underline{j}$$

AND TANGENTIAL COMPONENT (SHEAR DIRECTION)

$$\underline{s} \cdot \underline{t} = -20 \sin\theta \cos\theta + 30 \sin\theta \cos\theta = 5 \sin 2\theta$$

$\hookrightarrow$ MAXIMUM $\theta = \pm 45^\circ$

15) $$\varepsilon_x = \frac{1}{2\times10^{11}}\left[2\times10^7 - 0{,}3\times3\times10^7\right] = 5{,}5\times10^{-5}\ m/m$$

$$\varepsilon_y = \frac{1}{2\times10^{11}}\left[3\times10^7 - 0.3\times2\times10^7\right] = 12\times10^{-5}\ m/m$$

$$\varepsilon_z = \frac{1}{2\times10^{11}}\left[-0.3\times5\times10^7\right] = -7.5\times10^{-5}\ m/m$$

$$\varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{zx} = 0$$

16) PLACING THE COORDINATES ORIGIN AT THE CENTER OF THE PLATE

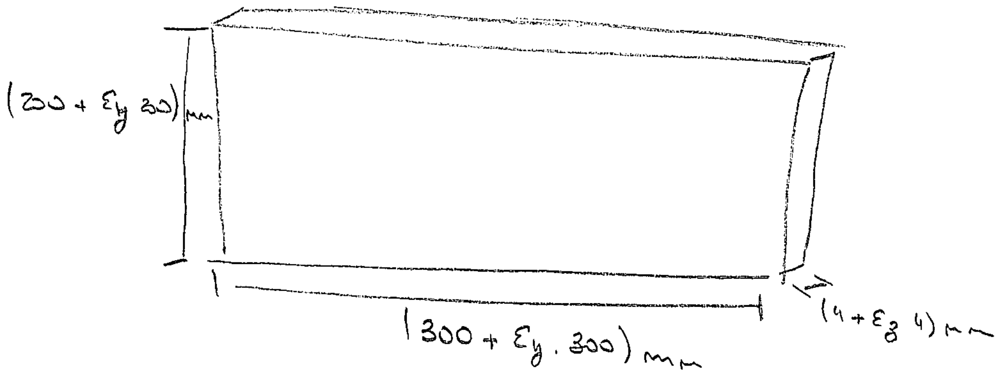
$$u_x = 5.5\times10^{-5}\,X + C_1$$

$$u_y = 12\times10^{-5}\,Y + C_2$$

$$u_z = -7.5\times10^{-5}\,Z + C_3$$

THE VECTOR (C_1, C_2, C_3) CORRESPONDS TO A POSSIBLE RIGID BODY TRANSLATION OF THE PLATE!

(7)



Question 2 :

(1) SMALL DEFORMATION THEORY:

$$\frac{\partial u_x}{\partial x} = 0 \quad ; \quad \frac{\partial u_y}{\partial y} = 0 \quad ; \quad \frac{\partial u_z}{\partial z} = 0$$

(NO EXTENSION COMPONENTS)

$$\frac{\partial u_y}{\partial x} = \alpha z \quad ; \quad \frac{\partial u_x}{\partial y} = -\alpha z \quad ; \quad \frac{\partial u_z}{\partial x} = \frac{\partial w}{\partial x}$$

$$\frac{\partial u_z}{\partial y} = \frac{\partial w}{\partial y} \qquad \frac{\partial u_x}{\partial z} = -\alpha y \quad ; \quad \frac{\partial u_y}{\partial z} = +\alpha x$$

$$\varepsilon = \begin{bmatrix} 0 & 0 & \frac{1}{2}\left(\frac{\partial w}{\partial x} - \alpha y\right) \\ & 0 & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \alpha x\right) \\ \text{Sim} & & 0 \end{bmatrix}$$

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LARGE DEFORMATION THEORY

$$\nabla_{\underline{u}}^T \nabla_{\underline{u}} = \begin{bmatrix} 0 & +\alpha z & \frac{\partial w}{\partial x} \\ -\alpha z & 0 & \frac{\partial w}{\partial y} \\ -\alpha y & +\alpha x & 0 \end{bmatrix} \begin{bmatrix} 0 & -\alpha z & -\alpha y \\ +\alpha z & 0 & +\alpha x \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} \alpha^2 z^2 + \left(\frac{\partial w}{\partial x}\right)^2 & \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} & \alpha^2 x z \\ \frac{\partial w}{\partial y}\frac{\partial w}{\partial x} & \alpha^2 z^2 + \left(\frac{\partial w}{\partial y}\right)^2 & \alpha^2 z y \\ \alpha^2 x z & \alpha^2 z y & \alpha^2 y^2 + \alpha^2 x^2 \end{bmatrix}$$

(2) SO THE DIFFERENCES BETWEEN BOTH THEORIES STRONGLY RELIES ON α BUT ALSO DEPENDS ON w!
(e.g: if $\alpha \ll 1$ AND $w = 0$ BOTH MEASURES TEND TO COINCIDE)

(3) $$\tau_{xz} = \frac{1}{2G}\left[\frac{\partial w}{\partial x} - \alpha y\right]$$

$$\tau_{yz} = \frac{1}{2G}\left[\frac{\partial w}{\partial y} + \alpha x\right]$$

(4) EQUILIBRIUM EQUATION

$$\cancelto{0}{\frac{\partial \sigma_{xx}}{\partial x}} + \cancelto{0}{\frac{\partial \sigma_{xy}}{\partial y}} + \frac{\partial \sigma_{xz}}{\partial z} = 0 \qquad (1)$$

$$\cancelto{0}{\frac{\partial \sigma_{yx}}{\partial x}} + \cancelto{0}{\frac{\partial \sigma_{yy}}{\partial y}} + \frac{\partial \sigma_{yz}}{\partial z} = 0 \qquad (2)$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \cancelto{0}{\frac{\partial \sigma_{zz}}{\partial x}} = 0 \qquad (3)$$

- AS σ_{xz} IS NOT A FUNCTION OF z $\rightarrow$ (1) IS SATISFIED

- SIMILARLY TO (2)

- THEN $\frac{1}{2G}\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right] = 0$

$$\hookrightarrow \quad \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0$$

↳ EQUILIBRIUM IS SATISFIED.