Relações Constitutivas – Elasticidade Linear:

Tensão X Deformação

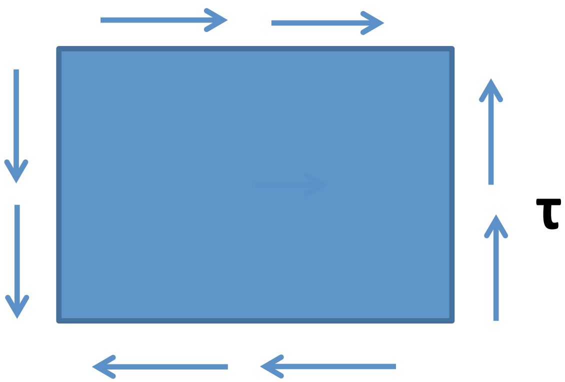
$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right] + \alpha\Delta T$$

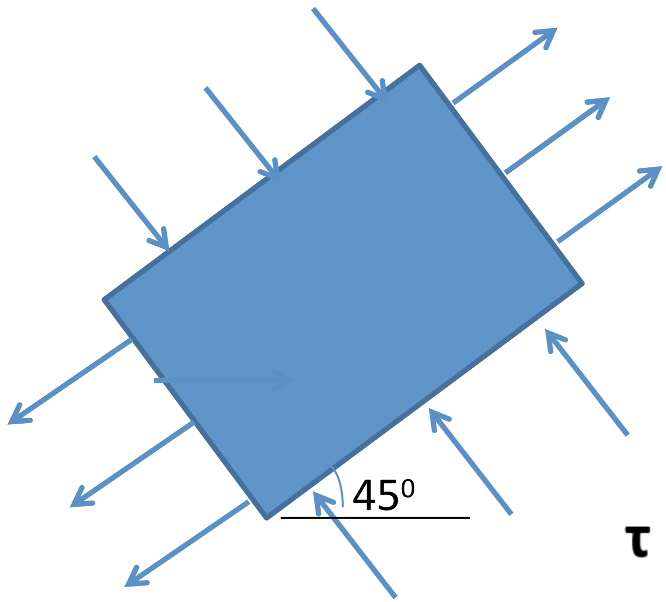
$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\left(\sigma_{xx} + \sigma_{zz}\right)\right] + \alpha\Delta T$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{yy} + \sigma_{xx}\right)\right] + \alpha\Delta T$$

$$\varepsilon_{xy} = \frac{\sigma_{xy}}{2G} \qquad \varepsilon_{xz} = \frac{\sigma_{xz}}{2G} \qquad \varepsilon_{zy} = \frac{\sigma_{zy}}{2G}$$

Independência : E, G, ν





$$\gamma_{xy} = \frac{\tau}{G}$$

$$\varepsilon_{x'} = \frac{1}{E}[\tau\nu + \tau] \qquad \varepsilon_{y'} = -\frac{1}{E}[\tau\nu + \tau]$$

$$\gamma_{xy} = \varepsilon_{x'} - \varepsilon_{y'} = \frac{2(1+\nu)}{E}\tau \longrightarrow G = \frac{E}{2(1+\nu)}$$

Mecânica dos Sólidos – Elasticidade Linear

Equilíbrio

Compatibilidade Geométrica – Geometria da Deformação

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} \qquad \varepsilon_{yy} = \frac{\partial u_y}{\partial y} \qquad \varepsilon_{zz} = \frac{\partial u_z}{\partial z}$$

$$\gamma_{xy} = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) \qquad \gamma_{xz} = \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) \qquad \gamma_{yz} = \left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial z}\right)$$

