HOMEWORK 5 - SOLUTIONS

1) 1-D PROBLEM

$$0 = \varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\underbrace{\cancel{\sigma_y + \sigma_z}}_{0}\right)\right] + \alpha \Delta T$$

$$\sigma_x = -\alpha E \Delta T$$

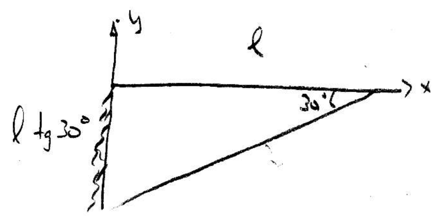
AND $\sigma_y = \sigma_z = \sigma_{yz} = \sigma_{yx} = \sigma_{zx} = 0$

$$\varepsilon_y = \varepsilon_z = -\nu \frac{\sigma_x}{E} + \alpha \Delta T$$

$$\varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0$$

2) CASE (a)

PLANE PROBLEM



$$u_x(0, y) = u_y(0, y) = 0$$

$$\sigma_y(x, 0) = \sigma_{xy}(x, 0) = 0$$

AND

$$\sigma_x \cos 30° - \sigma_{xy} \sin 30° = 0 \qquad 0 \leq x \leq l \text{ AND}$$

$$\sigma_{xy} \cos 30° - \sigma_y \sin 30° = 0 \qquad y = \operatorname{tg} 30° (x - l)$$

CASE (b)

$$u_x(0,y)=0 \quad ; \quad u_y(0,y)=0$$

$$\sigma_x(l,y) = \sigma_{xy}(l,y) = 0$$

$$\sigma_y(x,h) = \sigma_y(x,-h) = 0$$

$$\sigma_{xy}(x,h) = -\tau \quad \text{AND} \quad \sigma_{xy}(x,-h) = 0$$

3) # BOUNDARY CONDITIONS

LET US FIRST DEAL WITH $y=0 - \{(0,0)\}$

$\sigma_x = \sigma_y = \sigma_{xy} = 0$ $\boxed{\text{OK}}$

THE POINT $X=0$ AND $Y=0$ CAN NOT BE ANALYZED DIRECTLY DUE TO A CONCENTRATE FORCE WILL ENGENDER A SINGULARITY IN THE STRESS FIELD.

EQUILIBRIUM

$$\operatorname{div} T = \underline{0}$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0$$

LET US CHECK ...

$$\frac{\partial \sigma_x}{\partial x} = -\frac{4Pxy}{\pi(x^2+y^2)^2} + \frac{8Px^3y}{\pi(x^2+y^2)^3}$$

$$\frac{\partial \tau_{xy}}{\partial y} = -\frac{4Pxy}{\pi(x^2+y^2)^2} + \frac{8Pxy^3}{\pi(x^2+y^2)^3}$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = \frac{-8Pxy}{\pi(x^2+y^2)^2} + \frac{8Px\cancel{(x^2+y^2)}}{\pi(x^2+y^2)^{\cancel{3}}} = 0$$

SIMILARLY TO THE OTHER COMPONENT.

COMPATIBILITY EQUATIONS : JUST TO DERIVE ....

SUPERPOSITION :

$$\sigma_x = \sigma_x^{(1)} + \sigma_x^{(2)}$$

WHERE $\sigma_x^{(1)} = -\frac{Px^2y}{\pi(x^2+y^2)^2}$; $\sigma_x^{(2)} = -\frac{P\tilde{x}^2\tilde{y}^2}{\pi(\tilde{x}^2+\tilde{y}^2)^2}$

WHERE $\tilde{x} = (x-a)$

$\tilde{y} = y$

THEN $$\sigma_x^{(2)} = \frac{-P\,(x^2 - 2ax + a^2)\,y^2}{\pi\,\left((x^2 - 2ax + a^2) + y^2\right)^2} =$$

$$= \frac{-P\,\{x^2y^2 - 2axy + a^2y\}}{\pi\,(x^2 + y^2 - 2ax + a^2)}$$

THEN FOR $x \gg a$ OR $y \gg a$ $\sigma_x^{(2)} \to \sigma_x^{(1)}$ AND

WE RETRIEVE THE SOLUTION WHEN THERE WAS ONLY ONE

CONCENTRATED FORCE