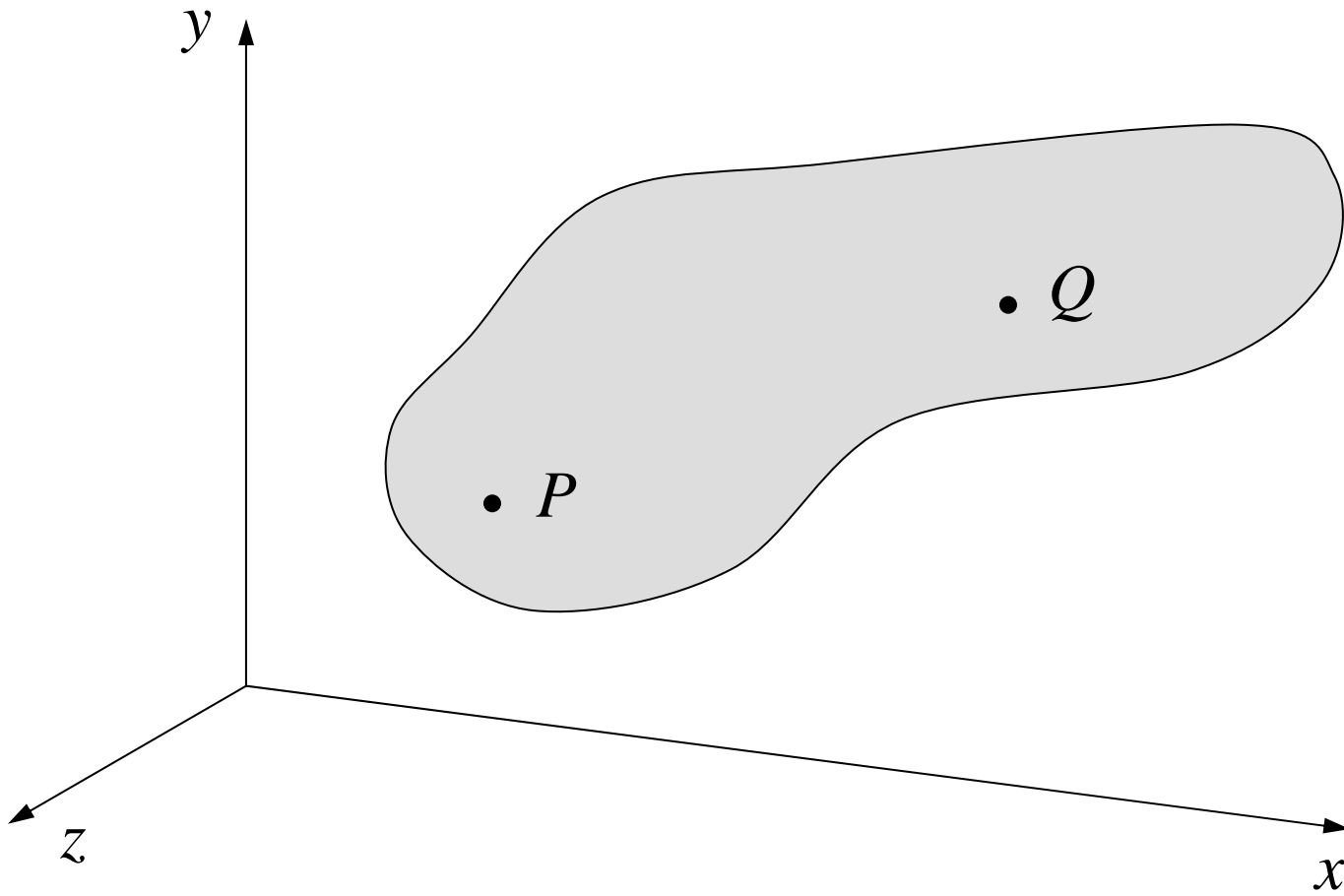
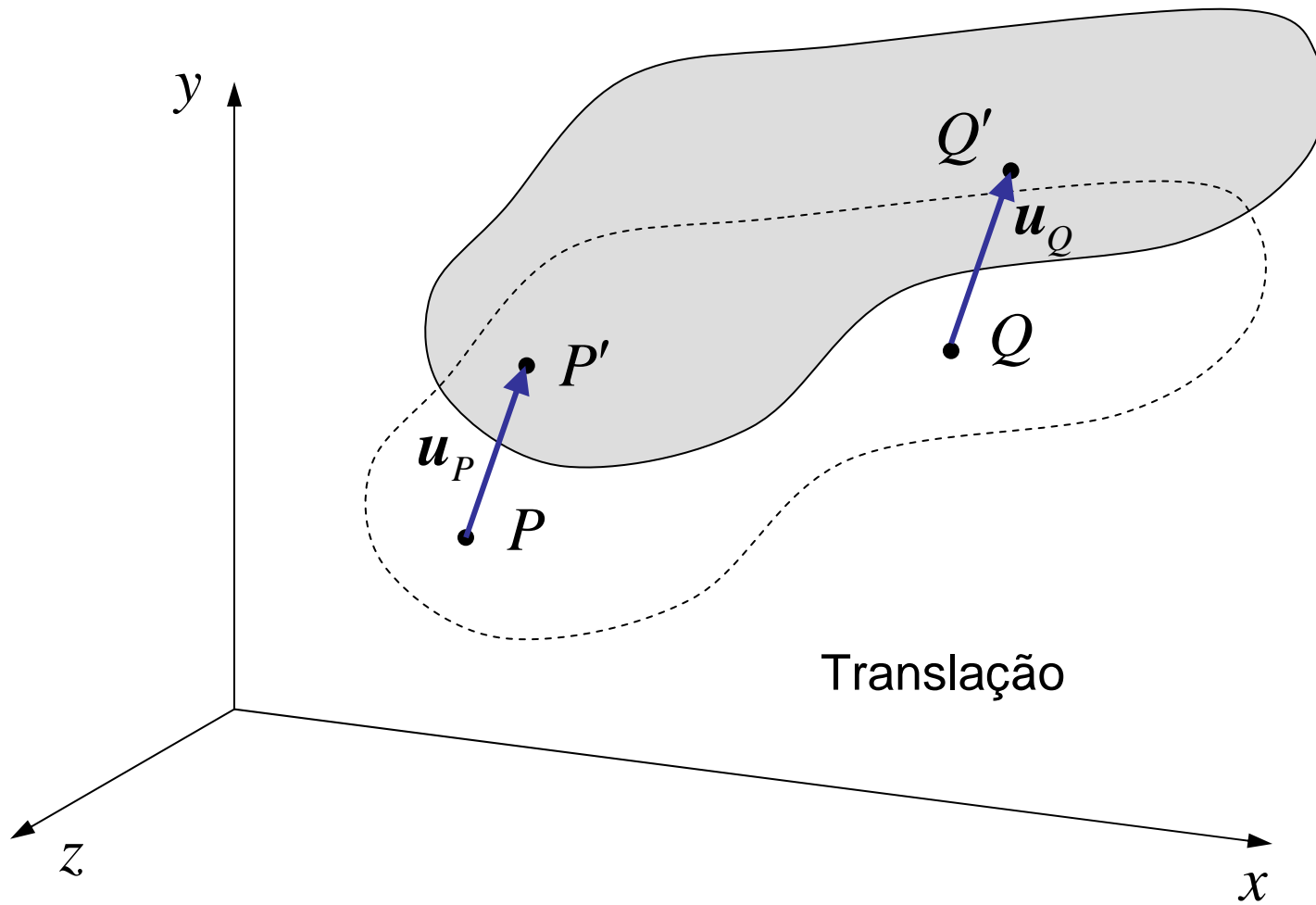