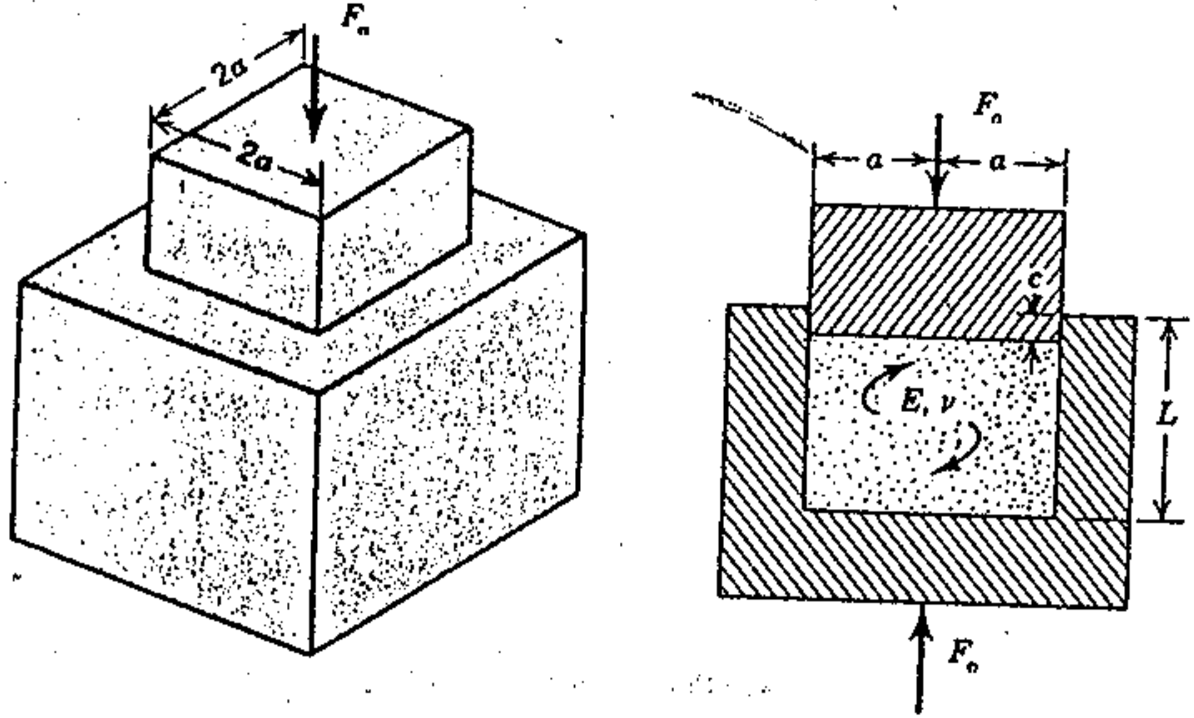
Comportamento Constitutivo

$$\varepsilon_{yy} = \frac{1}{E}[\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})] + \alpha\Delta T \qquad \varepsilon_{xx} = \frac{1}{E}[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})] + \alpha\Delta T$$

$$\varepsilon_{zz} = \frac{1}{E}[\sigma_{zz} - \nu(\sigma_{yy} + \sigma_{xx})] + \alpha\Delta T \qquad \varepsilon_{xy} = \frac{\sigma_{xy}}{2G} \qquad \varepsilon_{xz} = \frac{\sigma_{xz}}{2G} \qquad \varepsilon_{zy} = \frac{\sigma_{zy}}{2G}$$

Problema Linear → “Princípio da Superposição

Exemplo – Elasticidade 3D



Exercício 5.1 - An Introduction to the Mechanics of Solids

CINEMÁTICA : $\varepsilon_x = \varepsilon_z = 0$; $\varepsilon_y = \dfrac{-0.4}{25.4} = -0.016$ mm/mm

TENSÕES : $\sigma_y = \dfrac{-P}{A}$

EQ. CONSTITUTIVA : $\varepsilon_y = \dfrac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$

$$\left.\begin{aligned} 0 &= \sigma_x - \nu(\sigma_y + \sigma_z) \\ 0 &= \sigma_z - \nu(\sigma_y + \sigma_x) \end{aligned}\right\} \quad (1-\nu)(\sigma_x + \sigma_z) = 2\nu\sigma_y$$

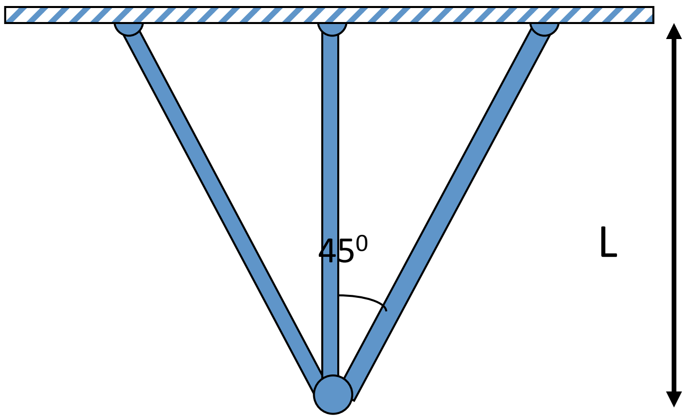
$$\therefore \quad -0.016 = \frac{1}{E}\left[\sigma_y - \frac{2\nu^2}{(1-\nu)}\sigma_y\right]$$

