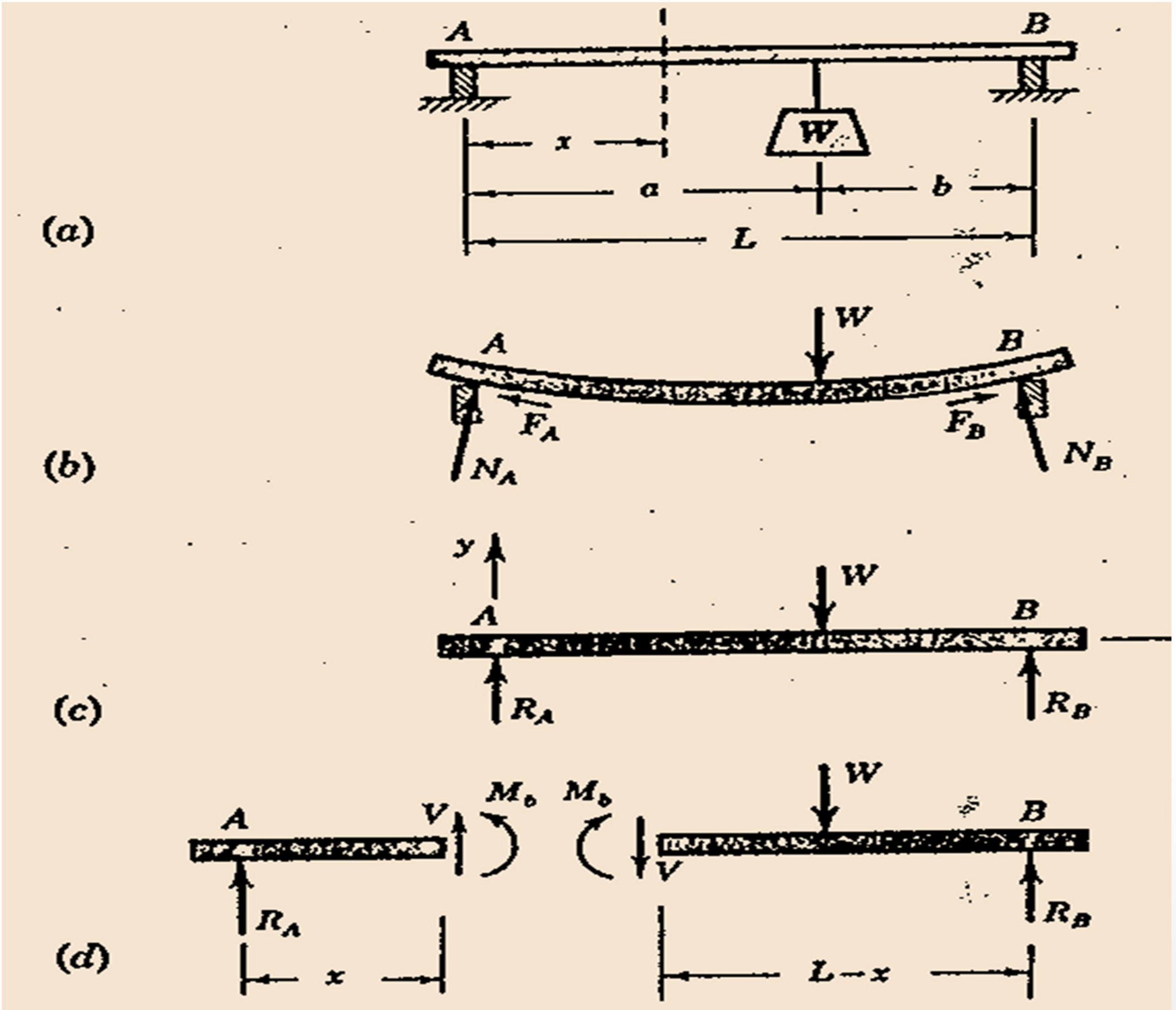
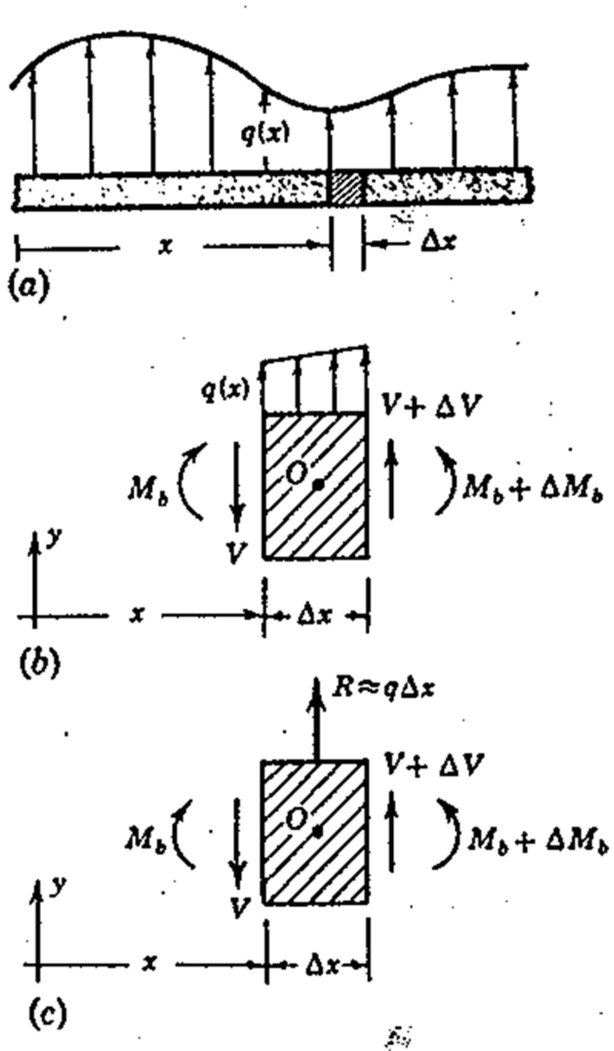
Flexão em Barras Esbeltas

