# CHAPTER 3

# STRESS IN A SOLID

# AND

# EQUILIBRIUM

THE PREVIOUS CHAPTER DEALS WITH THE KINEMATICS (GEOMETRY) OF DEFORMATION WITHOUT ADDRESSING THE AGENTS BEING RESPONSIBLE FOR IT. THE PRESENT CHAPTER IS DEVOTED TO INVESTIGATING WHAT CAUSES THE MOTION. INDEED, THIS STUDY PROVIDES A QUANTITATIVE METHOD TO DESCRIBE DEFORMABLE BODIES INTERACTION WITH THE HELP OF THE CONCEPTS OF FORCE, TRACTION VECTOR AND STRESS TENSOR.

AS OFTEN THE MAJOR CONTRIBUTER TO FAILURE IS THE MAXIMUM STRESS, ELASTICITY THEORY PRIMARY APPLICATION CONSISTS ON OBTAINING THE STRESS DISTRIBUTION ACROSS THE BODY. DUE THE CHARACTERISTICS OF MANY APPLICATIONS (THAT IS TRUE ALSO THAT IT IS MOTIVADED BY DIDACTICAL ISSUES), THE BODY IS CONSIDERED AT EQUILIBRIUM, WHICH IMPLIES THAT, CONSIDERING AN INFINITESIMAL ELEMENT AROUND ANY POINT, THE RESULTANTS MOMENT AND FORCE MUST BE ZERO.

IT IS WORTH MENTIONING THAT THE DEVELOPMENTS WITHIN THIS CHAPTER <u>DO NOT</u> REQUIRE THAT THE MATERIAL BE ELASTIC, AND THUS, IN PRINCIPLE, THE RESULTS APPLY TO A BROADER CLASS OF PROBLEMS.

## 3.1) Forces and Stress

During a motion mechanical interactions between parts of a body or between a body and its environment are expressed through forces. In general terms, we recognize three different types of forces, namely

(i) Contact forces between "separates" parts of a body (internal forces);

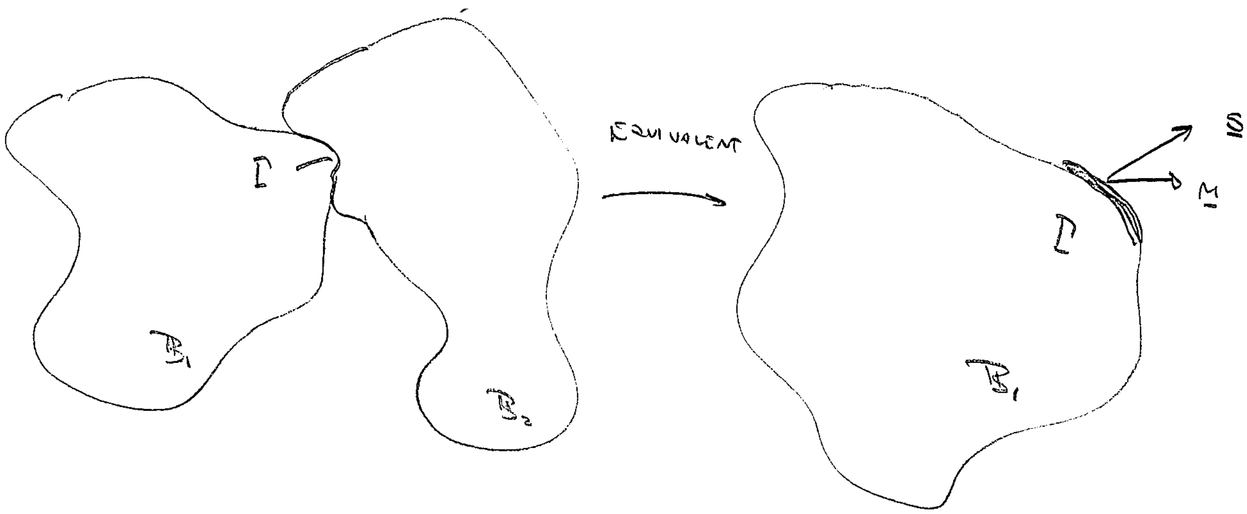
(ii) Contact forces exerted on the boundary of a body by external agents; (surface forces)

(iii) Forces exerted on the interior points of a body by the environment. (Body (volume) forces; e.g. gravity)