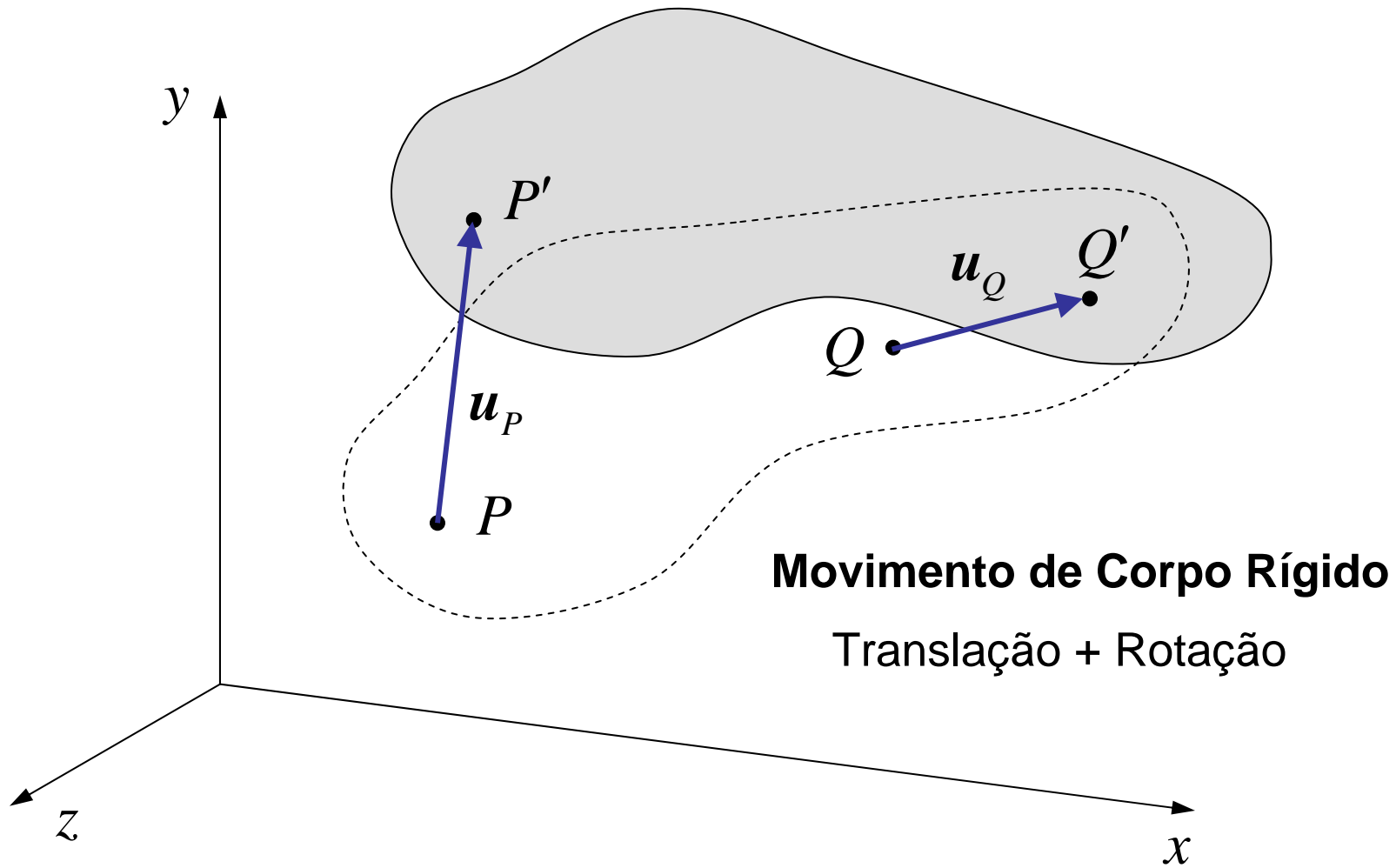
Deslocamento



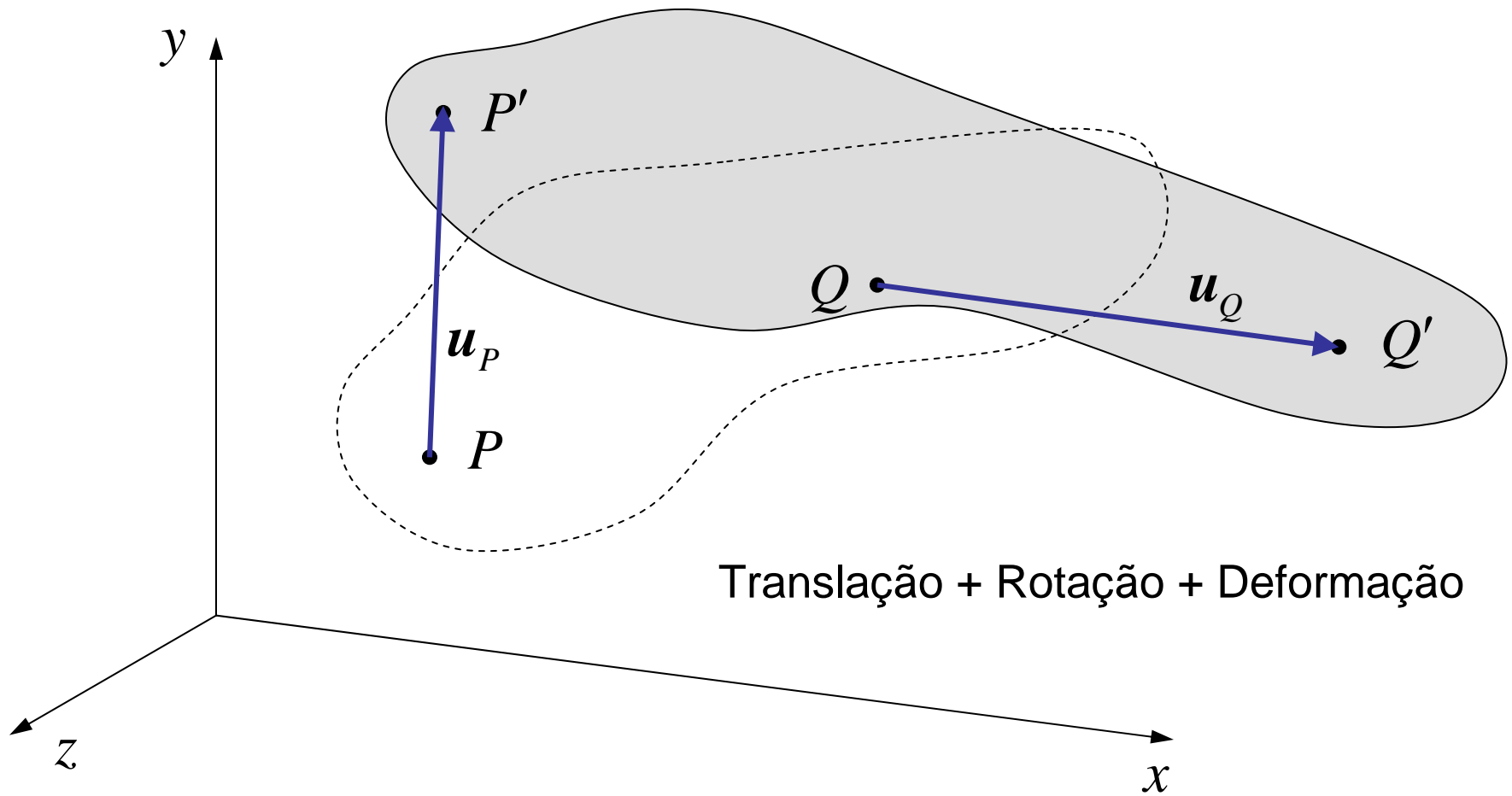
Deslocamento



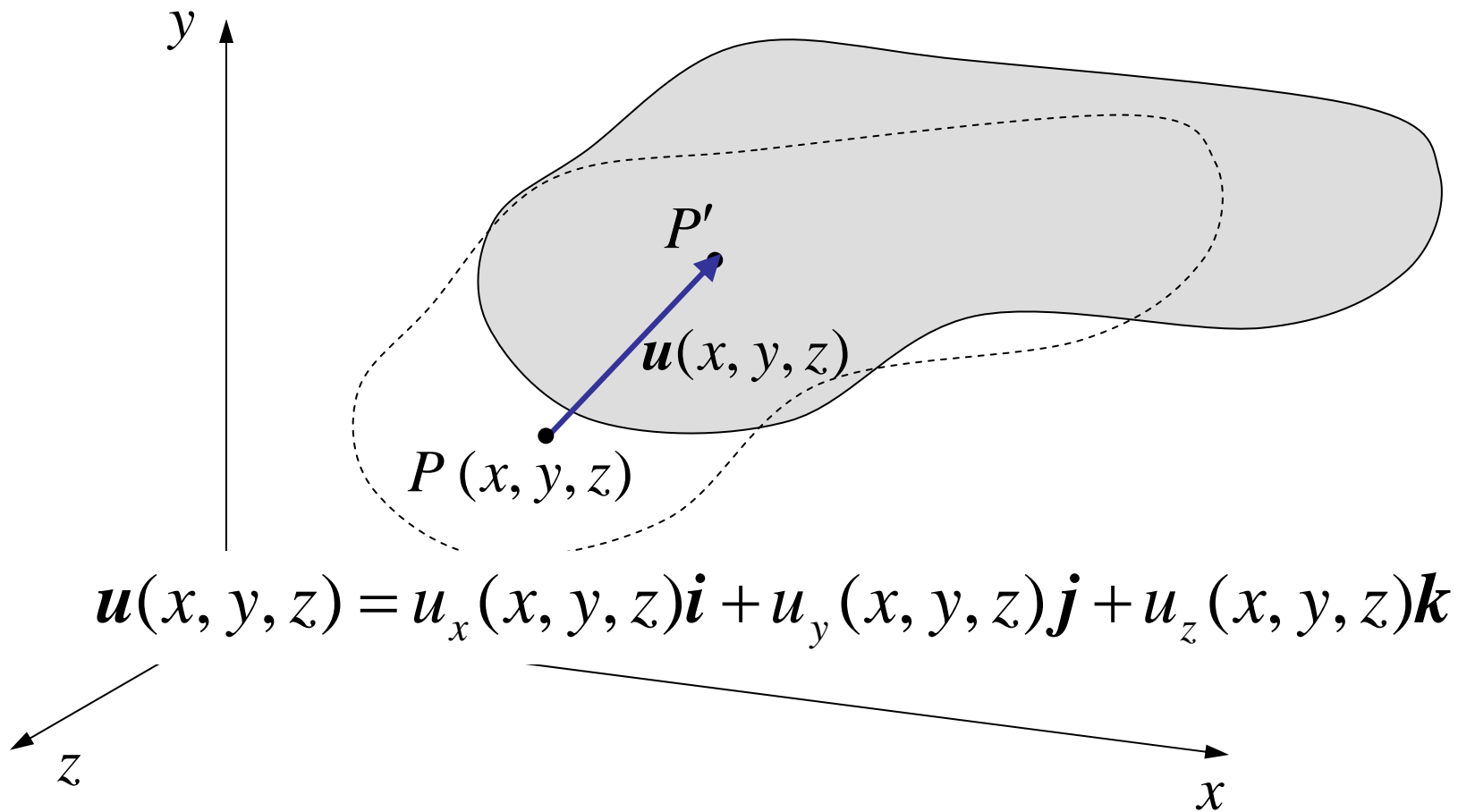
Deslocamento



Deslocamento



Deslocamento



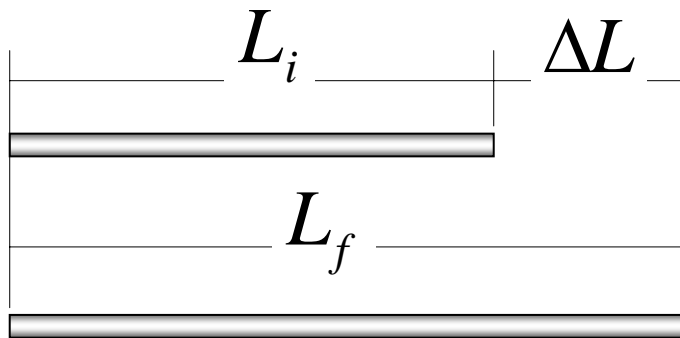
Deslocamento

Deslocamento é uma grandeza vetorial (o campo de deslocamentos):

$$\mathbf{u}(x, y, z) = u_x(x, y, z)\mathbf{i} + u_y(x, y, z)\mathbf{j} + u_z(x, y, z)\mathbf{k}$$

u_x , u_y , e u_z são as componentes do vetor deslocamento nas direções x , y e z .

Deformação – 1D



Medidas de Deformação

1) Lagrangeana

$$\varepsilon_L = \frac{L_f - L_i}{L_i} = \Delta L / L_i$$

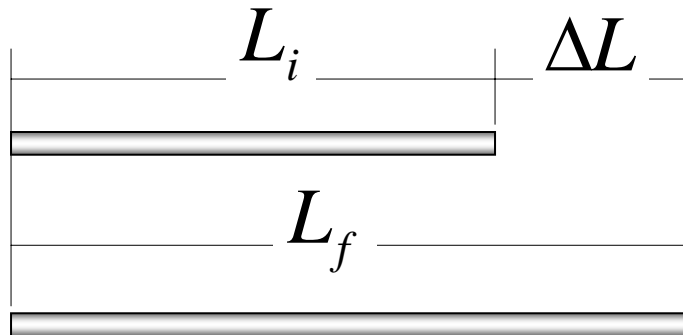
2) Euleriana

$$\varepsilon_E = \frac{L_f - L_i}{L_f} = \Delta L / L_f$$

3) Logaritimica

$$\varepsilon_{\log} = \ln\left(\frac{L_f}{L_i}\right)$$

Deformação – 1D



Grandes deformações

Pequenas deformações

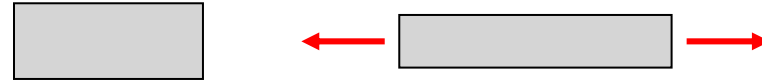
$$\varepsilon_L = \frac{L_f - L_i}{L_i} = \Delta L / L_i$$

$$\varepsilon_E = \frac{L_f - L_i}{L_f} = \Delta L / L_f$$

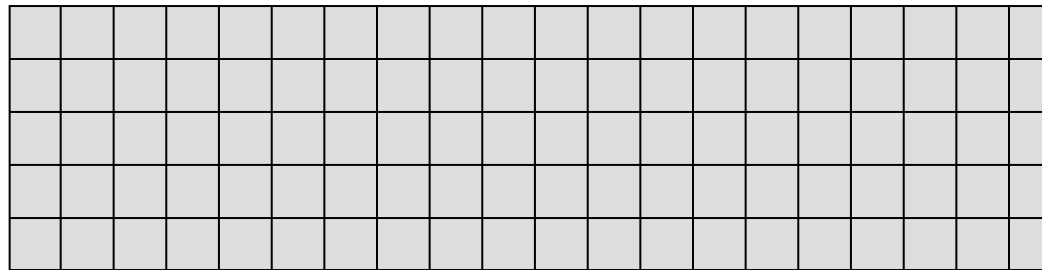
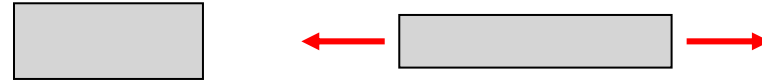
$$\varepsilon_{\log} = \ln\left(\frac{L_f}{L_i}\right)$$