Mecânica dos Sólidos I

Momento Fletor e Esforço Cortante



Equações Diferenciais de Equilíbrio



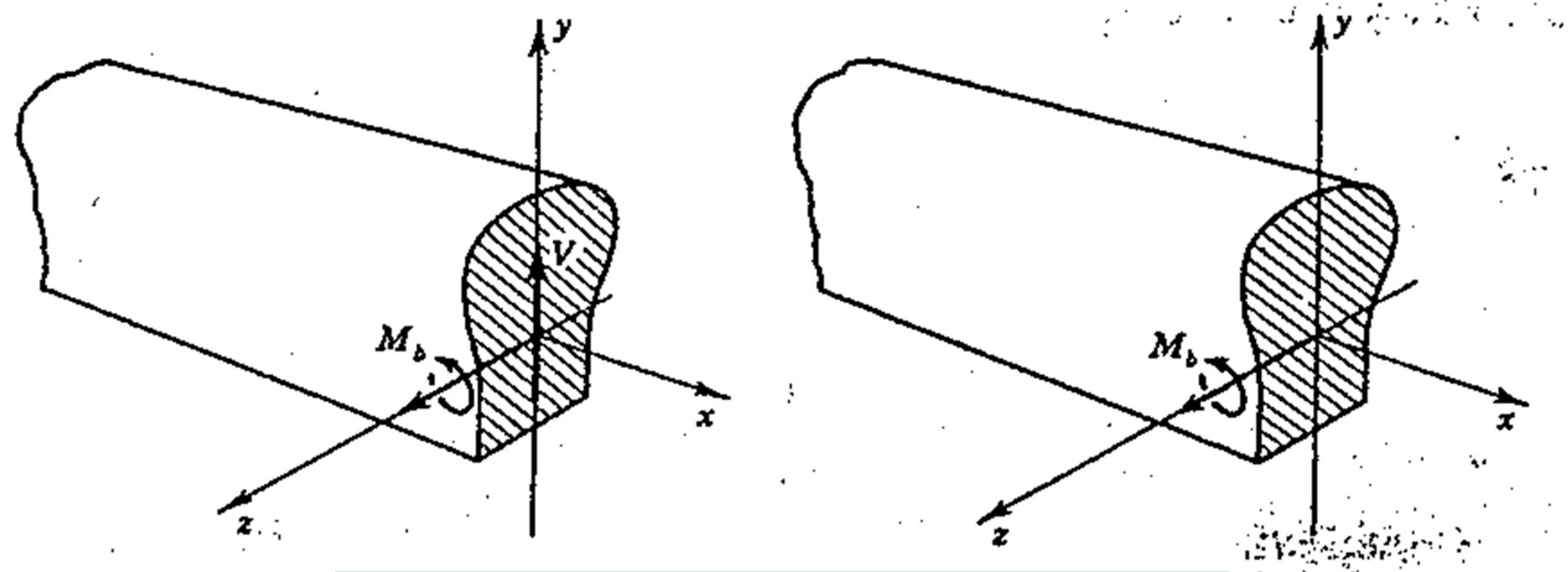
$$\Sigma F_y = V + \Delta V + q\,\Delta x - V = 0$$

$$\Sigma M_o = M_b + \Delta M_b + (V + \Delta V)\frac{\Delta x}{2} + V\frac{\Delta x}{2} - M_b = 0$$

$$\frac{dV}{dx} + q = 0$$

$$\frac{dM_b}{dx} + V = 0$$

Tensões de Flexões em Vigas (barras)



Seções Transversais Simétricas em relação à y

"Flexão Pura": Momento Fletor constante ao longo do eixo x

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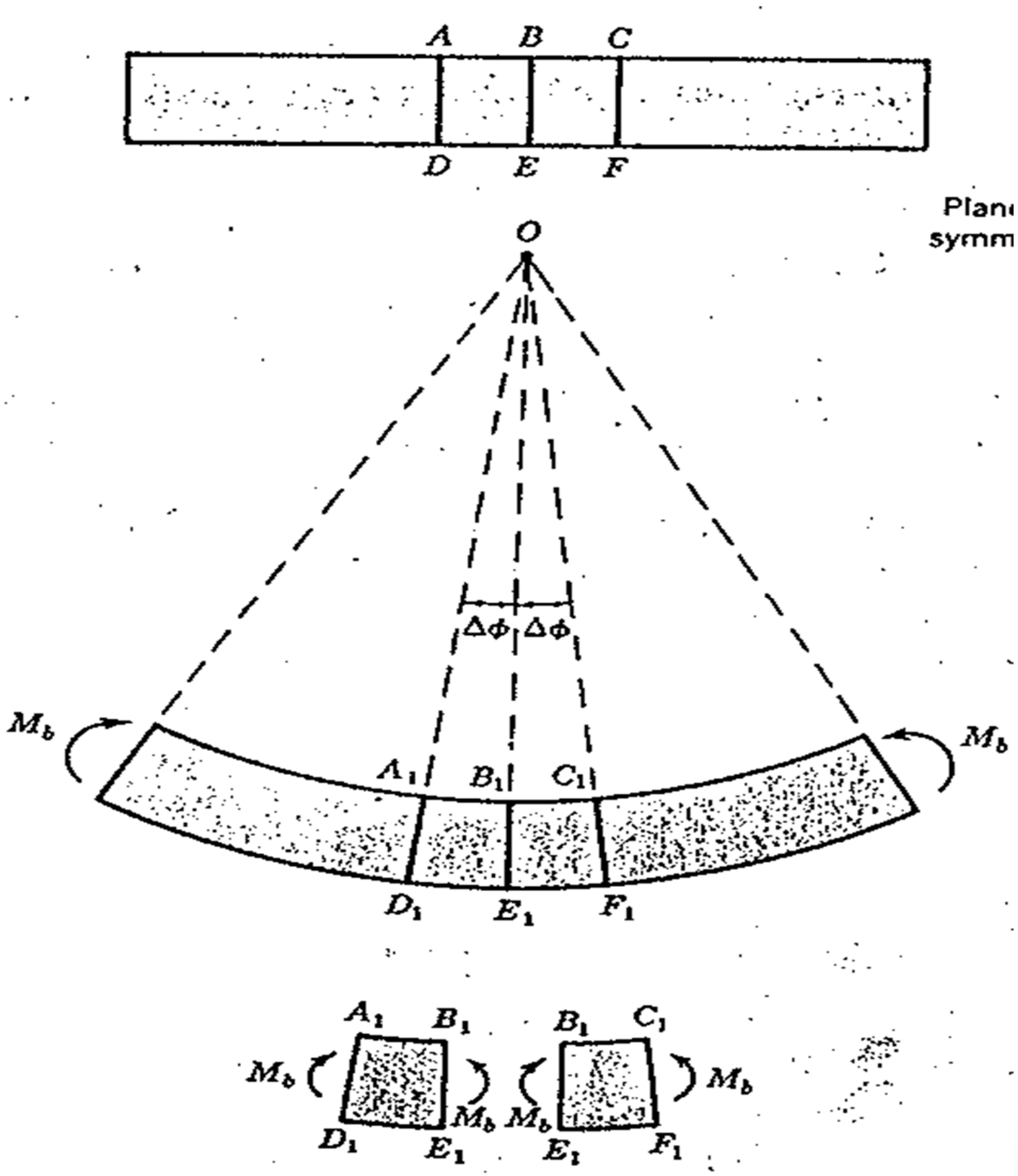
Geometria da Deformação