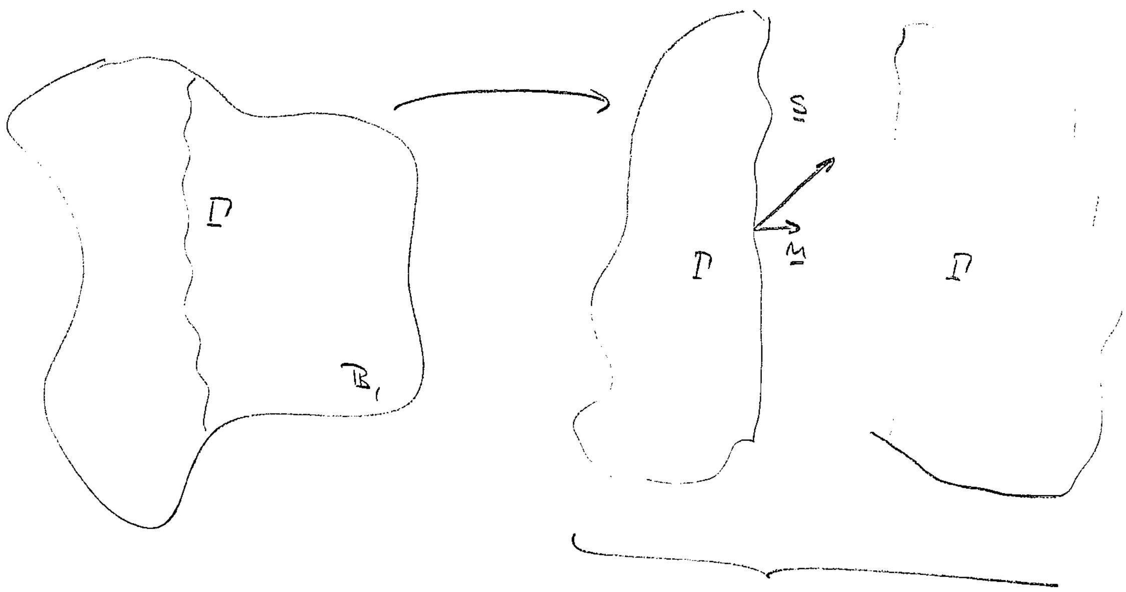
Cauchy's hypothesis: (contact forces) $\underline{S}(\underline{m}, x; t)$ is the force, per unit area, exerted across $\Gamma$ (a oriented surface with normal given by $\underline{m}$) upon the material on the negative side of $\Gamma$

# Cauchy's Hypothesis Illustration

## External Forces




## Internal Forces




Separate parts of $\mathbb{B}_1$

THEREFORE, THE RESULTANT FORCE AND MOMENT ON A PART $\mathcal{P}$ OF $\mathbb{B}$ (NOTE THAT $\mathcal{P}$ MAY COINCIDE WITH $\mathbb{B}$) IS GIVEN BY

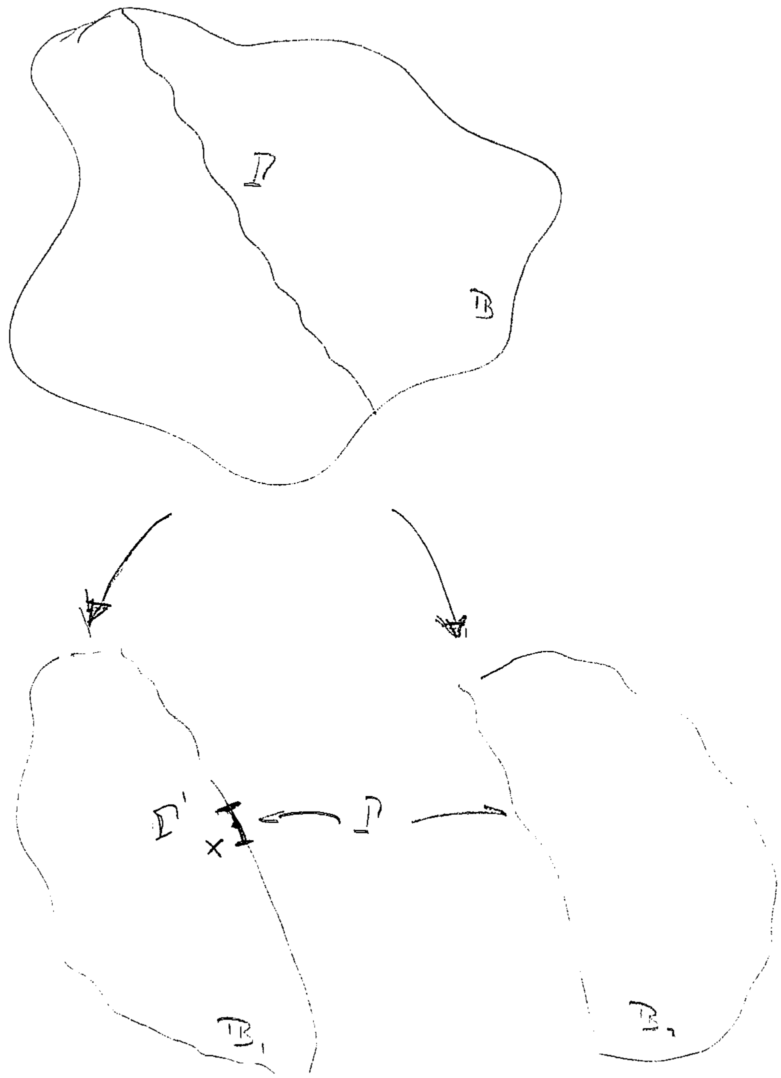
$$\underline{f}(\mathcal{P}) = \int_{\partial\mathcal{P}} \underline{s}\, d\Gamma + \int_{\mathcal{P}} \underline{b}\, dV$$

$$\underline{m}(\mathcal{P}) = \int_{\partial\mathcal{P}} (\underline{r} \wedge \underline{s})\, d\Gamma + \int_{\mathcal{P}} (\underline{r} \wedge \underline{b})\, dV$$