L_i	1,0 m	1,0 m	1,0 m	1,0 m
L_f	2,0 m	0,5 m	1,001 m	0,999 m
ε_L	100%	- 50%	0,10%	- 0,10%
ε_E	50%	-100%	0,10%	- 0,10%
ε_{\log}	69,3%	- 69,3%	0,10%	- 0,10%

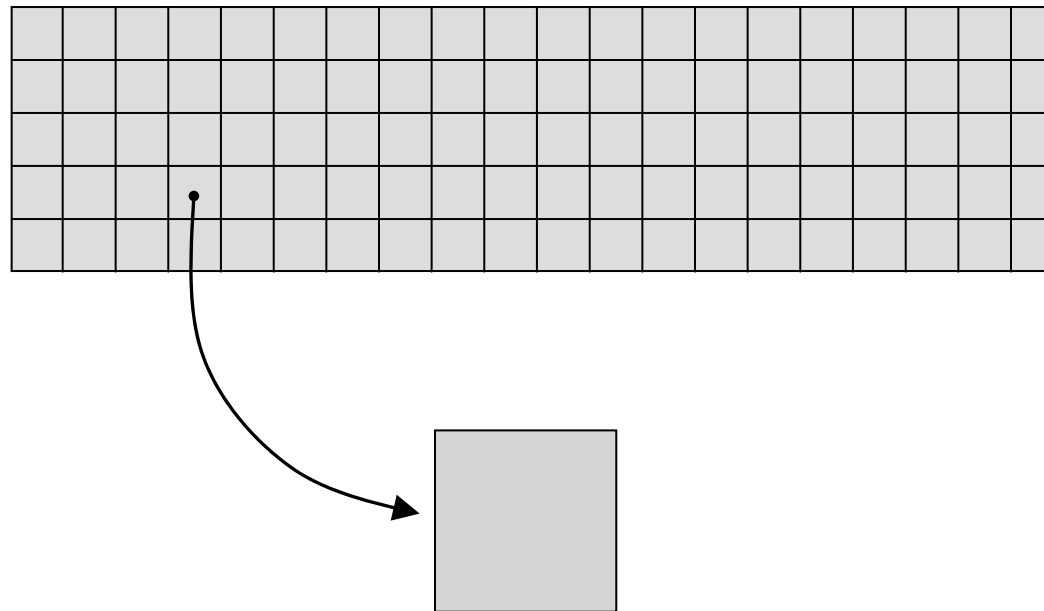
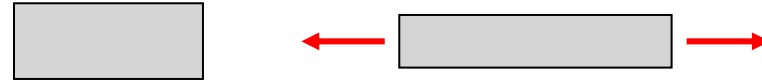
Deformação – 2D



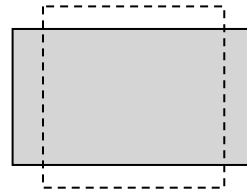
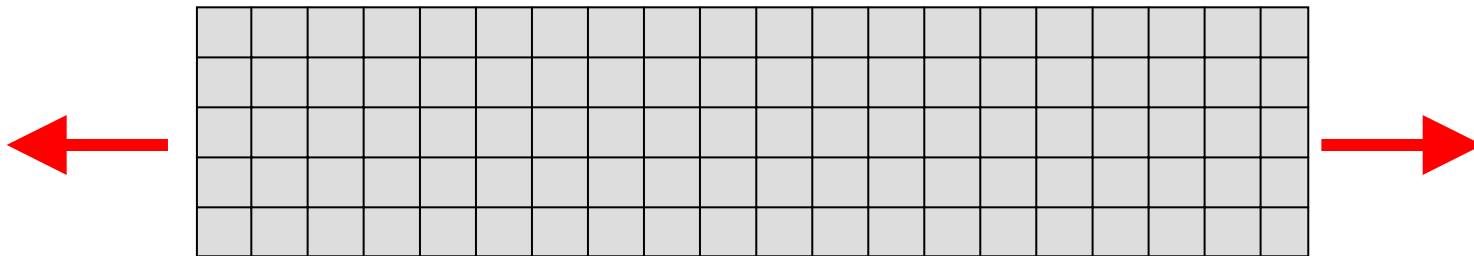
Deformação – 2D



Deformação – 2D

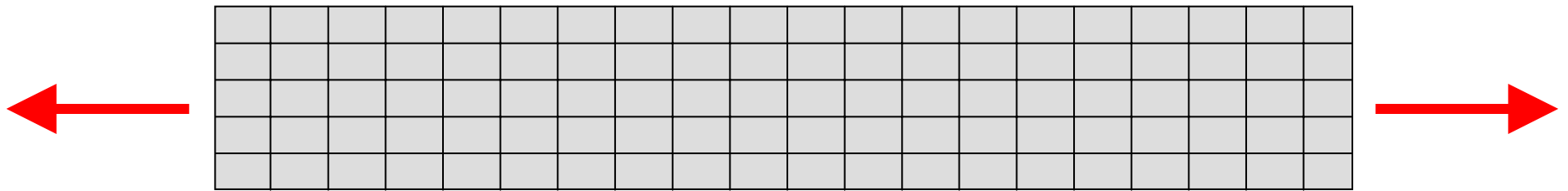
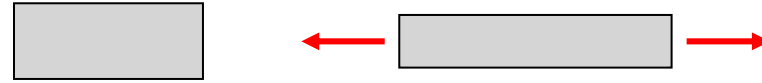


Deformação – 2D



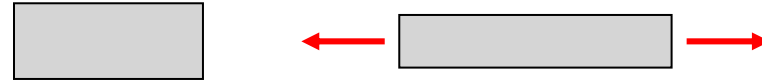
Não há distorção
do elemento

Deformação – 2D

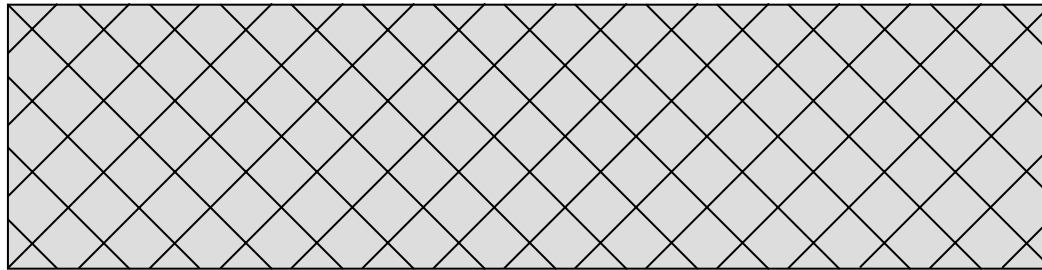
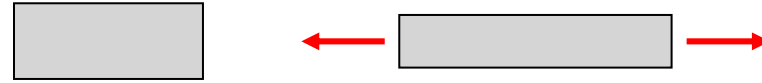


Não há distorção
do elemento

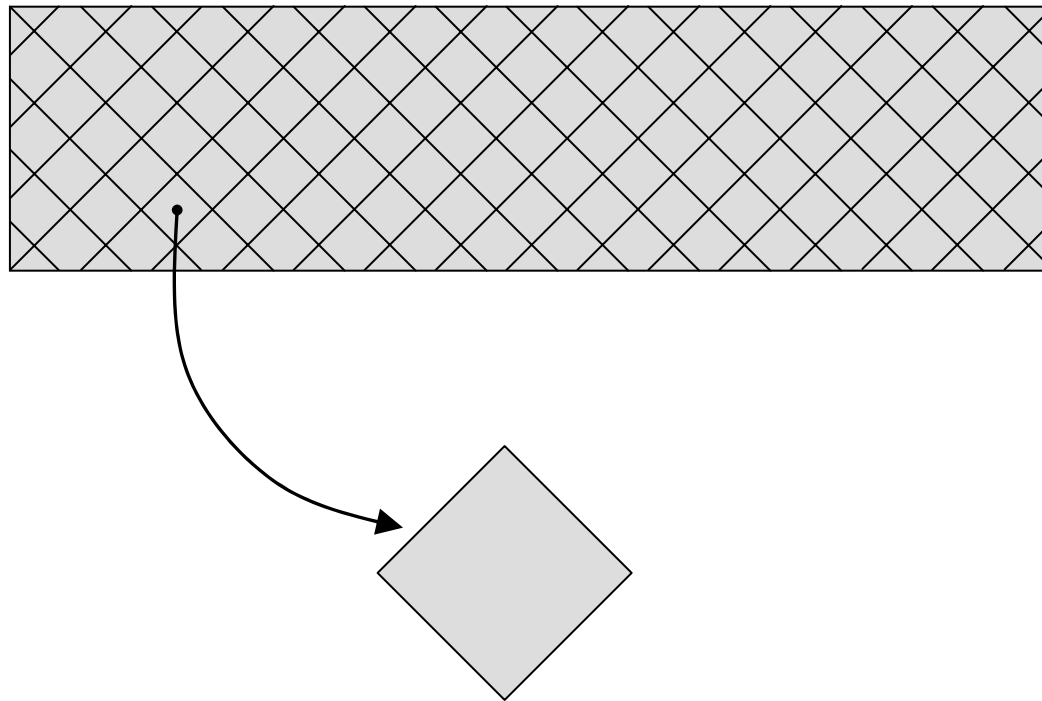
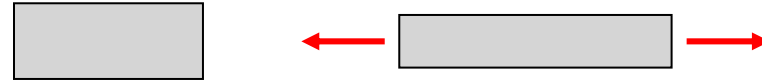
Deformação – 2D



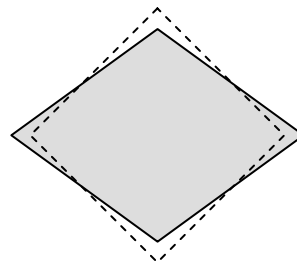
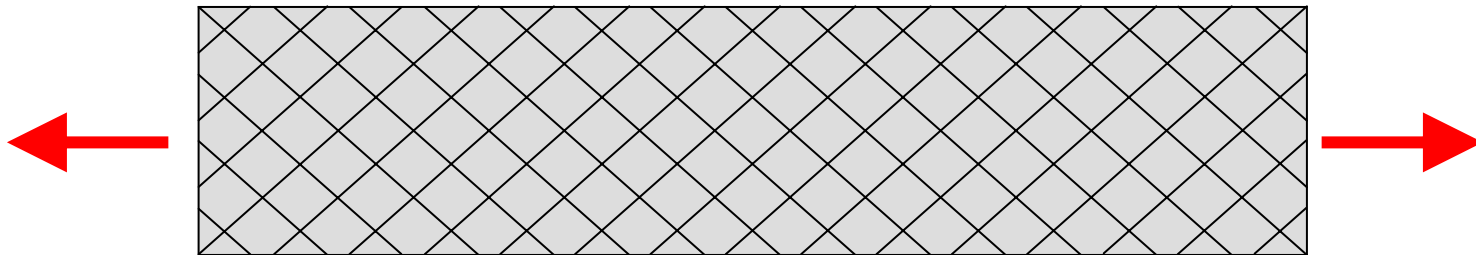
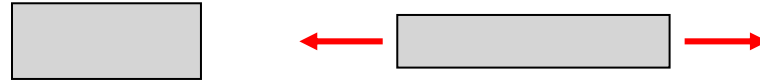
Deformação – 2D