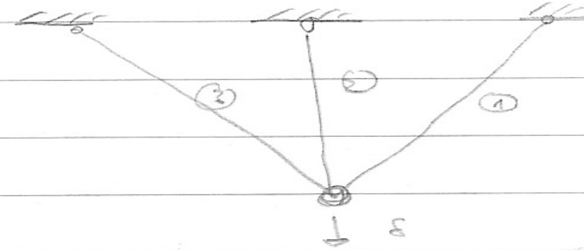
$$\boxed{\sigma_y = 47 \text{ MPa}}$$

LOGO $\boxed{P = 4.7 \text{ KN}}$

Tensões Térmicas

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right] + \alpha\Delta T$$





(SIMETRIA)

$\delta_1 = \delta_3$

$\delta = \delta_2$

COMPATIBILIDADE GEOMÉTRICA : $\delta = \dfrac{\delta^1}{\cos 45^\circ} = \delta^1 \sqrt{2}$ (1.0)

EQUILÍBRIO

$F_2 + 2F_1 \cos 45^\circ = 0$ (1.0)

LOGO : $\sigma_2 + \sigma_1 \sqrt{2} = 0$

COMPORTAMENTO CONSTITUTIVO :

$$\frac{\delta}{L} - \alpha \Delta T + \left(\frac{\delta^1}{\sqrt{2}\, L} - \alpha \Delta T \right) \sqrt{2} = 0$$

$$\frac{\delta}{L}\left(1 + \frac{\sqrt{2}}{2}\right) = \alpha \Delta T \left(1 + \sqrt{2}\right) \qquad (3.0)$$

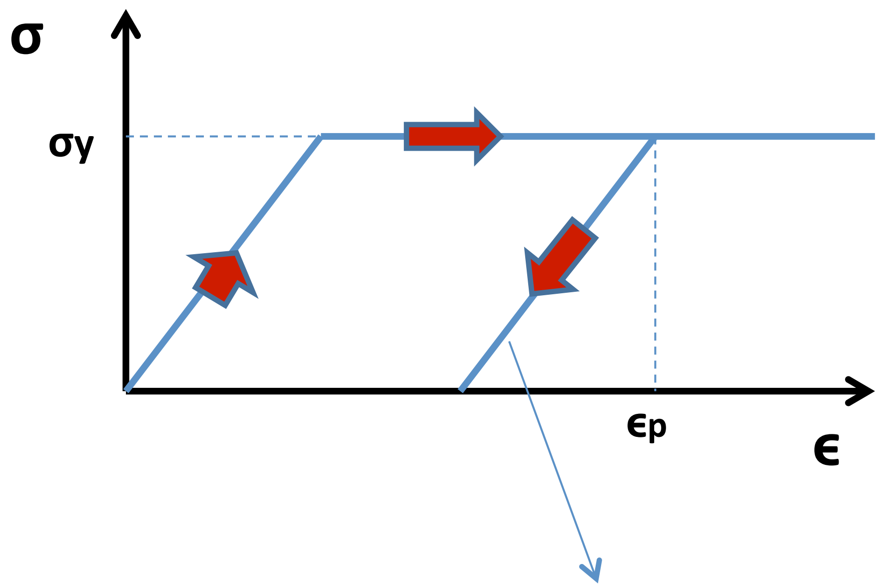
$$\delta = \alpha \Delta T L \, 2 \, \frac{(1+\sqrt{2})}{2+\sqrt{2}}$$

$$\sigma^2 = E\left(\alpha \Delta T \, 2 \, \frac{(1+\sqrt{2})}{2+\sqrt{2}} - \alpha \Delta T \right) = E \alpha \Delta T \, \frac{\sqrt{2}}{2+\sqrt{2}} > 0$$

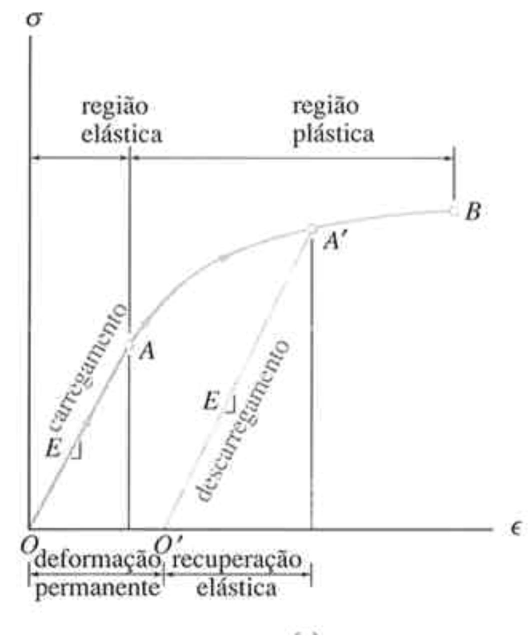
$$\sigma^1 = E\left(\alpha\Delta T \frac{2}{\sqrt{2}}\frac{(1+\sqrt{2})}{2+\sqrt{2}} - \alpha\Delta T\right) =$$