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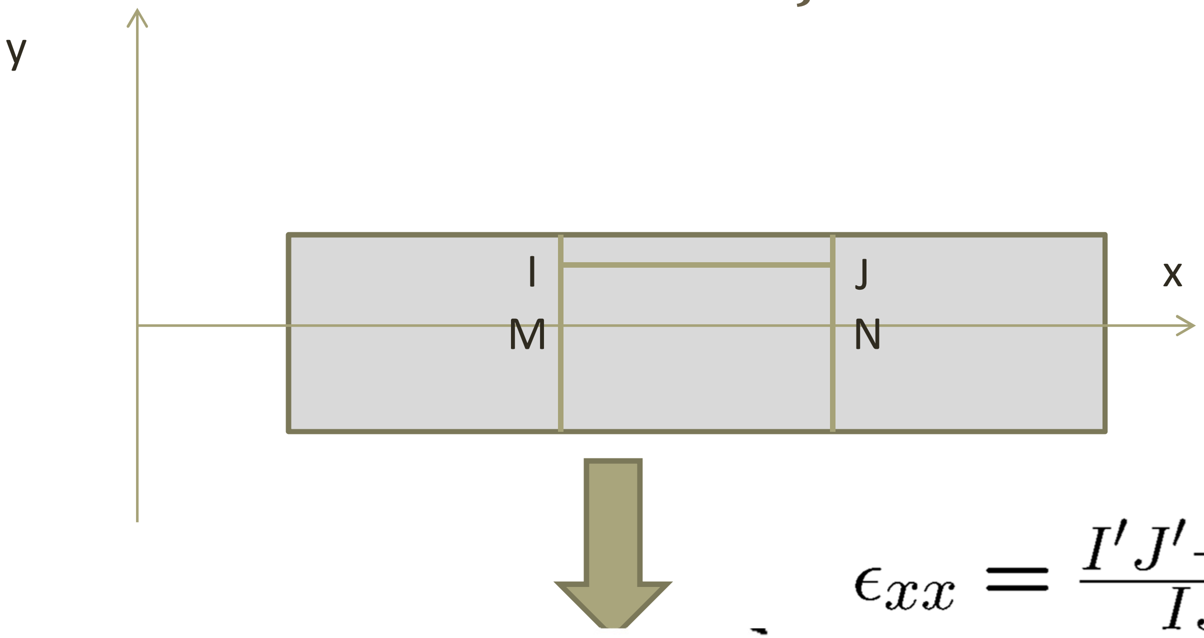
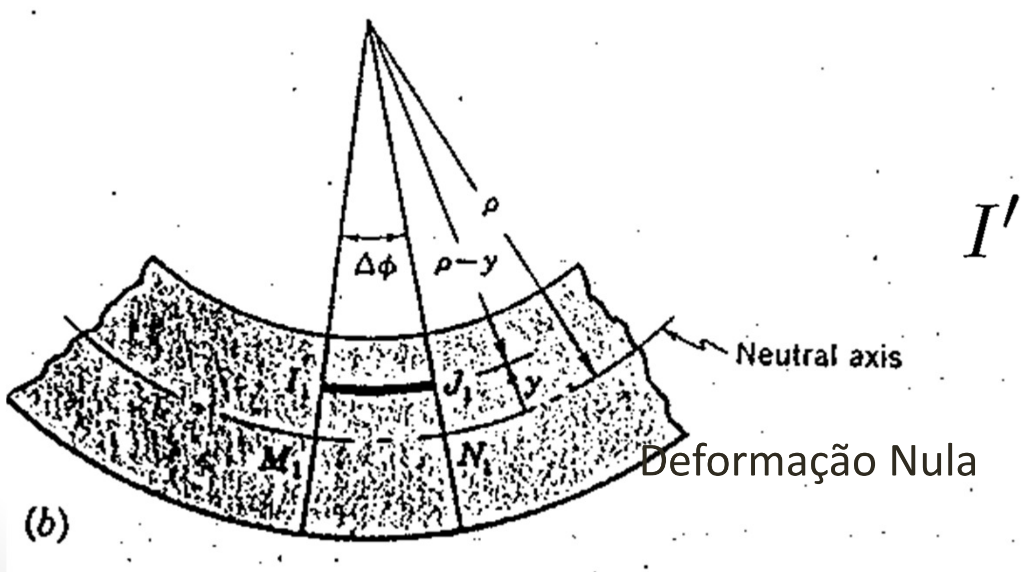
Geometria da Deformação





$$\epsilon_{xx} = \frac{I'J' - IJ}{IJ} = \frac{I'J' - M'N'}{M'N'}$$

$$M'N' = \rho\Delta\phi$$

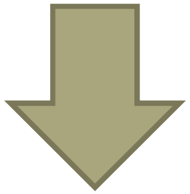
$$I'J' = (\rho - y)\Delta\phi$$

Estado de Deformações e Tensões

$$\epsilon_{xx} = -\frac{y}{\rho}$$

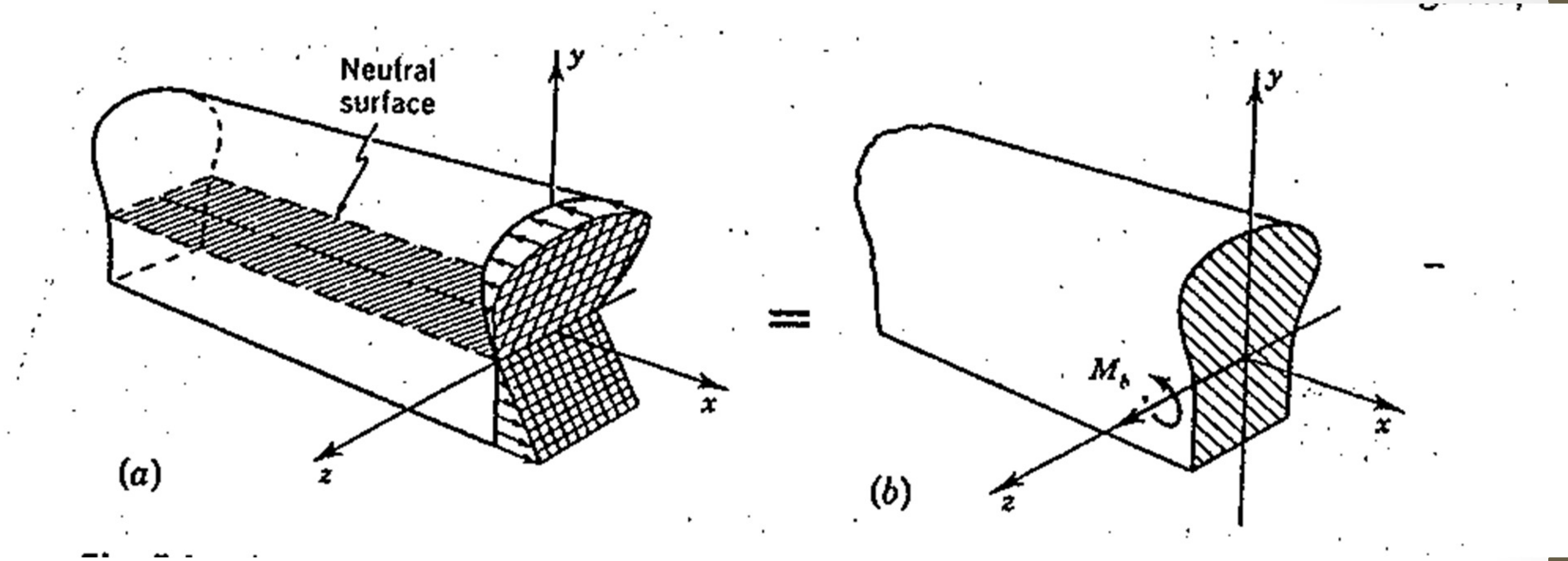
$$\epsilon_{xy} = 0$$

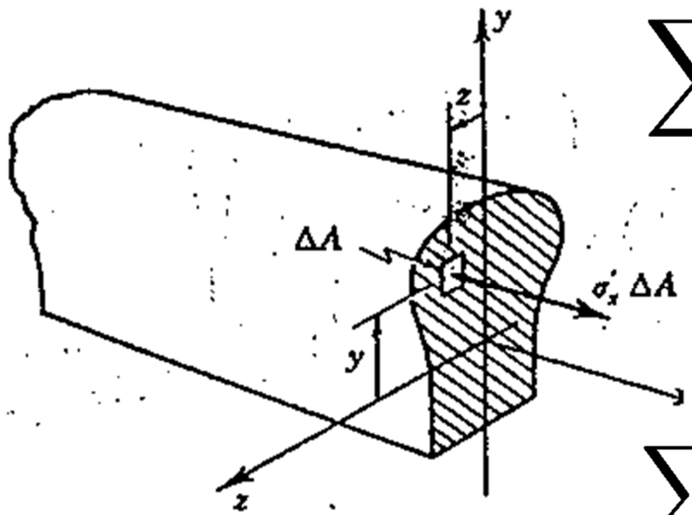
Simetria – Flexão Pura



$$\sigma_{xx} = -E\frac{y}{\rho}$$

Equilíbrio





$$\sum F_x = \int_A \sigma_{xx}\ dA = 0$$

$$\sum M_z = \int_A -y\ \sigma_{xx}\ dA = M$$

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Equilíbrio

$$\int_A \sigma_{xx} \; dA = -\int_A E \, \frac{y}{\rho} \; dA = 0$$

$$\Downarrow$$

$$\int_A y \; dA = \bar{y} = 0$$

Linha Neutra coincide com a linha que passa pelos centróides das seções transverais