WHERE $\underline{b}$ DENOTES THE BODY FORCE, PER UNIT VOLUME, AND $\underline{r}$ IS THE POSITION VECTOR WITH RESPECT TO THE ORIGIN OF COORDINATES

(REMEMEMBER $\times$ DENOTES THE VECTOR PRODUCT)

HEADING BACK TO INTERNAL FORCES, IT IS IMPORTANT TO INVESTIGATE DEEPER ITS NATURE AND, FURTHER, QUANTIFY THEIR DISTRIBUTION WITHIN THE SOLID BODY. SO, LET US AGAIN DIVIDE BODY $\mathbb{B}$ INTO TWO PARTS




(IT SEEMS SOMEHOW ARBITRARY, AND IT IS ALTOUGH OFTEN GUIDED BY THE CIRCUMSTANCES OF THE APPLICATION)

LET US NOW FIX THE ATTENTION TO THE NEIGHBOR REGION OF x ON $\Gamma$ (DENOTED AS $\Gamma'$). THE CONTACT RESULTANT FORCE ACTING ON $\Gamma'$ IS

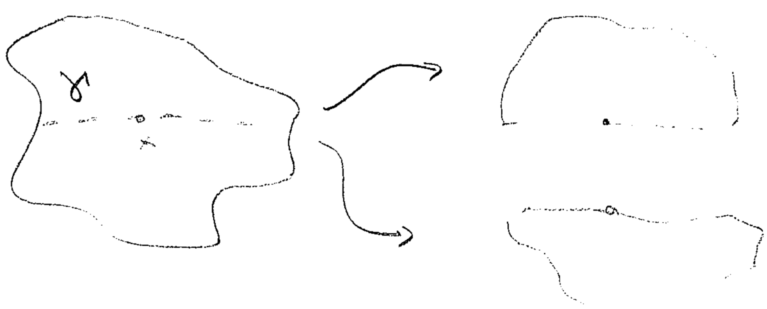
$$\underline{F}_{\Gamma'} = \int_{\Gamma'} \underline{s} \, d\Gamma$$

PASSIN THROUGH A LIMITING PROCESS

$$\underline{F}(x) = \lim_{\Gamma' \downarrow 0} \int_{\Gamma'} \underline{s} \, d\Gamma$$

RETRIEVING, IN THAT WAY, THE FIGURE OF A PUNCTUAL FORCE.

NOW, REMEMBER THE ABOVE RESULT WAS OBTAINED LAYING ON "AN ABRITARY CHOICE" FOR $\Gamma$. THUS



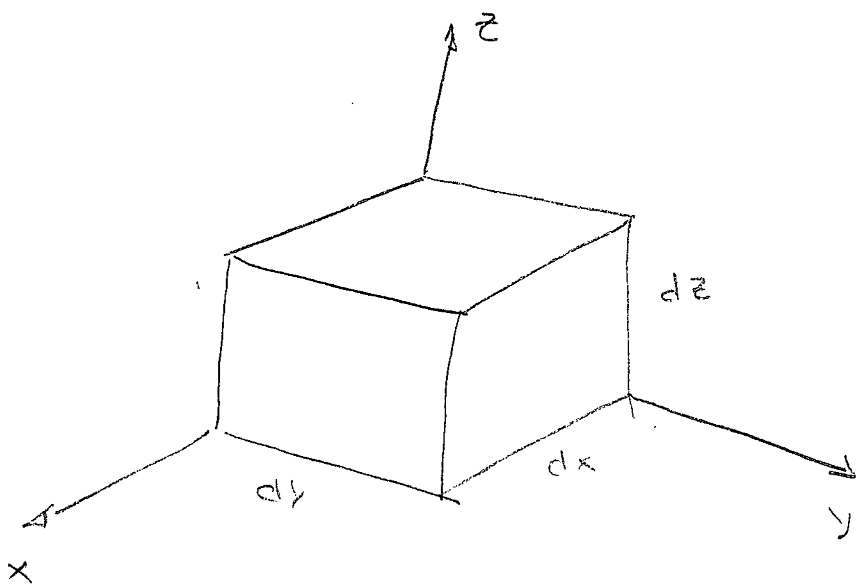

NOTE THAT, AS S DEPENDS ON x AN $\underline{n}$ WE WILL ARRIVE TO A DIFFERENT FORCE ACTING IN x...

THEREFORE, THE NATURE OF INTERNAL FORCES IS COMPLEX AND CAN'T BE REPRESENTED BY A VECTOR FIELD.