Deformação – 2D

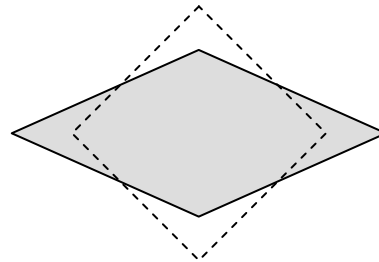
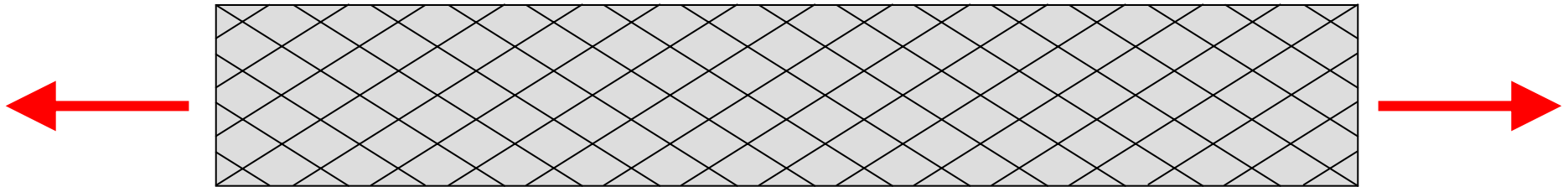


Deformação – 2D



Há distorção no elemento (cisalhamento)

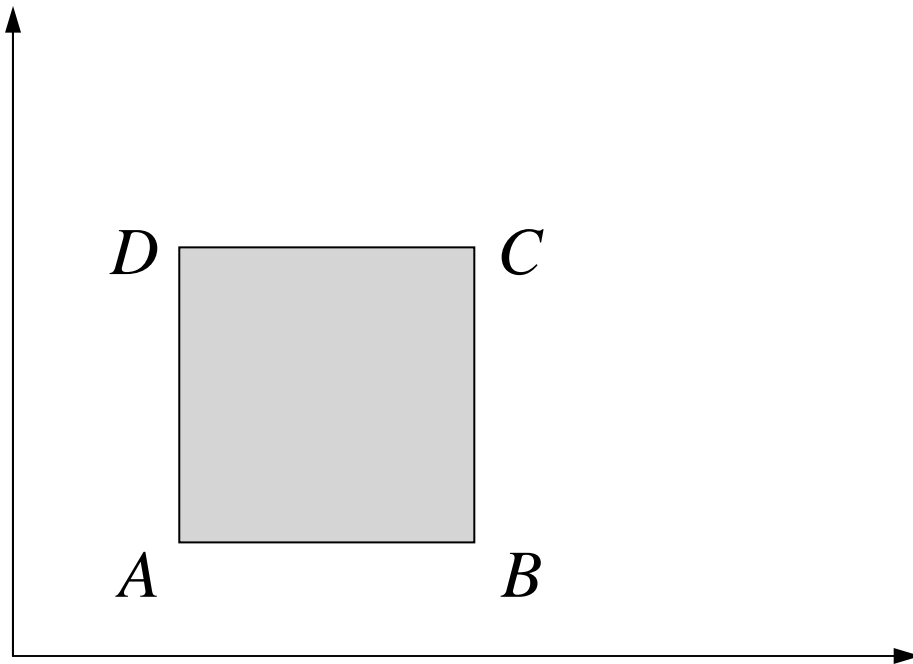
Deformação – 2D



Há distorção no elemento (cisalhamento)

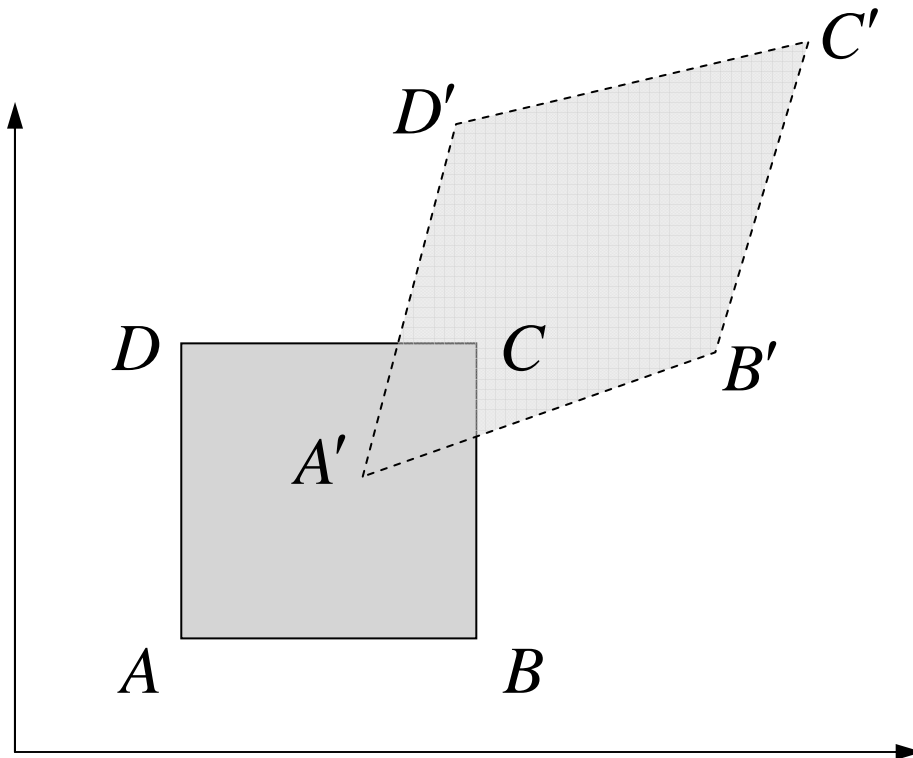
Deformação – 2D

Medida de Deformação – *Relações entre deslocamentos e deformações*



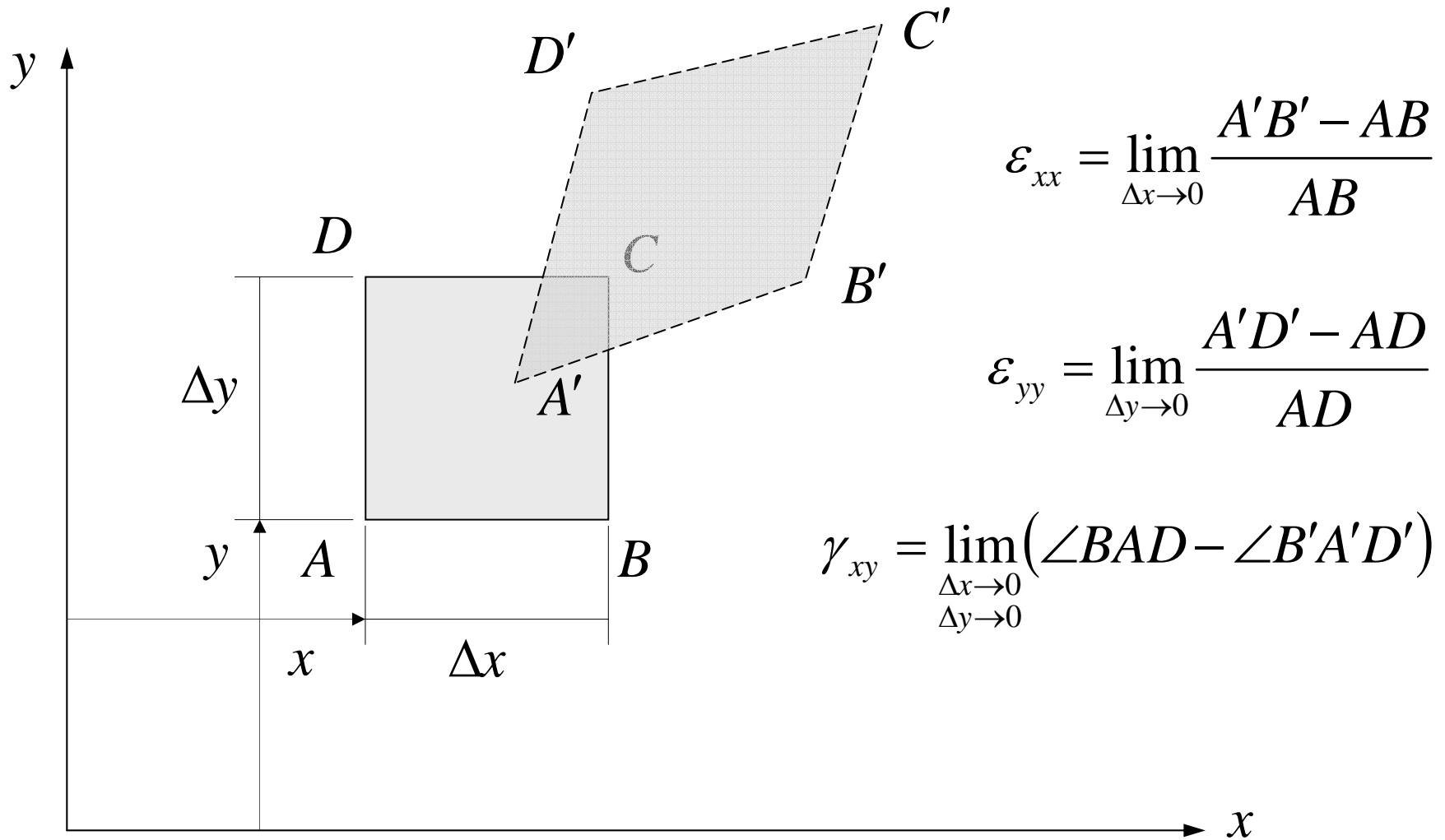
Deformação – 2D

Medida de Deformação – *Relações entre deslocamentos e deformações*

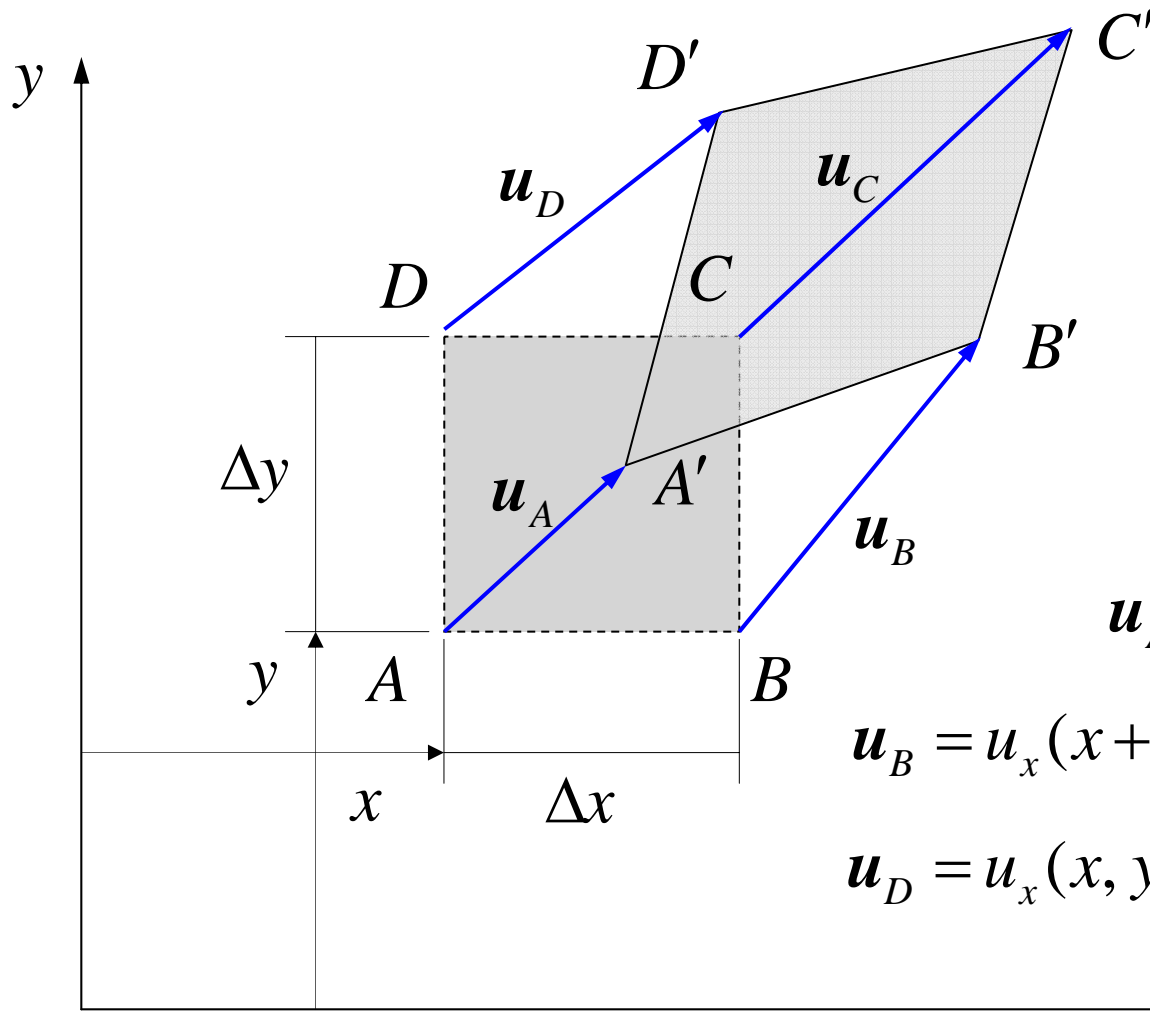


HIPÓTESE
Pequenas
Deformações e
Rotações!