Equilíbrio

$$-\int_A y\ \sigma_{xx}\ dA = \int_A E\ \frac{y^2}{\rho}\ dA = M$$

$$\frac{E}{\rho}\int_A\ y^2\ dA = M$$

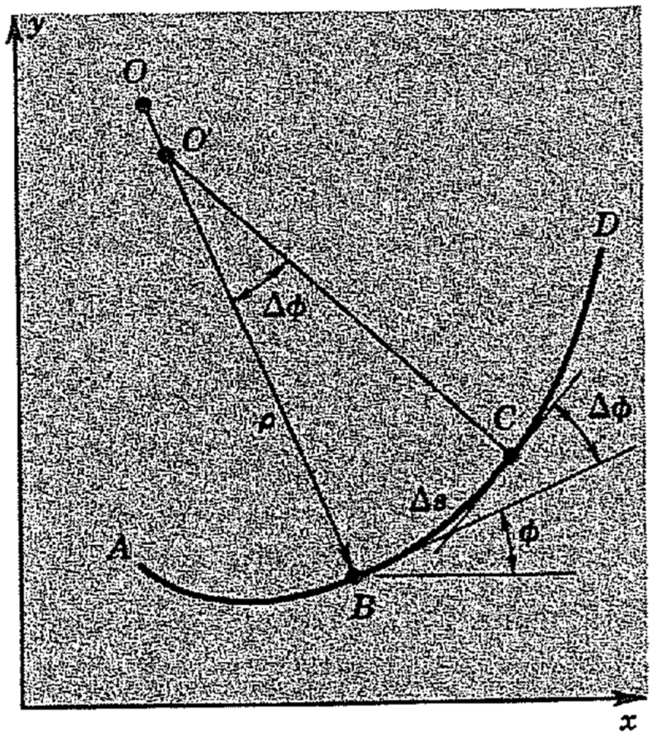
$$EI_{zz}\frac{d\phi}{dx} = M$$

Flexão

$$\epsilon_y = -\frac{M(x)y}{EI}$$

$$\sigma_y = -\frac{M(x)y}{I}$$

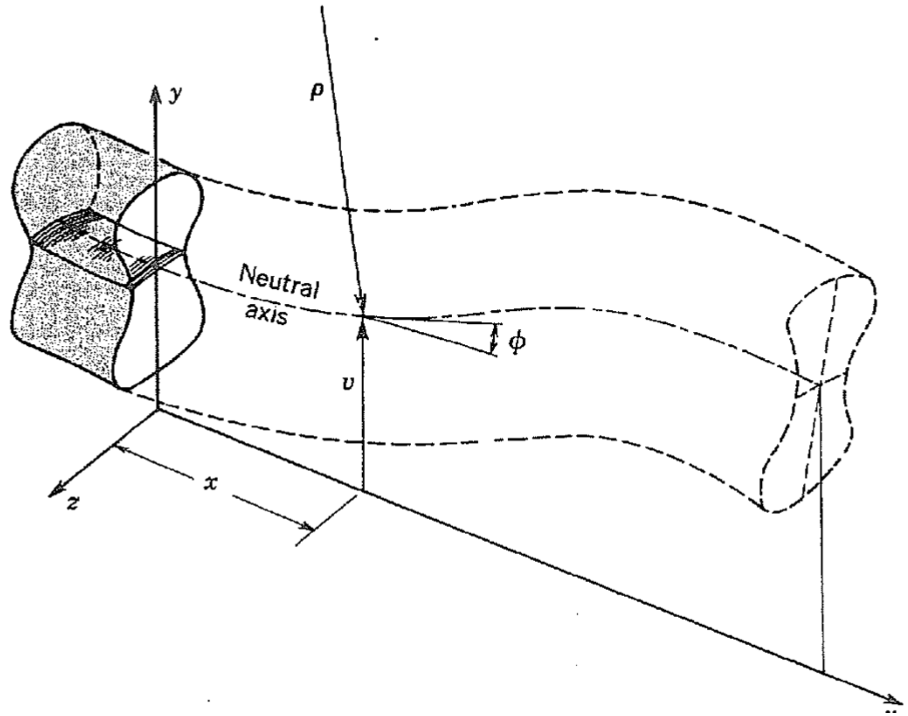
Geometria da Deformação

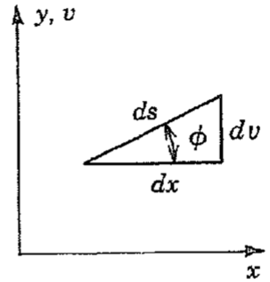


$$EI\frac{d\phi}{dS} = M$$

Geometria da Deformação: Campo de Deslocamentos

$$\frac{dv}{dx} = \tan \phi$$





$$\frac{dv}{dx} = \tan \phi$$

$$\frac{d^2v}{dx^2}\frac{dx}{ds} = \sec^2 \phi \frac{d\phi}{ds}$$

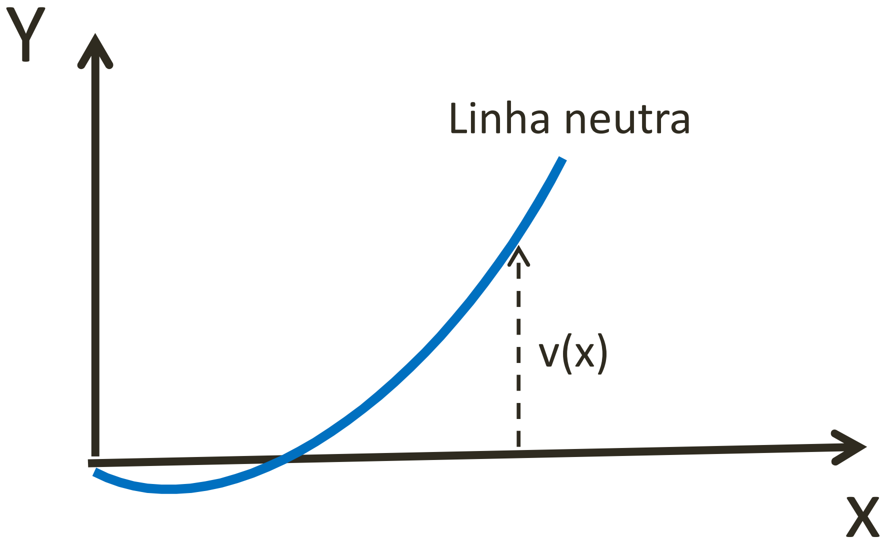
$$\frac{d\phi}{ds} = \frac{d^2v}{dx^2}\frac{dx}{ds}\cos^2 \phi$$