INDEED, WE INTRODUCE THE VECTOR $\underline{t}$ WHICH IS A FUNCTION OF THE MATERIAL POINT X AND OF THE NORMAL VECTOR $\underline{m}$ (WHICH, IN TURN, DEFINES THE SECTION OF THE BODY). NOTE THAT, FROM THE NEWTON'S ACTION-REACTION LAW

$$\underline{t}(x, \underline{m}) = - \underline{t}(x, -\underline{m})$$

LET US, ONCE AGAIN, INVESTIGATE THE NATURE OF A DEFORMABLE SOLID BY LOOKING AT AN INFINITESIMAL NEIGBORHOOD OF A POINT X TAKING A CUBIC FORM



THEN WE CAN COMPUTE $\underline{t}$ VECTOR FOR ANY OF THE 6 FACES OF THE CUBE, LABELLING EACH ONE BY ITS NORMAL (e.g.: FACE X IS THE ONE WITH $\underline{n} = \underline{e}_x$; AND FACE $-X$ CORRESPONDS TO $\underline{n} = -\underline{e}_x$):

$$\underline{t}(x, \underline{e}_x) = t_x^{e_x} \underline{e}_x + t_y^{e_x} \underline{e}_y + t_z^{e_x} \underline{e}_z$$

$$\underline{t}(x, \underline{e}_y) = t_x^{e_y} \underline{e}_x + t_y^{e_y} \underline{e}_y + t_z^{e_y} \underline{e}_z$$

$$\underline{t}(x, \underline{e}_z) = t_x^{e_z} \underline{e}_x + t_y^{e_z} \underline{e}_y + t_z^{e_z} \underline{e}_z$$

WHERE EACH SCALAR $t_i^j$ IS THE $i$-th COMPONENT OF THE $\underline{t}$ VECTOR COMPUTED IN THE $j$-th DIRECTION. WE WILL DISTINGUISH TWO SUB-SETS: NORMAL COMPONENTS (THAT WILL GIVE RAISE TO THE LABEL NORMAL STRESS) AND SHEAR COMPONENTS (⇒ SHEAR STRESS). THE FORMER CONTAINS THE COMPONENTS IN THE $\underline{n}$ DIRECTION. THE SHEAR COMPONENTS LIE ON THE SURFACE OF THE SECTION.

CONSIDERING NOW THAT $\underline{t} = \lim_{\Gamma \downarrow 0} \int_\Gamma \underline{s} \, d\Gamma$ (SO $\underline{s}$ WILL BE THE TRACTION VECTOR, WHICH REFLECTS DISTRIBUTION OF FORCE WITHIN $\Gamma$) THE ABOVE EXPRESSIONS, WE INTRODUCE THE STRESS tensor $T$ SUCH THAT ITS REPRESENTATION IN THE COORDINATE FRAME IS GIVEN BY THE MATRIX

$$[T]_{ij} = \begin{bmatrix} s_x^{\underline{e}x} & s_x^{\underline{e}y} & s_x^{\underline{e}z} \\ s_y^{\underline{e}x} & s_y^{\underline{e}y} & s_y^{\underline{e}z} \\ s_z^{\underline{e}x} & s_z^{\underline{e}z} & s_z^{\underline{e}z} \end{bmatrix}$$

(THEREFORE IT HAS UNITS OF FORCE PER AREA)

IT IS STRAIGHTFORWARD TO PROVE THAT FOR ANY SECTION OF THE BODY DEFINED BY THE NORMAL $\underline{m}$

$$\boxed{\underline{s}(x, m) = T(x)\, \underline{m}}$$

(IT IS A GOOD IDEA CHECK THE ABOVE RESULT IN ONE BOOK OF THE REFERENCE LIST)

P.S.: IT WILL BE SHOWN LATER THAT THE STRESS TENSOR IS SYMMETRICAL AND, BY USING THE SPECTRAL THEOREM, IT CAN BE THUS REPRESENTED IN THE COORDINATE SYSTEM FORMED BY THE PRINCIPAL DIRECTIONS (GIVEN BY THE EIGENVECTORS) AS

$$[T] = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

WHERE $\sigma_1$, $\sigma_2$ AND $\sigma_3$ ARE REAL NUMBERS AND THE EIGENVALUES, e.g.

$$T \phi_i = \sigma_i \phi_i \qquad (i = 1, \ldots 3)$$

AND, MOREOVER, THEY ARE THE PRINCIPAL STRESSES, WHICH MEANS THAT AMONG $\sigma_1$, $\sigma_2$ AND $\sigma_3$ WE FIND THE HIGHEST AND LOWEST VALUE FOR THE STRESSES COMPONENTS. NOTE THAT IN THOSE DIRECTIONS THE SHEAR COMPONENTS ARE ZERO.

## 3.2 BALANCE LAWS AND EQUILIBRIUM

WE RESTRICT THE PRESENT STUDY TO THE MOTIONS OBSERVING EQUILIBRIUM, WHICH IMPLIES THAT IN EVERY AND EACH ARBITRARY PART OF THE BODY THE APPLIED LOADINGS MUST SATISFY THE EQUATIONS OF EQUILIBRIUM (VIZ.: THE SUM OF ALL FORCES AND THE SUM OF ALL MOMENTS MUST VANISH). IF WE, FOR INSTANCE, TAKE THE EQUATIONS IN PAGE (3.4) AND APPLY THEM TO ANALYSE THE EQUILIBRIUM OF THE WHOLE BODY ($\mathcal{P}$ COINCIDES WITH $\mathcal{B}$) ALL WE CAN FIND ARE FORCES ACTING ON SUPPORTS (REACTION FORCES). THE MAIN OBJECTIVE OF ELASTICITY IS TO COMPUTE THE STRESS AND STRAIN DISTRIBUTION WITHIN THE BODY UNDERGOING A DEFORMABLE MOTION. THUS, WE CAN PARTITION IT INTO APPROPRIATE SUBDOMAIN AND REQUIRE THAT THE PART IS IN EQUILIBRIUM. SO, LET $\mathcal{P}$ A GENERIC PART INSIDE $\mathcal{B}$, THAN

$$\int_{\partial \mathcal{P}} \underline{s} \, d\Gamma + \int_{\mathcal{P}} \underline{b} \, dV = \underline{0}$$