$$= -E\alpha\Delta T\frac{2\sqrt{2}}{2\sqrt{2}+2} \leq 0$$

Comportamento Inelástico : Plasticidade Ideal



Descarregamento Elástico

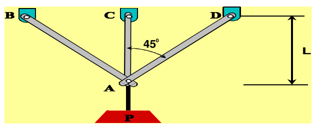


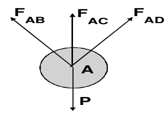
Hibeller. Resistência dos Materiais

Exemplo : Tensões Elasto-plásticas e residuais (prof. Lavinia Borges)

Treliça elasto plástica

Comportamento elastoplástico





EQUILÍBRIO

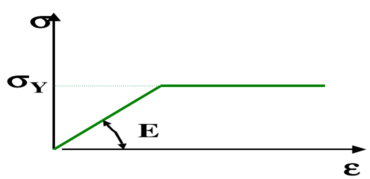
$$\Sigma F_x = 0 \quad -F_{AB}\frac{1}{\sqrt{2}} + F_{AD}\frac{1}{\sqrt{2}} = 0 \quad \Rightarrow F_{AB} = F_{AD}$$

$$\Sigma F_y = 0 \quad F_{AB}\frac{1}{\sqrt{2}} + F_{AC} + F_{AD}\frac{1}{\sqrt{2}} = P \quad \Rightarrow F_{AC} = P - \frac{2F_{AB}}{\sqrt{2}}$$

Cinemática

$$\delta_{AB} = \delta_{AC}\frac{1}{\sqrt{2}}$$

Material :Elástico linear, idealmente plástico



$\sigma < \sigma_y \qquad \varepsilon = \varepsilon^e$

$\sigma = \sigma_y \qquad \varepsilon = \varepsilon^e + \varepsilon^p$

Solução Elástica

$$\delta_{AC} = \frac{F_{AC} \cdot L}{E \cdot A} \qquad \delta_{AB} = \frac{F_{AB} \cdot L \cdot \sqrt{2}}{E \cdot A}$$

A=Área da Seção transversal

Da cinemática: $\delta_{AB} = \delta_{AC} \cdot \frac{1}{\sqrt{2}}$ => $F_{AB} = \frac{1}{2} \cdot F_{AC}$

Do equilíbrio: $F_{AC} = P - \frac{2 \cdot F_{AB}}{\sqrt{2}}$ => $F_{AC} = .586 \cdot P$ e $F_{AB} = .293 \cdot P$

$$\delta_{AC} = V = \frac{.586 \cdot P \cdot L}{E \cdot A}$$

Carga Limite elático P_{le}: Barra mais solicitada é a primeira a plastificar

$$\frac{F_{AC}}{A} = \sigma_Y \quad \Rightarrow \quad P_{LE} = 1.71 \cdot \sigma_Y \cdot A \qquad \delta_{AC}{}^{le} = \sigma_Y \cdot \frac{L}{E}$$

Solução Elastoplástica: $P > P_{le}$

$$F_{AC} = \sigma_Y \cdot A$$

Material idealmente plástico

Equilíbrio: $F_{AB} = F_{AD}$

$$\frac{F_{AB}}{\sqrt{2}} + F_{AC} + \frac{F_{AB}}{\sqrt{2}} = P \quad \Rightarrow \quad 2 \cdot \frac{F_{AB}}{\sqrt{2}} + \sigma_Y \cdot A = P$$

Problema agora é estaticamente determinado: $F_{AB} = \frac{1}{\sqrt{2}} \cdot (P - \sigma_Y \cdot A)$

Logo, da relação constitutiva: $\delta_{AB} = \frac{(P - \sigma_Y \cdot A) \cdot L}{E \cdot A}$

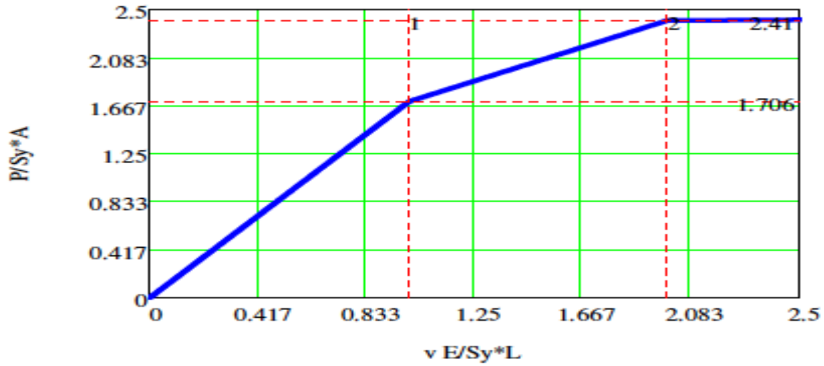
Da cinemática: $\delta_{AB} = \delta_{AC} \cdot \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \delta_{AC} = V = \sqrt{2} \cdot \frac{(P - \sigma_Y \cdot A) \cdot L}{E \cdot A}$