$$\cos \phi = \frac{dx}{ds} = \frac{1}{[1 + (dv/dx)^2]^{1/2}}$$

$$\frac{d\phi}{ds} = \frac{d^2v/dx^2}{[1 + (dv/dx)^2]^{3/2}}$$

$$\frac{d\phi}{ds} \approx \frac{d^2v}{dx^2}$$

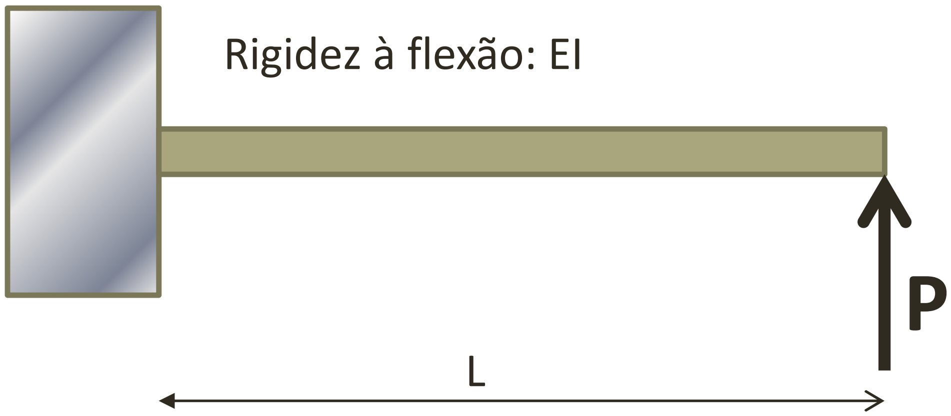
Geometria da Deformação : Configuração Deformada

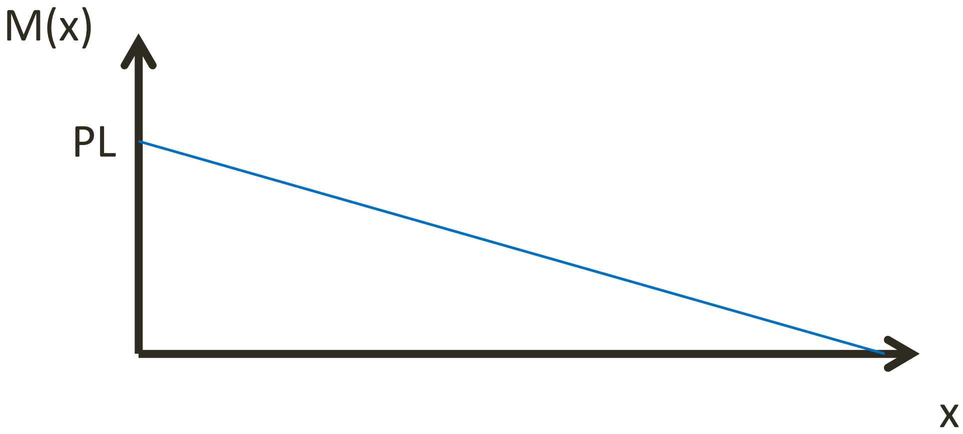


$$\frac{d^2v}{dx^2} = \frac{M_b}{EI}$$

A configuração deformada da viga é definida completamente pela posição da linha neutra, ou seja pela função v(x) (seções transversais permanecem planas e ortogonais à linha neutra)

Exemplo
(caso estaticamente determinado)





Integrando a relação Momento-Curvatura

$$\frac{dM}{dx} = \frac{M}{EI} \qquad M(x) = P(L - x)$$

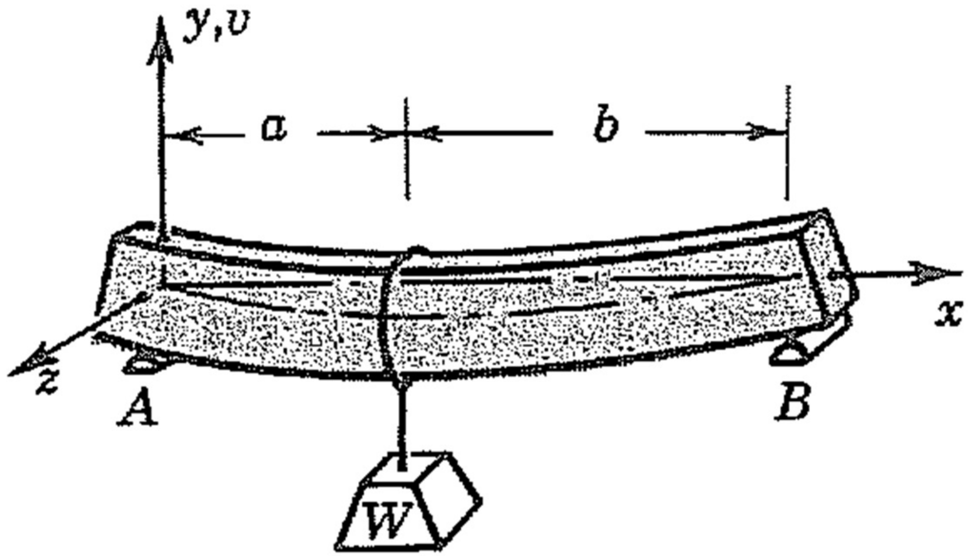
Condições de apoio (contorno):

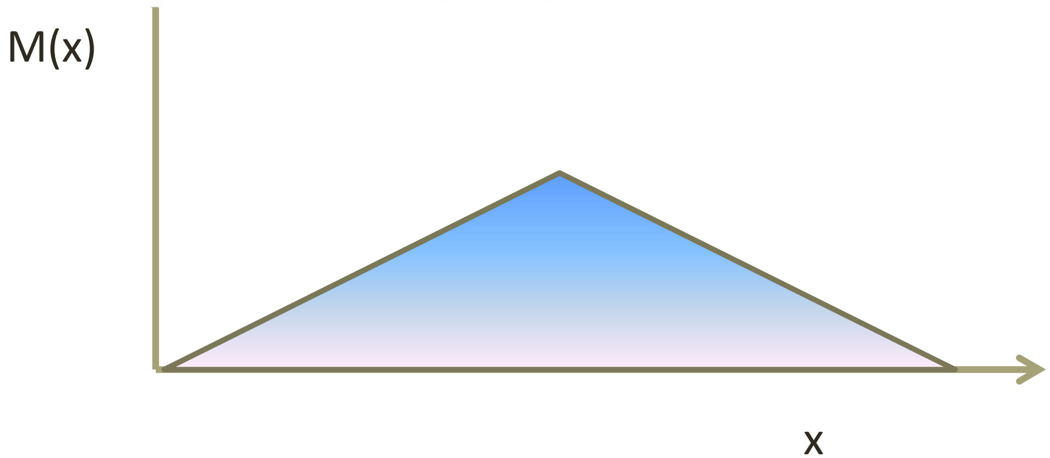
$$v(0) = 0 \qquad \frac{dv}{dx}(0) = 0$$

$$\frac{dv}{dx} = -\frac{1}{2}P(L - x)^2 + C_1 \rightarrow C_1 = \frac{PL^2}{2}$$