AND

$$\int_{\partial \mathcal{P}} \underline{s} \, d\Gamma = \int_{\partial \mathcal{P}} T \underline{n} \, d\Gamma$$

BY THE DIVERGENCE THEOREM

$$\int_{\partial \mathcal{P}} T \underline{n} \, d\Gamma = \int_{\mathcal{P}} \operatorname{div} T \, dV$$

So $$\int_{\mathcal{P}} (\operatorname{div} T + \underline{b}) \, dV = \underline{0}$$

BUT THE ABOVE RESULT IS VALID FOR $\forall \, \mathcal{P} \subset \mathcal{B}$, SO, ONCE AGAIN, USING THE LOCALIZATION THEOREM:

$$\operatorname{div} T + \underline{b} = \underline{0} \qquad \forall \; x \in \mathcal{B}$$

WHICH IS THE LOCAL EQUILIBRIUM EQUATION (INDEED IT IS THE BALANCE OF LINEAR MOMENTUM DISREGARDING INERTIAL FORCES).

REPHRASING THE STRESS TENSOR IN COORDINATES, WE ADOPT THE FOLLOWING CONVENTION

$$[T] = \begin{bmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$$

WHERE $\sigma_{ij}$ IS THE STRESS COMPONENT ACTING ON THE $i$-th SURFACE AND IN $j$-th DIRECTION.

USUALLY, WE USE $\sigma_i$ INSTEAD OF $\sigma_{ii}$ ($\sigma_x \sim \sigma_{xx}$; $\sigma_y \sim \sigma_{yy}$ AND $\sigma_z \sim \sigma_{zz}$)

VERY SOON, WE WILL SEE THAT $\sigma_{ij} = \sigma_{ji}$

THE ABOVE EQUATION CORRESPONDS TO THREE SCALAR EQUATION IN A CARTESIAN COORDINATE FRAME

$$\frac{\partial \sigma_{x}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + b_x = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + b_y = 0$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{z}}{\partial z} + b_z = 0$$

INVOLVING NINE UNKNOWNS.

NOW CONSIDERING THE ANGULAR MOMENTUM EQUATION (THE BALANCE OF MOMENTS LOADING $\mathbb{B}$ MUST VANISH IN EQUILIBRIUM) WE HAVE

$$\int_{\partial P} (\underline{r} \wedge \underline{s})\, d\Gamma + \int_{P} (r \wedge \underline{b})\, dV = \underline{0}$$

$$\int_{\partial P} (\underline{r} \wedge \underline{s})\, d\Gamma = \int_{\partial P} (\underline{r} \wedge T\underline{n})\, d\Gamma =$$

$$= \int_{\partial P} (RT)\,\underline{n}\, d\Gamma = \int_{P} \operatorname{div}(RT)\, dV$$

DIVERGENCE THEOREM!

WHERE $R$ IS AN ANTISYMMETRIC SUCH THAT $R\underline{n} = \underline{r} \wedge \underline{n}$ $\forall \underline{n}$

WITH $[R] = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}$ AND THEN

$$[RT] = \begin{bmatrix} -z\,\sigma_{xy} + y\,\sigma_{xz} & -z\,\sigma_{yy} + y\,\sigma_{yz} & -z\,\sigma_{zy} + y\,\sigma_{zz} \\ z\,\sigma_{xx} - x\,\sigma_{xz} & z\,\sigma_{yx} - x\,\sigma_{yz} & z\,\sigma_{zx} - x\,\sigma_{zz} \\ -y\,\sigma_{xx} + x\,\sigma_{xy} & -y\,\sigma_{yx} + x\,\sigma_{yy} & -y\,\sigma_{zx} + x\,\sigma_{zy} \end{bmatrix}$$

