Homework 1 - Vectors and Tensors

Handed out: Thurs., 20-09-2007 **Due to: Thurs., 27-09-2007**

1. Prove the following propositions

(a) If $\mathbf{S}$ is a symmetric tensor,

$$\mathbf{S}.\mathbf{T} = \mathbf{S}.\mathbf{T}^T = \mathbf{S}.\frac{1}{2}(\mathbf{T} + \mathbf{T}^T)$$

(b) If $\mathbf{W}$ is skew-symmetric ($\mathbf{W} = -\mathbf{W}^T$)

$$\mathbf{W}.\mathbf{T} = -\mathbf{W}.\mathbf{T}^T = \mathbf{W}.\frac{1}{2}(\mathbf{T} - \mathbf{T}^T)$$

(c) If $\mathbf{S}$ is symmetric and $\mathbf{W}$ skew

$$\mathbf{W}.\mathbf{S} = 0$$

2.Let $\underline{n}$ be a unit vector normal to a plane $\mathcal{P}$. Consider $\mathbf{P}$ and $\mathbf{R}$ two tensors which, respectively, project and reflect a vector in the plane $\mathcal{P}$.(remember the notes contain a characterization of a projection; moreover reflection is, from the geometrical standpoint, the specular image of a vector $\underline{v}$ when the plane is saw as a mirror, $R\underline{v}= \underline{v} - 2(\underline{v}.\underline{n})\underline{n}$).

(a) Draw a sketch representing a geometry view of both tensors. Write a formal expression to them based on $\underline{n}$ and using tensor product.
(b) Show that both tensors are really linear transformations.
(c) Verify that $\mathbf{P}$ is symmetric and $\mathbf{R}$ is an orthogonal tensor.

3. Compute the invariants, eigenvalue and eigenvectors of the second order tensor represented by the following matrix

$$[A] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & -3 \end{bmatrix}$$