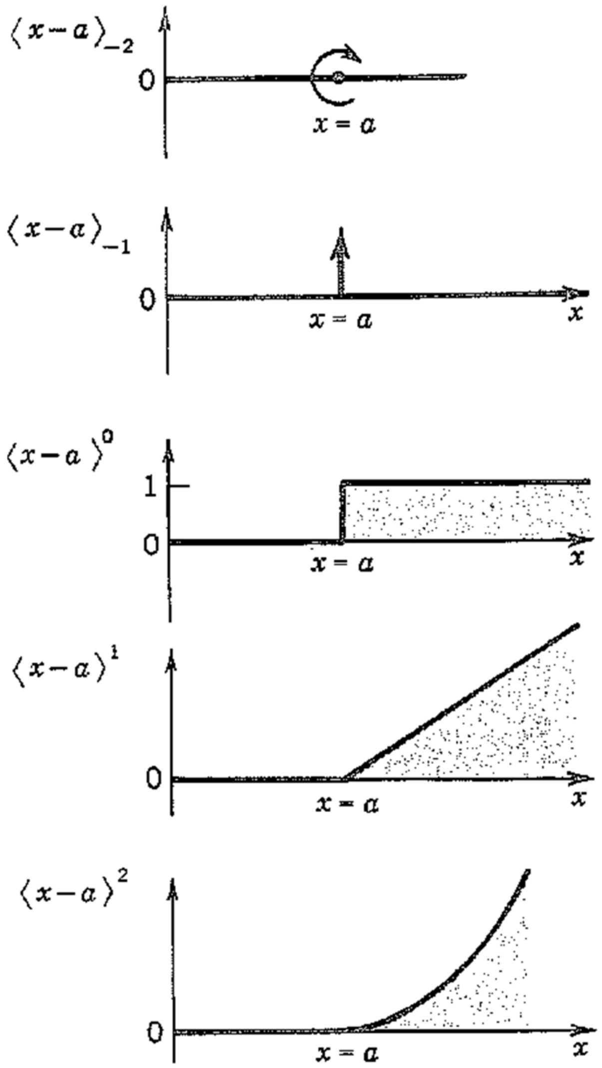
$$v = \frac{1}{6}P(L - x)^3 + C_1 x + C_2 \rightarrow C_2 = -\frac{PL^3}{6}$$

Exemplo
(funções de singularidade)





Funções de Singularidade

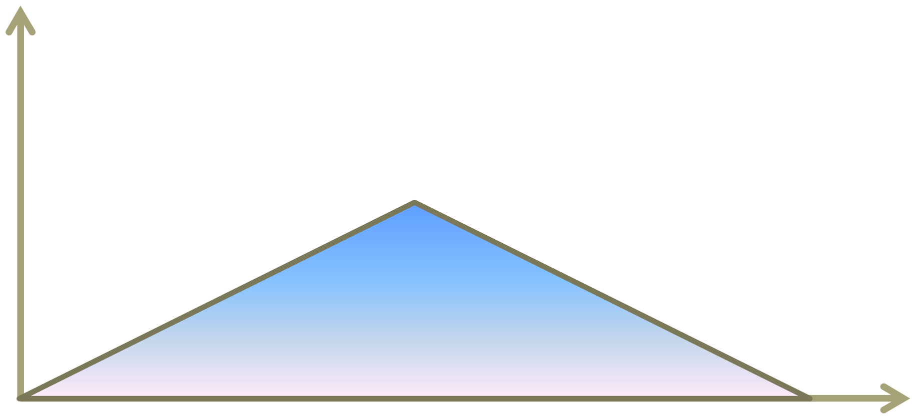


$$< y >= 0, se \;\; y < 0$$

$$< y >= y, se \;\; y \geq 0$$

$$\int_{-\infty}^{x} \langle x-a \rangle^{n} \, dx = \frac{\langle x-a \rangle^{n+1}}{n+1} \qquad n \geqq 0$$

Distribuição de Momento Fletor



$$M(x) = \frac{Wb}{L}x, se\ x < a$$

$$M(x) = \frac{Wb}{L}x - W(x-a), se\ x \geq a$$

$$M(x) = \frac{Wb}{L}x - W < x - a >$$

Integrando a relação momento curvatura

$$EI\frac{dv}{dx} = \frac{Wb}{L}\frac{x^2}{2} - W\frac{\langle x-a\rangle^2}{2} + c_1$$

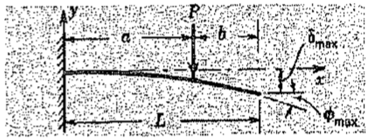
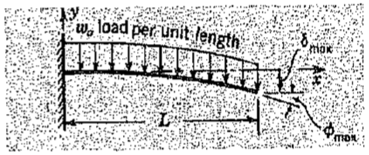
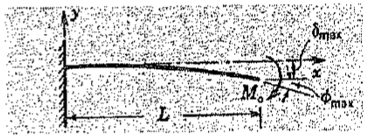
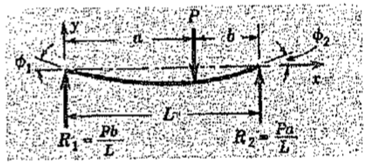
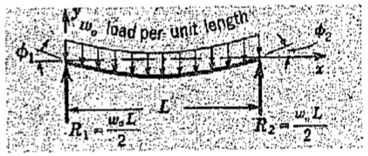
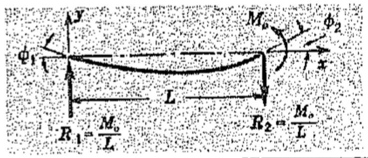
$$EIv = \frac{Wb}{L}\frac{x^3}{6} - W\frac{\langle x-a\rangle^3}{6} + c_1x + c_2$$

$v = 0$ at $x = 0$ and at $x = L$

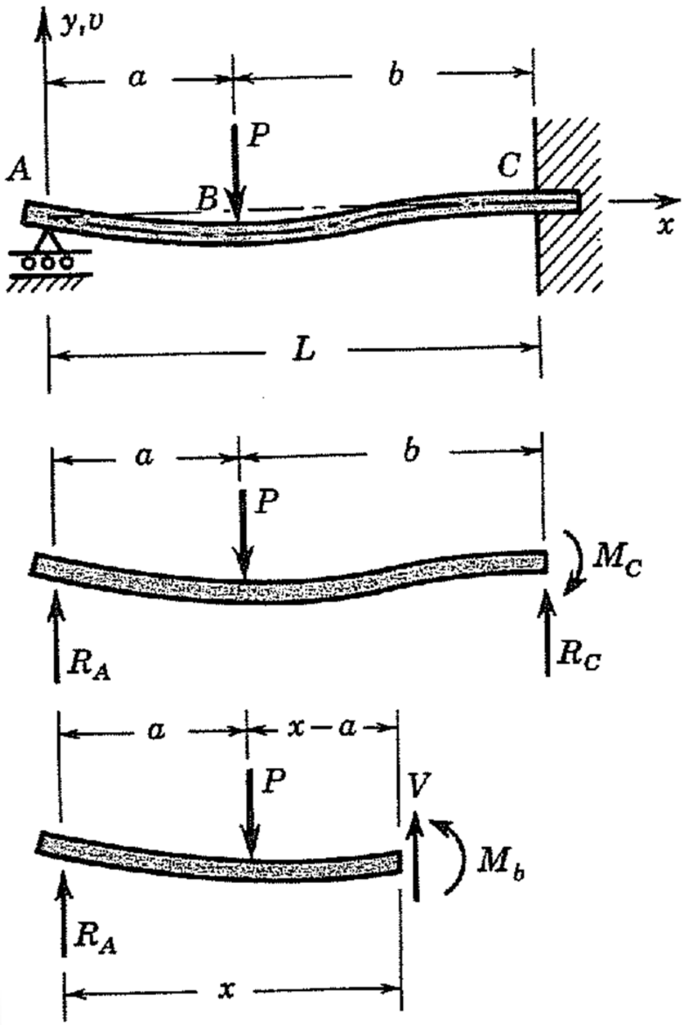
$$v = -\frac{W}{6EI}\left[\frac{bx}{L}(L^2 - b^2 - x^2) + \langle x-a\rangle^3\right]$$