AND

$$\operatorname{div}(RT) = \begin{bmatrix} -z\dfrac{\partial\sigma_{xy}}{\partial x} + y\dfrac{\partial\sigma_{xz}}{\partial x} - z\dfrac{\partial\sigma_{yy}}{\partial y} + \sigma_{yz} + y\dfrac{\partial\sigma_{yz}}{\partial y} - \sigma_{zy} - z\dfrac{\partial\sigma_{zy}}{\partial z} + y\dfrac{\partial\sigma_{zz}}{\partial z} \\[3em] z\dfrac{\partial\sigma_{xx}}{\partial x} - \sigma_{xz} - x\dfrac{\partial\sigma_{xz}}{\partial x} + z\dfrac{\partial\sigma_{yx}}{\partial y} - x\dfrac{\partial\sigma_{yz}}{\partial y} + \sigma_{zx} + z\dfrac{\partial\sigma_{zx}}{\partial z} - x\dfrac{\partial\sigma_{zz}}{\partial z} \\[3em] -y\dfrac{\partial\sigma_{xx}}{\partial x} + \sigma_{xy} + x\dfrac{\partial\sigma_{xy}}{\partial x} - \sigma_{yx} - y\dfrac{\partial\sigma_{yx}}{\partial y} + x\dfrac{\partial\sigma_{yy}}{\partial y} - y\dfrac{\partial\sigma_{zx}}{\partial z} + x\dfrac{\partial\sigma_{zy}}{\partial z} \end{bmatrix}$$

WHICH CAN BE DECOMPOSED INTO

$$\operatorname{div}(RT) = \overbrace{\begin{bmatrix} \sigma_{yz} - \sigma_{zy} \\ -\sigma_{xz} + \sigma_{zx} \\ \sigma_{xy} - \sigma_{yx} \end{bmatrix}}^{\tilde{\sigma}} + (\underline{r} \wedge \operatorname{div} T)$$

PLUGGING THIS EXPRESSION IN THE ANGULAR MOMENTUM BALANCE WE HAVE

$$\int_{\mathcal{P}} \underline{r} \times (\operatorname{div} T + \underline{b})\, dV + \int_{\mathcal{P}} \underset{\sim}{\underline{\sigma}}\, dV = \underline{0}$$

BUT NOTE THAT IN THE FIRST INTEGRAL WE HAVE THE BALANCE OF LINEAR MOMENTUM, SO THE INTEGRAL VANISHES AND

$$\int_{\mathcal{P}} \underset{\sim}{\underline{\sigma}}\, dV = \underline{0} \qquad \Downarrow \qquad \forall \mathcal{P} \in \mathcal{B}$$

$$\longrightarrow \underset{\sim}{\sigma} = \underline{0} \rightarrow \begin{cases} \sigma_{yz} = \sigma_{zy} \\ \sigma_{xz} = \sigma_{zx} \\ \sigma_{xy} = \sigma_{yx} \end{cases}$$

$$\boxed{T = T^T}$$

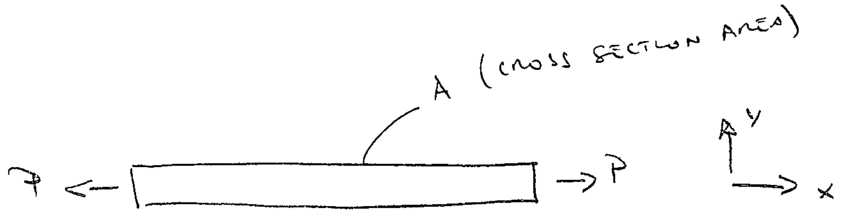
COMMENT: THE PROOF OF $T$ BEING SYMMETRIC SHOULD BE CARRIED OUT BY USING THE RESULT CONTAINED IN THE EXAMPLE OF pg. 1.47. HOW?

P.S.: (i) DUE TO THE ADOPTION OF THE "LAGRANGIAN VIEW" OF THE DEFORMATION THE MASS BALANCE IS AUTOMATICALLY SATISFIED

(ii) INDEED, ALL THE INTEGRALS ASSOCIATE TO THE BALANCE LAWS MUST BE PERFORMED OVER $\chi(p) \in \chi(\mathbb{B})$ (REGIONS ASSIGNED IN THE DEFORMED CONFIGURATION). BUT, WITHIN THE REALM OF SMALL DEFORMATIONS, $P$ AND $\chi(p)$ (AS $\mathbb{B}$ AND $\chi(\mathbb{B})$) ARE CLOSE AND, FOR CERTAIN PURPOSES, CAN BE IDEALIZED AS THE SAME REGION.

## 3.3) EXAMPLES

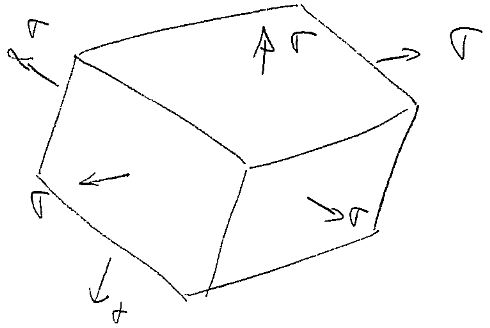
### (1) UNIAXIAL STRESS




$$[T] = \begin{bmatrix} P/A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

PRINCIPAL STRESSES: $\sigma_I = P/A > \sigma_{II} = \sigma_{III} = 0$

### (2) HYDROSTATIC STRESS

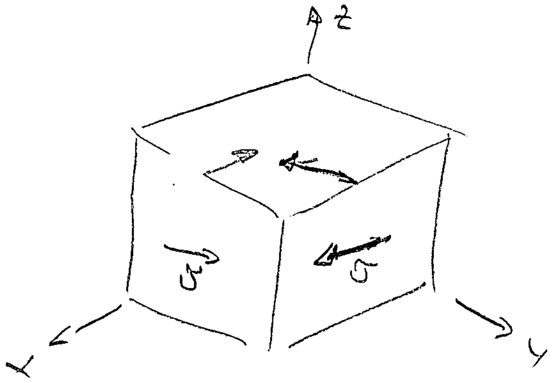



PRINCIPAL STRESSES:

$\sigma_I = \sigma_{II} = \sigma_{III} = \sigma$

$$[T] = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix} = \sigma I$$

(3) PURE SHEAR




$$[T] = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

PRINCIPAL STRESSES:

$$[T]\,\Phi_i = \sigma_i\,\Phi_i$$

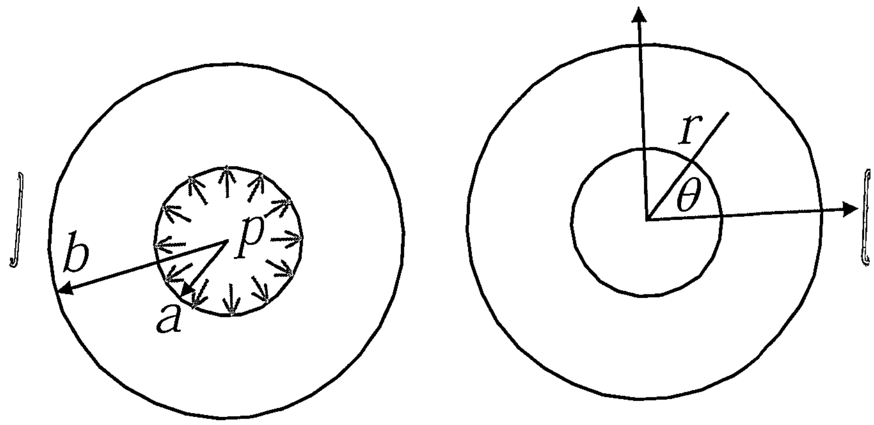
CHARACTERISTIC EQUATION:

$$\det\left[T - \sigma_i I\right] = 0$$

$$-\sigma_i\left(\sigma_i^2 - \tau^2\right) = 0 \Longrightarrow \sigma_I = \tau \;;\; \sigma_{II} = 0 \;;\; \sigma_{III} = -\tau$$

THEREFORE THE PRINCIPAL STRESSES ARE LOCALIZED IN THE XY PLANE POINTING TOWARDS A 45° DIRECTION.

(4)




AT THE INNER BOUNDARY ($r=a$) THE TRACTION VECTOR IS GIVEN BY

$$\underline{s} = T(-\underline{e_r}) = P\,\underline{e_r}$$

↳ (WHY?)

AND AT $r=b$

$$\underline{s} = T(\underline{e_r}) = \underline{0}$$

THEREFORE:

AT $r=a \longrightarrow \sigma_{rr} = \underline{e_r} \cdot T\underline{e_r} = -p$

$\sigma_{r\theta} = \underline{e_\theta} \cdot T\underline{e_r} = 0$

AT $r=b \longrightarrow \sigma_{rr} = 0 \quad ; \quad \sigma_{r\theta} = 0$

# PRINCIPAL DIRECTIONS (EIGENVECTORS)

I $(\sigma_I = \tau)$

$$\begin{bmatrix} -\tau & \tau & 0 \\ \tau & -\tau & 0 \\ 0 & 0 & -\tau \end{bmatrix} \begin{bmatrix} \phi_1^x \\ \phi_1^y \\ \phi_1^z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\longrightarrow \phi_1^z = 0 \quad ; \quad \phi_1^x = \phi_1^y$

IF WE TAKE $\| \phi_i \|^2 = 1 \rightarrow \phi_1 = \begin{bmatrix} \pm 1/\sqrt{2} \\ \pm 1/\sqrt{2} \\ 0 \end{bmatrix}$

II $(\sigma_{II} = 0)$

$\phi_2^x = \phi_2^y = 0$

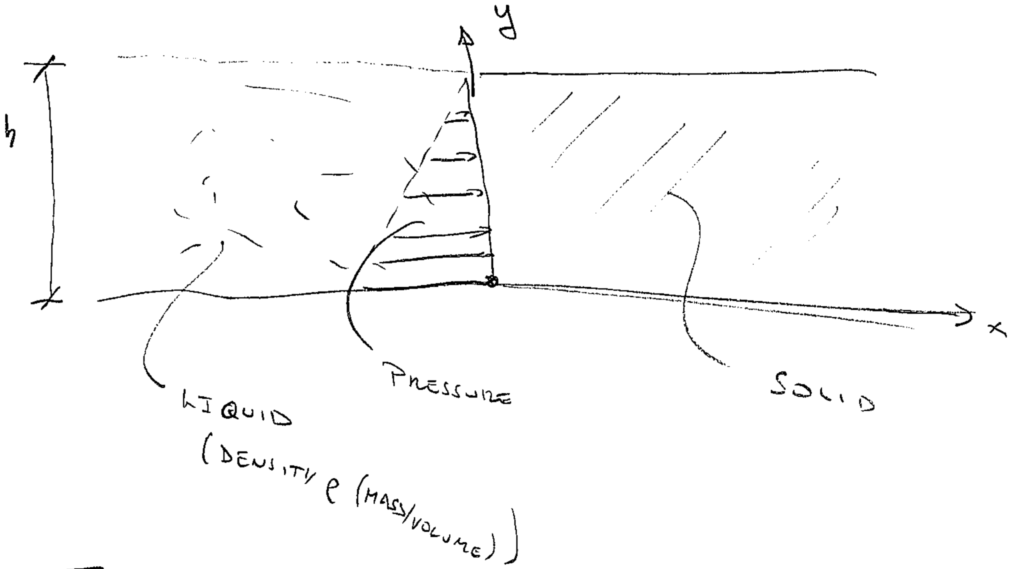
AND THUS, FROM THE NORMALIZATION, $\phi_2^z = \pm 1$