Carga de Colapso P_c : Quando a barra AB plastificar $F_{AB} = \sigma_Y \cdot A$

$$P_C = 2.41 \cdot \sigma_Y \cdot A \qquad \delta_{AC}{}^{c} = 2 \cdot \sigma_Y \cdot \frac{L}{E}$$

Representação gráfica da solução até o colapso

$$v(P) := \text{if}\left[P \le 1.706, 0.586\cdot P, \text{if}\left[P \le 2.41, \sqrt{2}\cdot(P-1), 2\cdot(P-1)\right]\right] \qquad P := 0, 0.01 .. 2.5$$



Descarrecamento elástico em $P = 2\,\sigma_Y A > P_{le}$

(a)Soluçao elasto-plástica neste nível de carregamento

$$F_{AC} = \sigma_Y\cdot A \qquad F_{AD} = F_{AB} = \frac{1}{\sqrt{2}}\cdot\left[2(\sigma_Y\cdot A) - \sigma_Y\cdot A\right] = \frac{1}{\sqrt{2}}\cdot\sigma_Y\cdot A$$

Elásticas !

$$\delta_{AD} = \delta_{AB} = \delta_{AC}\cdot\frac{1}{\sqrt{2}}$$

$$\delta_{AC} = V = \sqrt{2}\cdot\frac{\left[2(\sigma_Y\cdot A) - \sigma_Y\cdot A\right]\cdot L}{E\cdot A} = \sqrt{2}\cdot\frac{\sigma_Y\cdot L}{E} \qquad \delta_{AB} = \frac{\sigma_Y\cdot L}{E}$$

Descarga : $\Delta P = -2\sigma_Y\cdot A \qquad P_r = 0$

$$\Delta F_{AC} = -(.586\cdot\Delta P) = -1.172\sigma_Y\cdot A \qquad \Delta F_{AD} = \Delta F_{AB} = -.586\cdot\sigma_Y\cdot A$$

$$\Delta\delta_{AC} = \Delta V = -\frac{1.172\sigma_y\cdot L}{E} \qquad \Delta\delta_{AD} = \Delta\delta_{AB} = -\frac{0.8287\cdot\sigma_Y\cdot L}{E}$$

$$F_{AC}{}^{r} = F_{AC} + \Delta F_{AC} = \sigma_Y\cdot A - 1.172\cdot\sigma_Y\cdot A = -0.172\cdot\sigma_Y\cdot A$$

$$F_{AD}{}^{r} = F_{AB}{}^{r} = F_{AD} + \Delta F_{AD} = \frac{1}{\sqrt{2}}\cdot\sigma_Y\cdot A - 0.586\cdot\sigma_Y\cdot A = 0.1211\cdot\sigma_Y\cdot A$$

Observação : Tensões residuais são auto-equilibradas

$$\frac{2F_{AB}{}^{r}}{\sqrt{2}} + F_{AC}{}^{r} = 0$$

$$\delta_{AC}{}^{r} = V^{r} = \delta_{AC} + \Delta\delta_{AC} = \sqrt{2}\cdot\frac{\sigma_Y\cdot L}{E} - \frac{1.172\cdot\sigma_Y\cdot L}{E} = 0.2422\cdot\frac{\sigma_Y\cdot L}{E}$$

$$\delta_{AB}{}^{r} = \delta_{AB} + \Delta\delta_{AB} = \frac{\sigma_Y\cdot L}{E} - \frac{0.8287\cdot\sigma_Y\cdot L}{E} = 0.1713\cdot\frac{\sigma_Y\cdot L}{E}$$

Representação gráfica da solução com descarga antes do colapso

$$v(P) := \text{if}\left[P \le 1.706, 0.586\cdot P, \text{if}\left[P \le 2.41, \sqrt{2}\cdot(P-1), 2\cdot(P-1)\right]\right] \qquad P := 0, 0.01 .. 2$$

$$v_r(P_r) := \sqrt{2} - 0.586(2 - P_r) \qquad P_r := 2, 1.99 .. 0$$