Table 8.1 Deflection formulas for uniform beams

δ is positive downward

1.		$\delta = \frac{P}{6EI}(\langle x-a\rangle^3 - x^3 + 3x^2 a)$	$\delta_{max} = \frac{Pa^2(3L-a)}{6EI}$	$\phi_{max} = \frac{Pa^2}{2EI}$
2.		$\delta = \frac{w_o x^2}{24EI}(x^2 + 6L^2 - 4Lx)$	$\delta_{max} = \frac{w_o L^4}{8EI}$	$\phi_{max} = \frac{w_o L^3}{6EI}$
3.		$\delta = \frac{M_o x^2}{2EI}$	$\delta_{max} = \frac{M_o L^2}{2EI}$	$\phi_{max} = \frac{M_o L}{EI}$
4.		$\delta = \frac{Pb}{6LEI}\left[\frac{L}{b}\langle x-a\rangle^3 - x^3 + (L^2-b^2)x\right]$	$\delta_{max} = \frac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}\,LEI}$ at $x = \sqrt{\frac{L^2-b^2}{3}}$	$\phi_1 = \frac{Pab(2L-a)}{6LEI}$ $\phi_2 = \frac{Pab(2L-b)}{6LEI}$
5.		$\delta = \frac{w_o x}{24EI}(L^3 - 2Lx^2 + x^3)$	$\delta_{max} = \frac{5w_o L^4}{384EI}$	$\phi_1 = \phi_2 = \frac{w_o L^3}{24EI}$
6.		$\delta = \frac{M_o L x}{6EI}\left(1 - \frac{x^2}{L^2}\right)$	$\delta_{max} = \frac{M_o L^2}{9\sqrt{3}\,EI}$ at $x = \frac{L}{\sqrt{3}}$	$\phi_1 = \frac{M_o L}{6EI}$ $\phi_2 = \frac{M_o L}{3EI}$

Exemplos:
Problemas Estaticamente Indeterminados



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Equilíbrio :

$$R_C = P - R_A$$
$$M_C = Pb - R_A L$$

Distribuição de Momento Fletor

$$M_b = R_A x - P\langle x - a\rangle^1$$

Condições de Apoio (contorno):

$$v = 0 \qquad \text{at } x = 0$$

$$v = 0 \qquad \text{at } x = L$$

$$\frac{dv}{dx} = 0 \qquad \text{at } x = L$$

3 condições : 2 constantes de integração + “indeterminação”

Integrando a relação momento-curvatura

$$EI\,\frac{d^2v}{dx^2} = M_b = R_A x - P\langle x - a\rangle^1$$

$$EI\,\frac{dv}{dx} = R_A\,\frac{x^2}{2} - P\,\frac{\langle x - a\rangle^2}{2} + c_1$$

$$\frac{dv}{dx} = 0 \qquad \text{at } x = L$$

$$c_1 = \frac{Pb^2}{2} - \frac{R_A L^2}{2}$$

$$EIv = R_A \frac{x^3}{6} - P\frac{\langle x-a\rangle^3}{6} + \frac{Pb^2x}{2} - \frac{R_A L^2 x}{2} + c_2$$