III $(\sigma_{III} = -\tau)$

$\phi_3^x = -\phi_3^y$ AND $\phi_3^z = 0$

$$\phi_3 = \pm \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$$

(5)



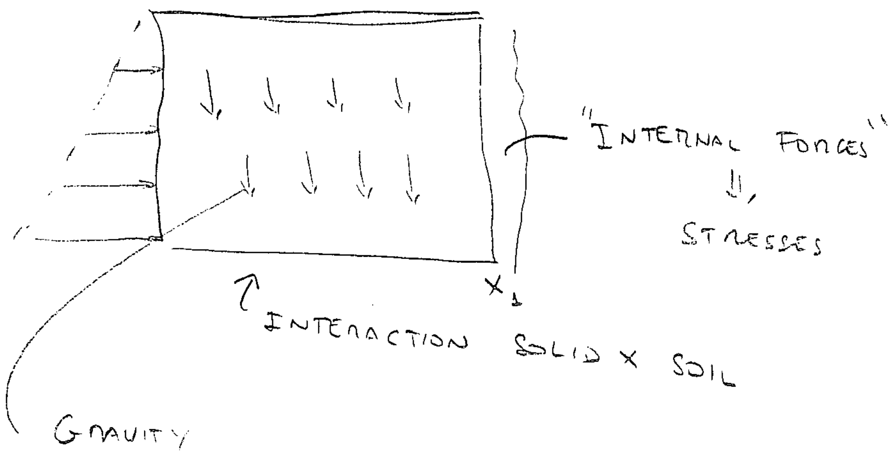

THUS, AT $x = 0$

$$\underline{s}(0, y) = T(0, y)(-\underline{e}_1) = \rho g (h - y)\, \underline{e}_1$$

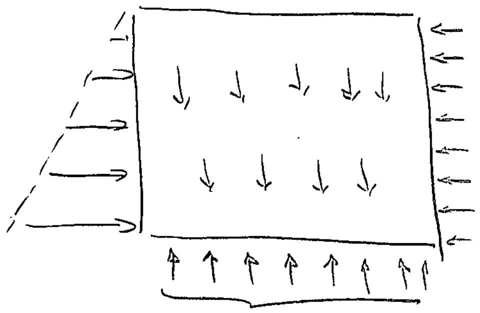
↳ AT $x = 0$ ⟶ $\sigma_{xx} = -\rho g (h - y)$ (COMPRESSION)

$$\sigma_{xy} = 0$$

WHAT HAPPENS IN THE SECTION OF THE SOLID DEFINED BY $x = x_1$



ASSUMING THAT THERE IS NO FRICTION BETWEEN SOLID AND SOIL AND UNIFORMITY ALONG Z DIRECTION ($\Rightarrow$ 2D PROBLEM)



REACTION = FORCE LINEAR DISTRIBUTION $r$

* EQUILIBRIUM IN THE Y-DIRECTION

WITH $\underline{b} = -\rho_s \, g \, \underline{e}_y$

$\rho_s$ (SOLID DENSITY)

$$\rho_s g \, (X_1 h) = \int_V \underline{b} \cdot \underline{e}_y \, dV = \int_\Gamma \underline{s} \cdot \underline{e}_y \, d\Gamma = \int_0^{X_1} r \, dx = r X_1$$

AND THEREFORE : $\boxed{\sigma_{yy}(x,0) = -\rho_s g h}$

IF $x < X_1$

INDEED $\sigma_{yy}(x,0) = -\rho_s g h$ ALWAYS!

AND MORE:

$$\sigma_{yy}(x,y) = -\rho_s g y$$

# EQUILIBRIUM IN THE X-DIRECTION

$$\int_{\underline{\Gamma}} \underline{S} \cdot \underline{e}_x \, d\underline{\Gamma} = 0$$

AND THEN

$$-\int_0^h \sigma_{xx}\Big|_{x=0} dy + \int_0^h \sigma_{xx} \, dy = 0$$

THEN $\sigma_{xx}(x_1, y) = -\rho g (h-y)$

AND AS $x_1$ IS ARBITRARY

$$\sigma_{xx}(x,y) = -\rho g (h-y)$$