Deformação – 2D



Deformação – 2D



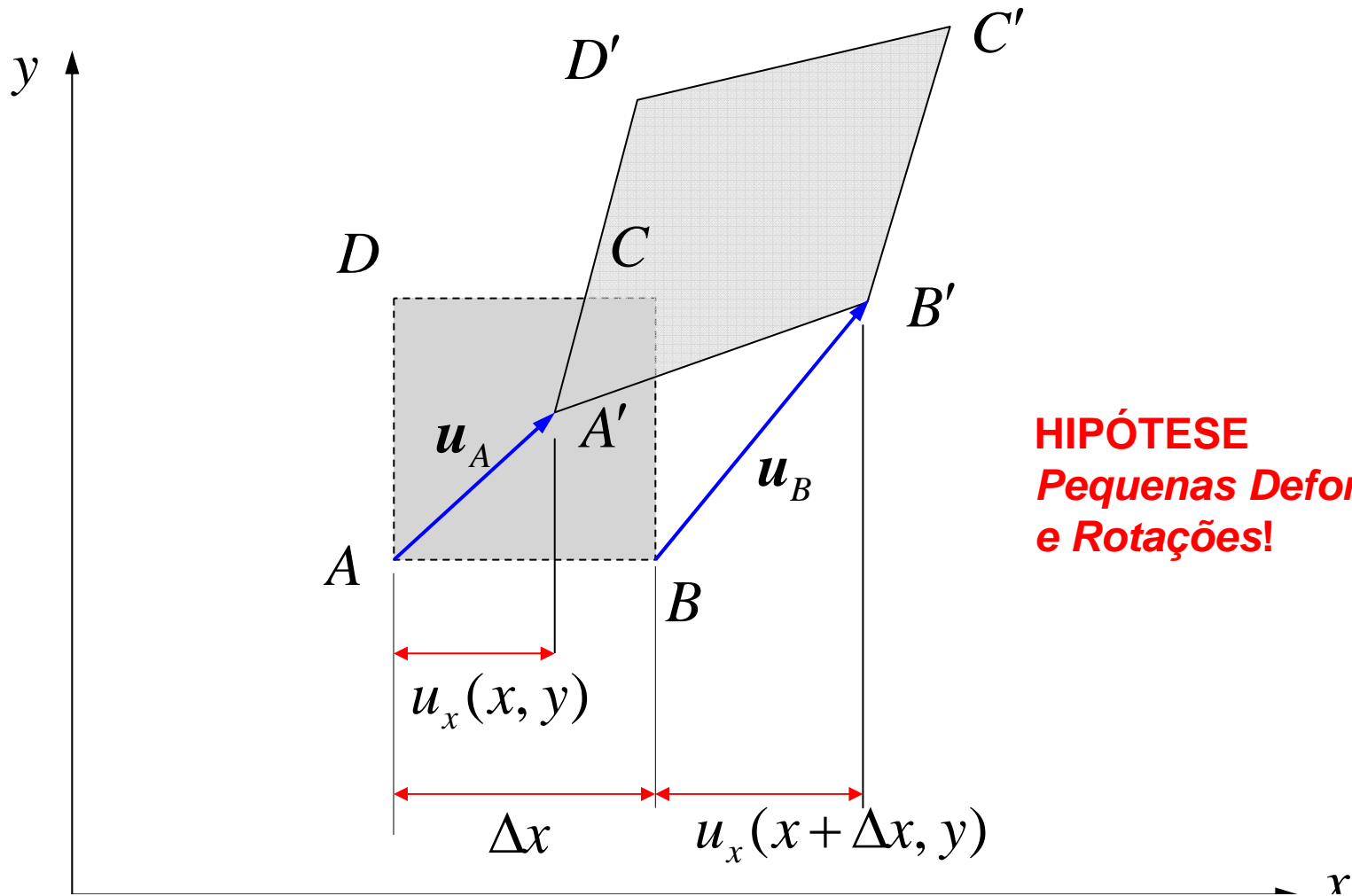
Deslocamentos

$$\mathbf{u}_A = u_x(x, y)\mathbf{i} + u_y(x, y)\mathbf{j}$$

$$\mathbf{u}_B = u_x(x + \Delta x, y)\mathbf{i} + u_y(x + \Delta x, y)\mathbf{j}$$

$$\mathbf{u}_D = u_x(x, y + \Delta y)\mathbf{i} + u_y(x, y + \Delta y)\mathbf{j}$$

Deformação – 2D



HIPÓTESE
*Pequenas Deformações
e Rotações!*

Deformação – 2D

$$\varepsilon_{xx} = \lim_{\Delta x \rightarrow 0} \frac{A'B' - AB}{AB}$$

$$AB = \Delta x$$

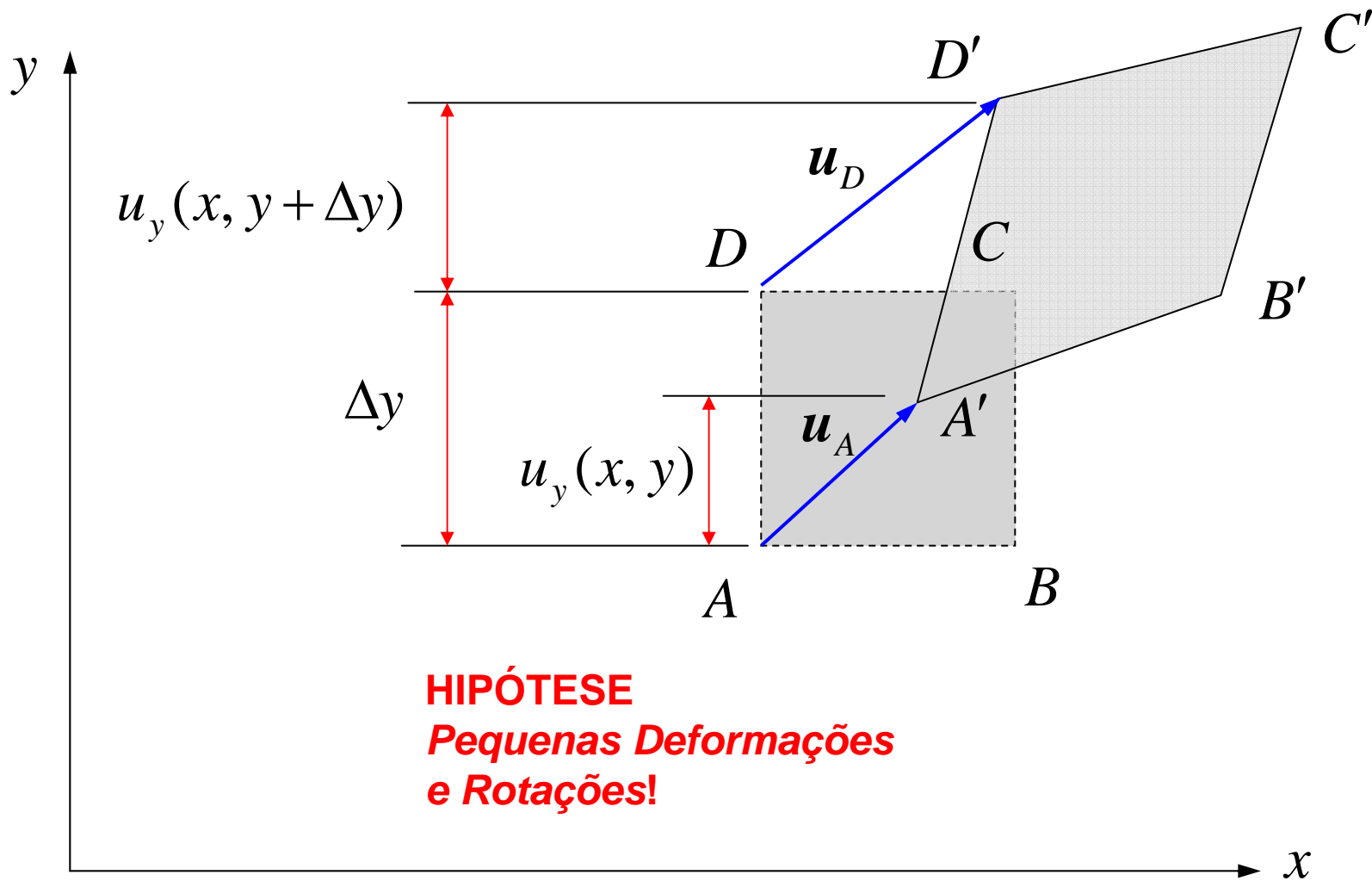
$$A'B' = [\Delta x + u_x(x + \Delta x, y)] - u_x(x, y)$$

$$u_x(x + \Delta x, y) \approx u_x(x, y) + (\partial u_x / \partial x) \Delta x$$

$$A'B' = [\Delta x + u_x(x, y) + (\partial u_x / \partial x) \Delta x] - u_x(x, y) = [\Delta x + (\partial u_x / \partial x) \Delta x]$$

$$\varepsilon_{xx} = \lim_{\Delta x \rightarrow 0} \frac{[\Delta x + (\partial u_x / \partial x) \Delta x] - \Delta x}{\Delta x} = \frac{\partial u_x}{\partial x}$$