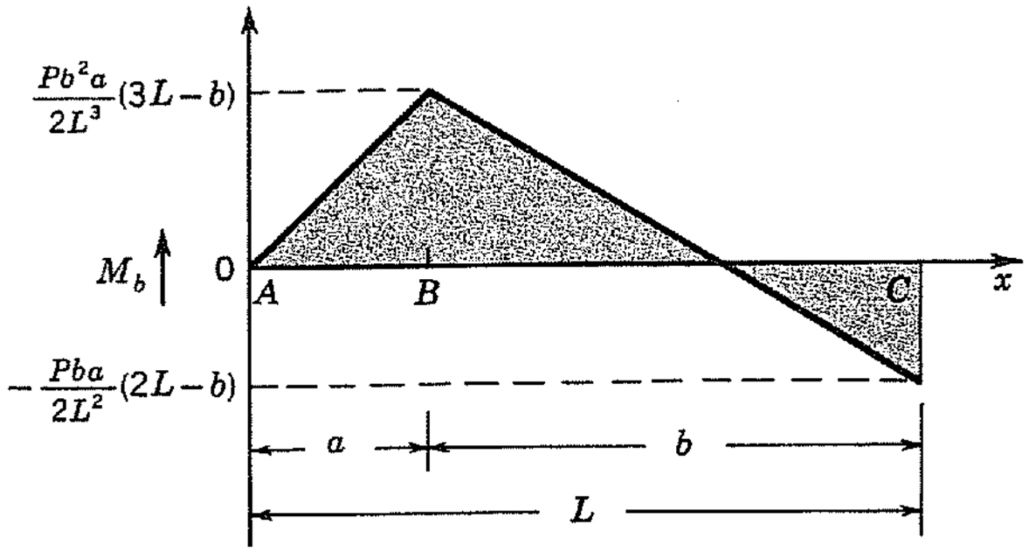
$v = 0$ at $x = 0$

$v = 0$ at $x = L$

$$c_2 = 0.$$

$$0 = R_A \frac{L^3}{6} - P\frac{b^3}{6} + \frac{Pb^2L}{2} - \frac{R_A L^3}{2}$$

$$R_A = \frac{Pb^2}{2L^3}(3L - b)$$



$\frac{Pb^2a}{2L^3}(3L-b)$
M_b
0
A
B
C
x
$-\frac{Pba}{2L^2}(2L-b)$
a
b
L

Superposição

$$M = EI\frac{dv^2}{dx^2}$$

$$M = M_1 + M_2 + M_3 + \ldots$$

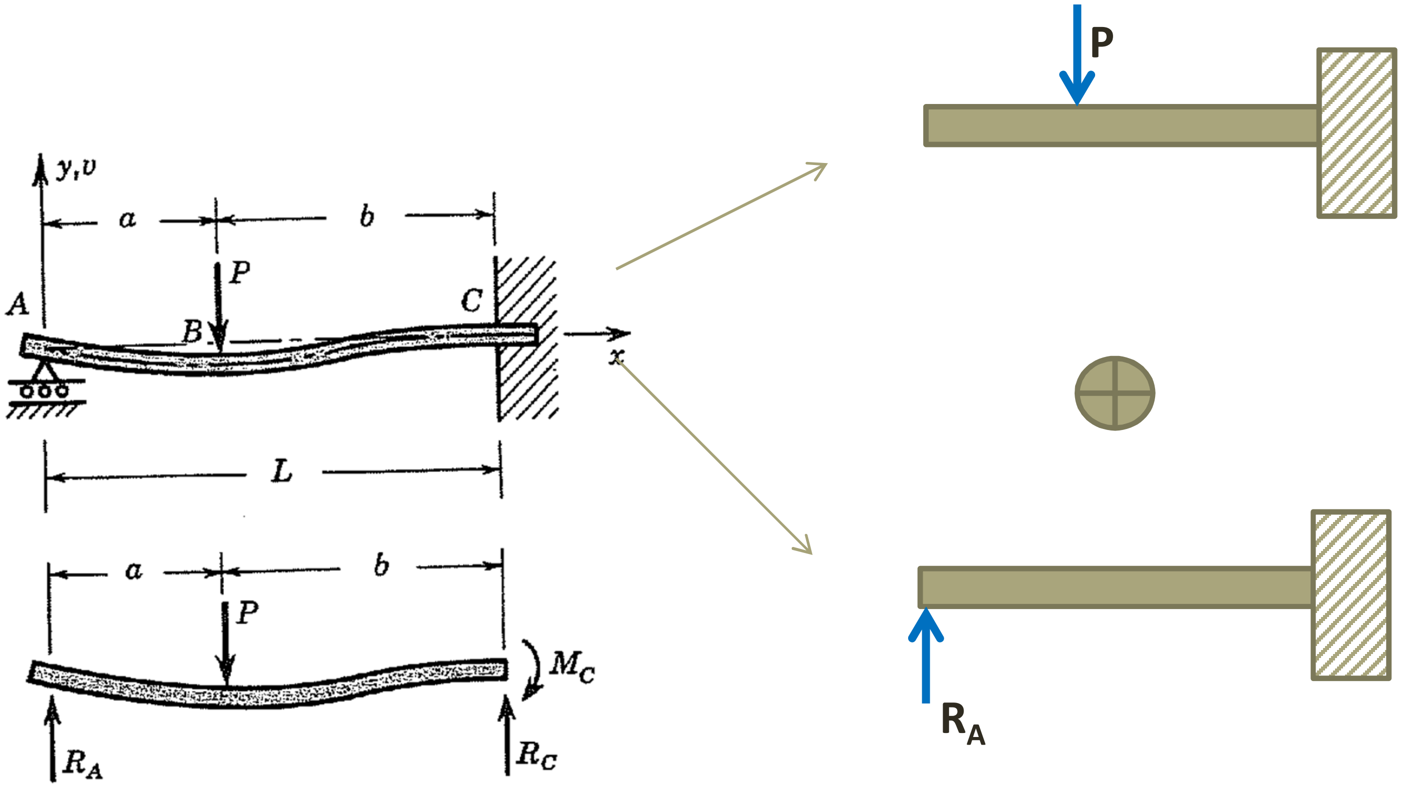
$$\Downarrow$$

$$v = v_1 + v_2 + v_3 + \ldots$$

$$M_i = EI\frac{dv_i^2}{dx^2} \Longrightarrow \sum_i M_i = EI\sum_i \frac{dv_i^2}{dx^2}$$

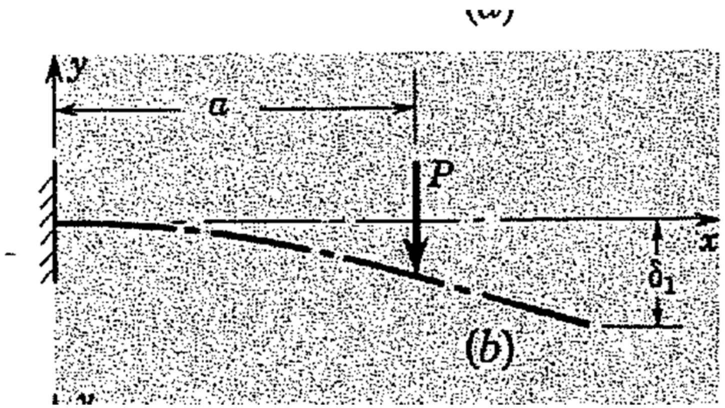
Observação Importante: Condições de apoio

Revisitando o exemplo anterior



Importante : vínculo cinemático   v(0) = 0

Dados da “ tabela “

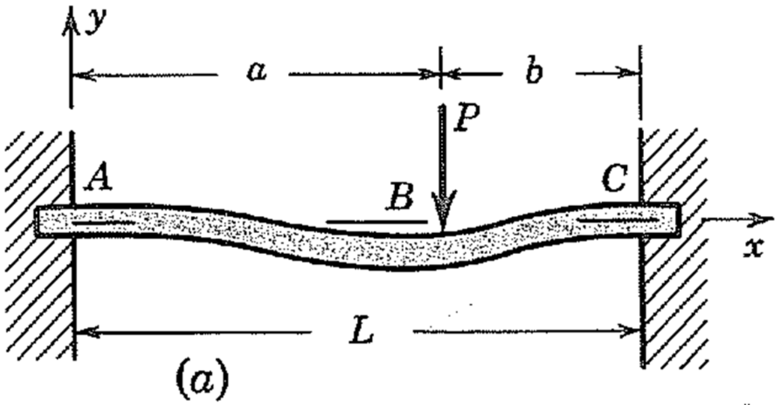


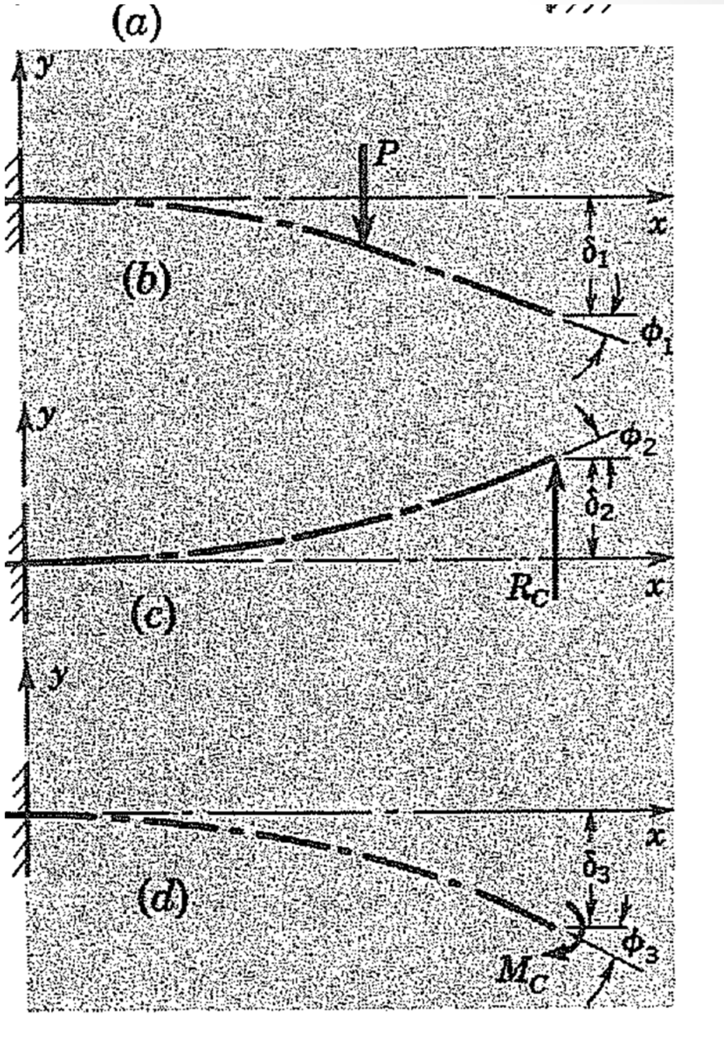
$$\delta = \frac{P}{6EI}(\langle x-a\rangle^3 - x^3 + 3x^2a)$$

$$v(x') = -\delta(L-x)$$

$$v = v_1 + v_2$$

$$v(0) = \frac{1}{6EI}(2L^3R_A - P[(L-a)^3 - L^3 + 3L^2a]) = 0$$





$$\delta_1 - \delta_2 + \delta_3 = 0$$

$$\phi_1 - \phi_2 + \phi_3 = 0$$