Deformação – 2D



Deformação – 2D

$$\varepsilon_{yy} = \lim_{\Delta y \rightarrow 0} \frac{A'D' - AD}{AD}$$

$$AD = \Delta y$$

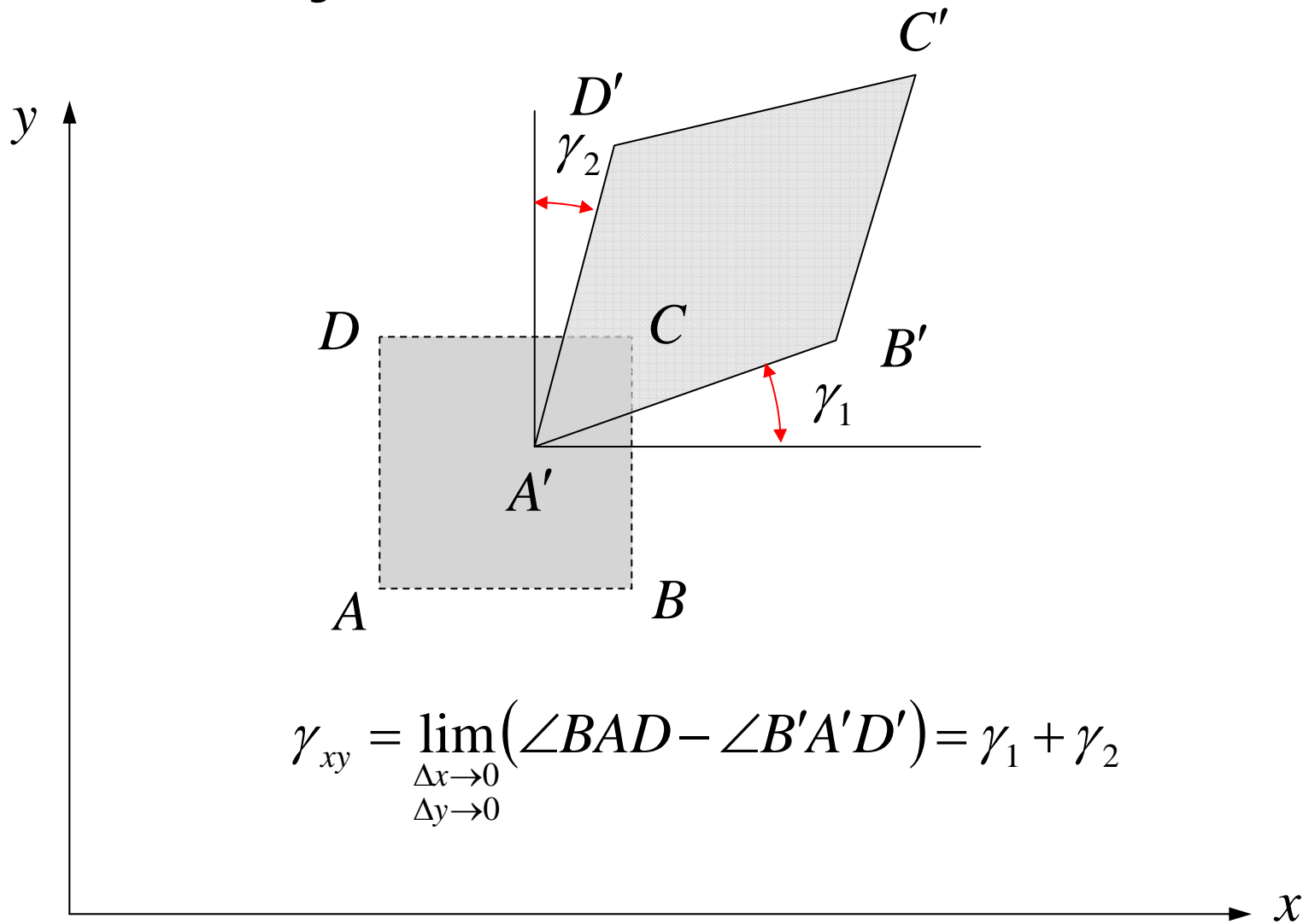
$$A'D' = [\Delta y + u_y(x, y + \Delta y)] - u_y(x, y)$$

$$u_y(x, y + \Delta y) \approx u_y(x, y) + (\partial u_y / \partial y) \Delta y$$

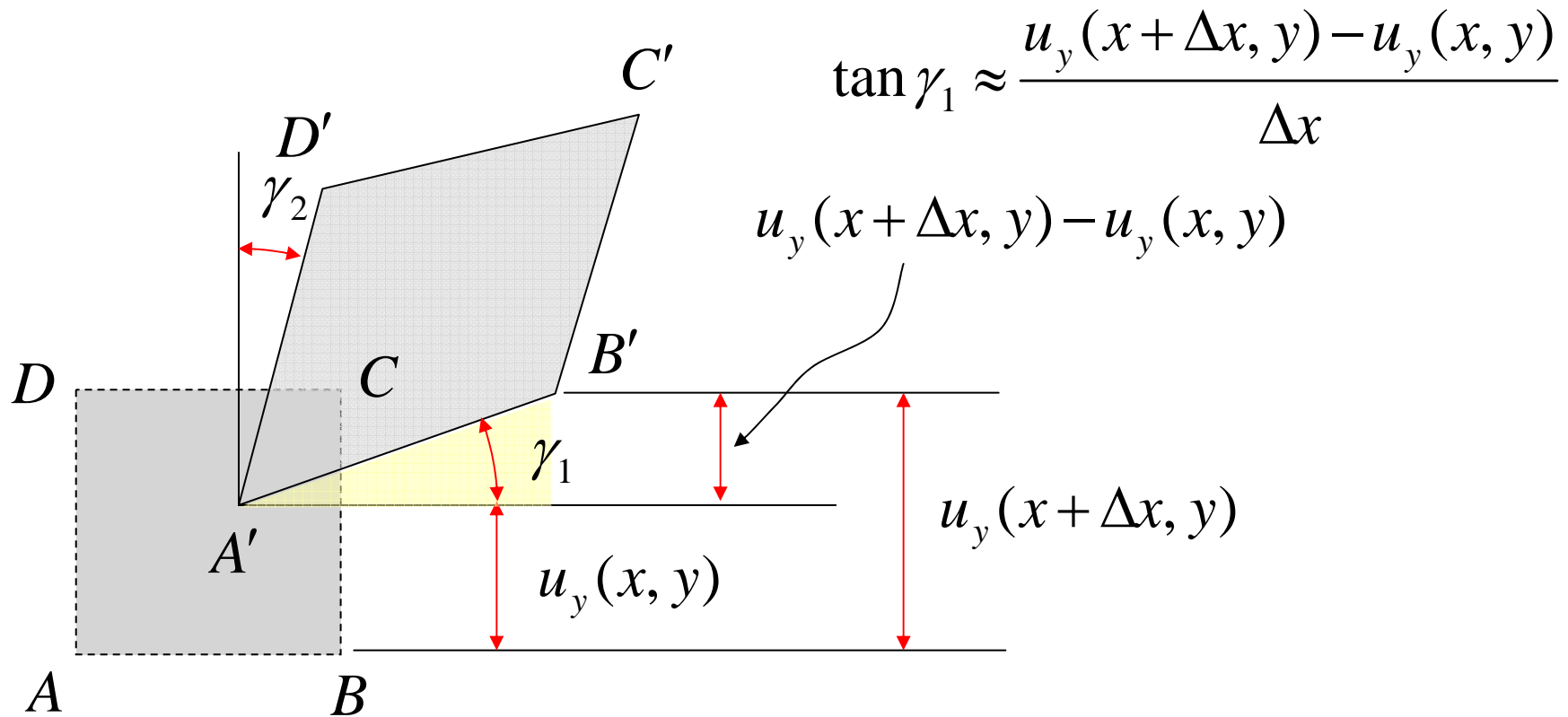
$$A'D' = [\Delta y + u_y(x, y) + (\partial u_y / \partial y) \Delta y] - u_y(x, y) = [\Delta y + (\partial u_y / \partial y) \Delta y]$$

$$\varepsilon_{yy} = \lim_{\Delta y \rightarrow 0} \frac{[\Delta y + (\partial u_y / \partial y) \Delta y] - \Delta y}{\Delta y} = \frac{\partial u_y}{\partial y}$$

Deformação – 2D

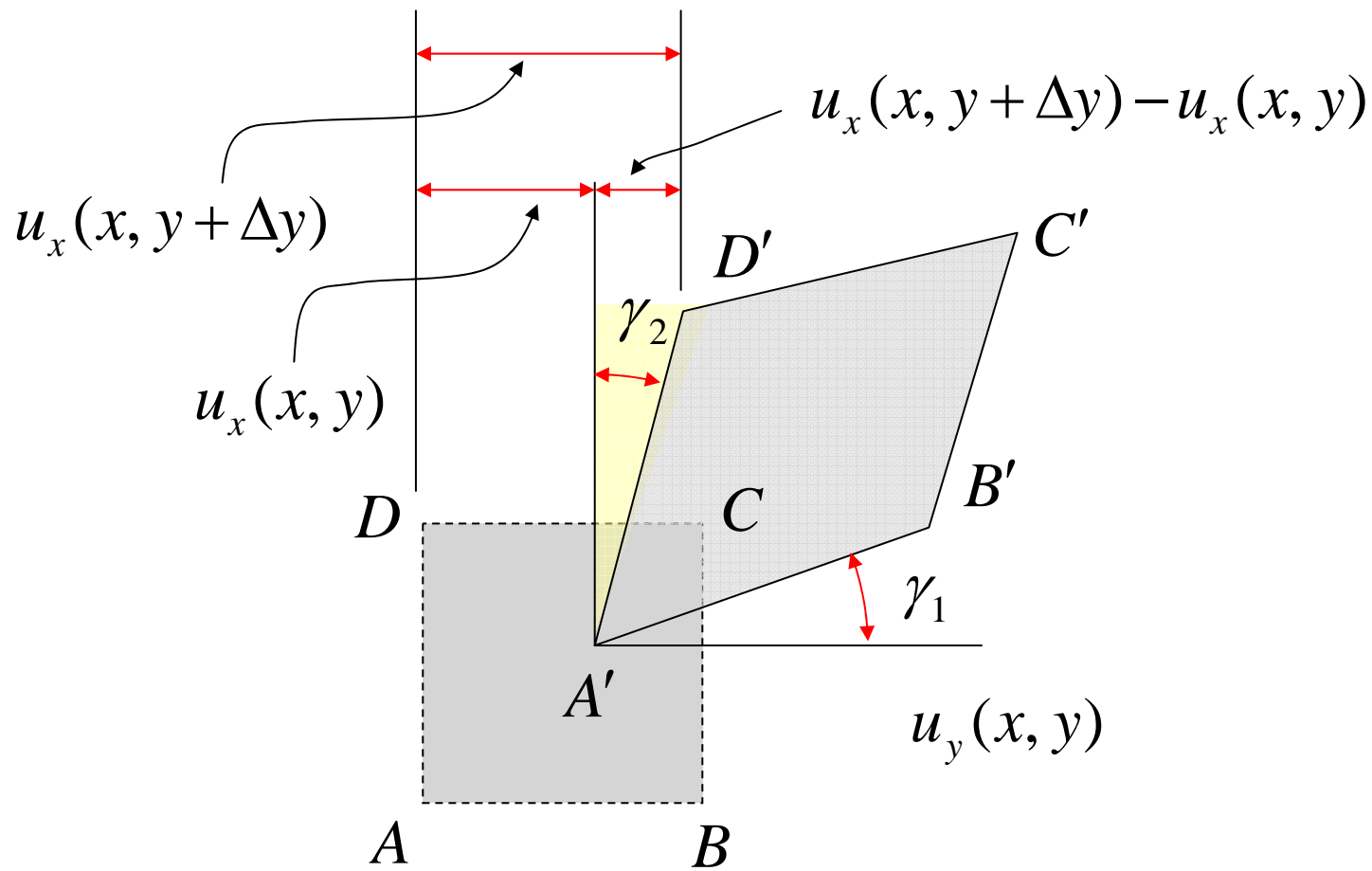


Deformação – 2D



Deformação – 2D

$$\tan \gamma_2 = \frac{u_x(x, y + \Delta y) - u_x(x, y)}{\Delta y}$$



Deformação – 2D

$$\gamma_1 \approx \tan \gamma_1 = \frac{u_y(x + \Delta x, y) - u_y(x, y)}{\Delta x}$$

$$u_y(x + \Delta x, y) \approx u_y(x, y) + (\partial u_y / \partial x) \Delta x$$

$$\gamma_1 = \frac{\partial u_y}{\partial x}$$

$$\gamma_2 \approx \tan \gamma_2 = \frac{u_x(x, y + \Delta y) - u_x(x, y)}{\Delta y}$$

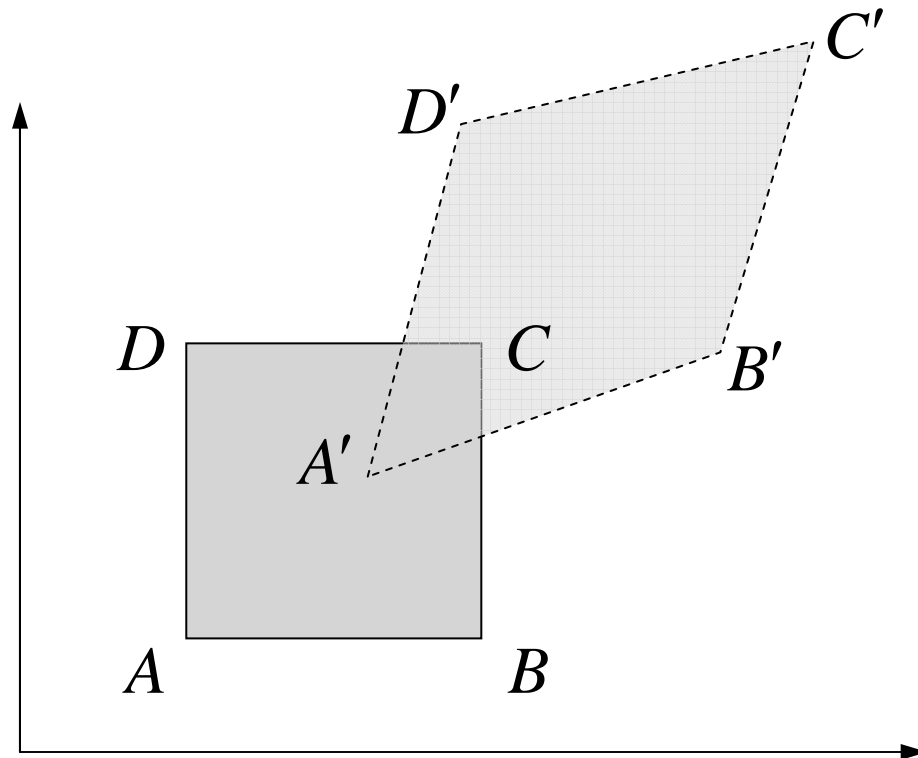
$$u_x(x, y + \Delta y) \approx u_x(x, y) + (\partial u_x / \partial y) \Delta y$$

$$\gamma_2 = \frac{\partial u_x}{\partial y}$$

$$\gamma_{xy} = \gamma_1 + \gamma_2 = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

Deformação – 2D

Medida de Deformação – *Relações entre deslocamentos e deformações*



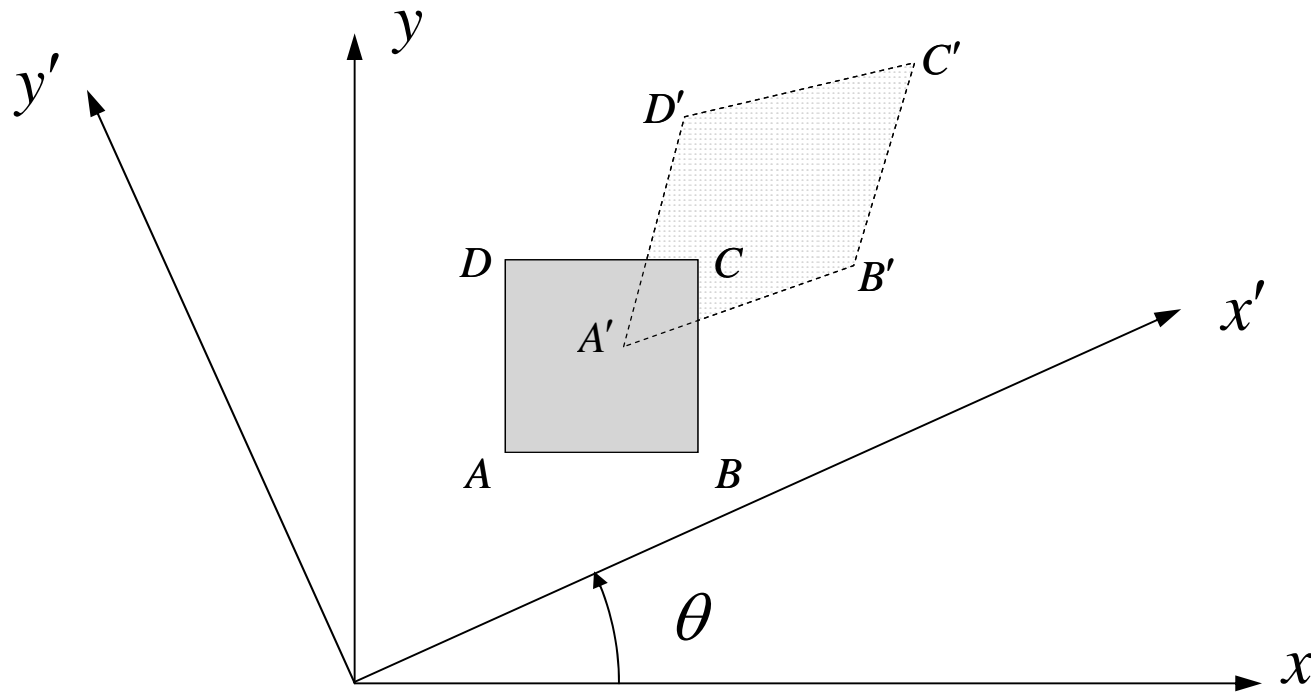
$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$$

$$\varepsilon_{yy} = \frac{\partial u_y}{\partial y}$$

$$\varepsilon_{xy} = \frac{1}{2} \gamma_{xy} = \frac{1}{2} \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right)$$

Deformação – 2D

Medida de Deformação – *Mudança do sistemas de coordenada*



$$\varepsilon_{x'x'} = \frac{\partial u'_x}{\partial x'}$$

$$\varepsilon_{y'y'} = \frac{\partial u'_y}{\partial y'}$$

$$\gamma_{x'y'} = \frac{\partial u'_x}{\partial y'} + \frac{\partial u'_y}{\partial x'}$$

Deformação – 2D

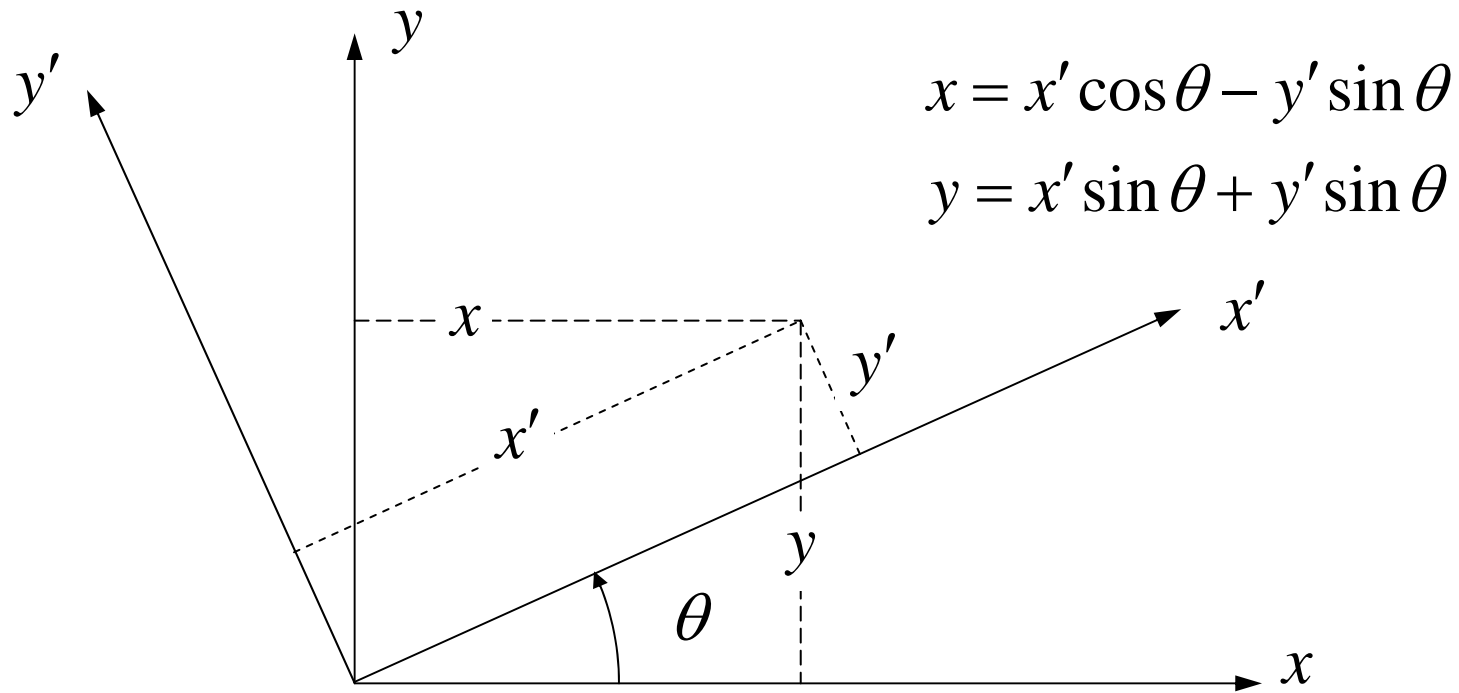
Medida de Deformação – *Mudança do sistemas de coordenada*

$$\varepsilon_{x'x'} = \frac{\partial u'_x}{\partial x'} = \frac{\partial u'_x}{\partial x} \frac{\partial x}{\partial x'} + \frac{\partial u'_x}{\partial y} \frac{\partial y}{\partial x'}$$

$$\varepsilon_{y'y'} = \frac{\partial u'_y}{\partial y'} = \frac{\partial u'_y}{\partial x} \frac{\partial x}{\partial y'} + \frac{\partial u'_y}{\partial y} \frac{\partial y}{\partial y'}$$

$$\gamma_{x'y'} = \frac{\partial u'_x}{\partial y'} + \frac{\partial u'_y}{\partial x'} = \frac{\partial u'_x}{\partial x} \frac{\partial x}{\partial y'} + \frac{\partial u'_x}{\partial y} \frac{\partial y}{\partial y'} + \frac{\partial u'_y}{\partial x} \frac{\partial x}{\partial x'} + \frac{\partial u'_y}{\partial y} \frac{\partial y}{\partial x'}$$

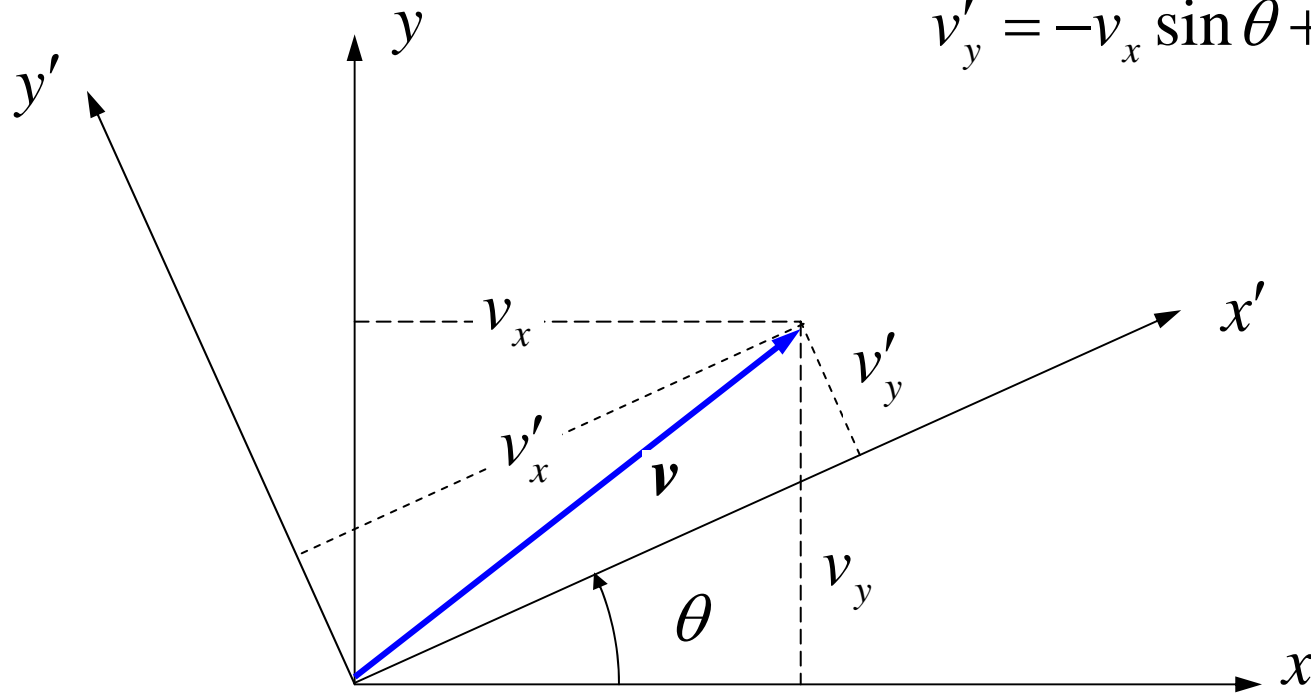
Deformação – 2D



Deformação – 2D

$$v'_x = v_x \cos \theta + v_y \sin \theta$$

$$v'_y = -v_x \sin \theta + v_y \cos \theta$$



Deformação – 2D

$$\begin{aligned}\varepsilon_{x'x'} &= \left(\frac{\partial u_x}{\partial x} \cos \theta + \frac{\partial u_y}{\partial x} \sin \theta \right) \cos \theta + \left(\frac{\partial u_x}{\partial y} \cos \theta + \frac{\partial u_y}{\partial y} \sin \theta \right) \sin \theta \\ &= \frac{\partial u_x}{\partial x} \cos^2 \theta + \frac{\partial u_y}{\partial y} \sin^2 \theta + \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) \sin \theta \cos \theta\end{aligned}$$

$$\varepsilon_{x'x'} = \varepsilon_{xx} \cos^2 \theta + \varepsilon_{yy} \sin^2 \theta + \gamma_{xy} \sin \theta \cos \theta$$

$$\varepsilon_{x'x'} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \cos 2\theta + \frac{1}{2} \gamma_{xy} \sin 2\theta$$

$$\varepsilon_{x'x'} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \cos 2\theta + \varepsilon_{xy} \sin 2\theta$$

Deformação – 2D

Medida de Deformação – *Mudança do sistemas de coordenada*

$$\varepsilon_{x'x'} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \cos 2\theta + \varepsilon_{xy} \sin 2\theta$$

$$\varepsilon_{y'y'} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} - \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \cos 2\theta - \varepsilon_{xy} \sin 2\theta$$

$$\varepsilon_{x'y'} = \frac{1}{2} \gamma_{x'y'} = -\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \sin 2\theta + \varepsilon_{xy} \cos 2\theta$$

Deformação – 3D

Tensor de Deformação $[\varepsilon] = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix}$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y} \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z}$$

$$\varepsilon_{xy} = \frac{1}{2} \gamma_{xy} = \frac{1}{2} \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) \quad \varepsilon_{xz} = \frac{1}{2} \gamma_{xz} = \frac{1}{2} \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right)$$

$$\varepsilon_{yz} = \frac{1}{2} \gamma_{yz} = \frac{1}{2} \left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} \right)$$

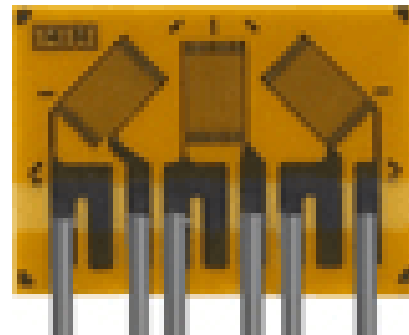
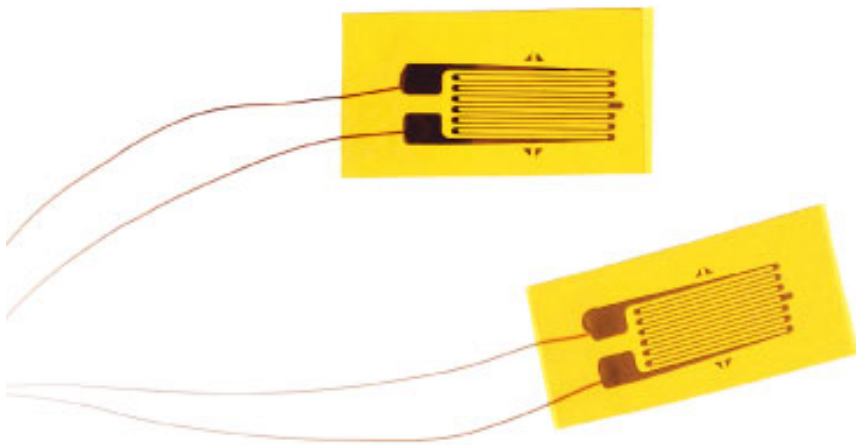
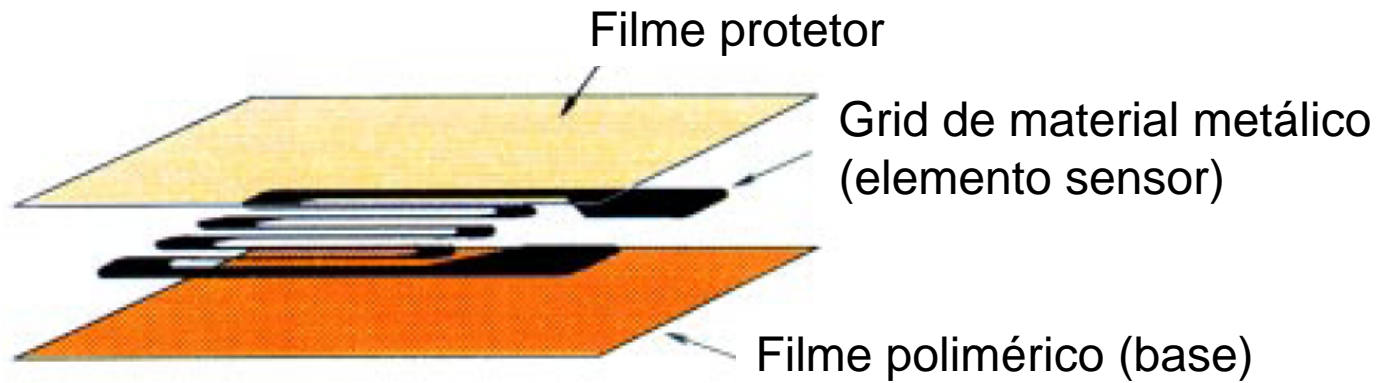
Medida Experimental de Deformações

Técnicas para medida de deformação

- Mecânicas
- Elétricas
- Ópticas
- Acústicas

Medida Experimental de Deformações

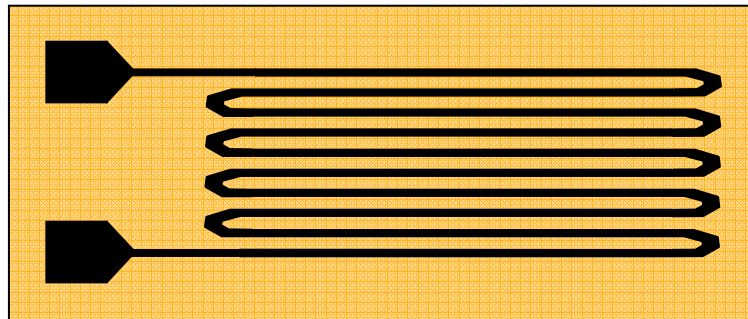
Extensômetros Elétricos Resistivos



Medida Experimental de Deformações

Extensômetros Elétricos Resistivos

Extensômetros uniaxiais



$$R = \rho \frac{L}{A}$$

R – Resistência

r – Resistividade

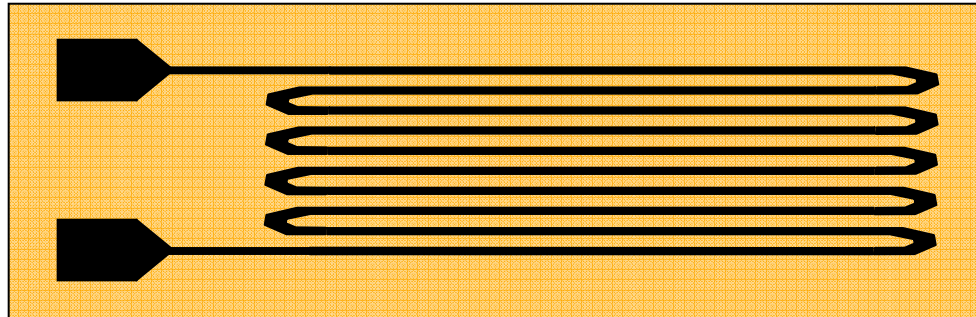
L – Comprimento do fio metálico

A – Área

Medida Experimental de Deformações

Extensômetros Elétricos Resistivos

Extensômetros uniaxiais

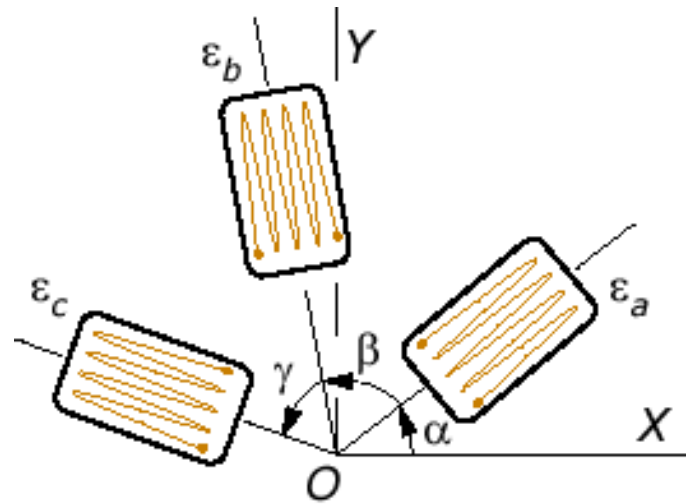


$$\frac{\Delta R}{R} = S_g \varepsilon$$

Medida Experimental de Deformações

Extensômetros Elétricos Resistivos

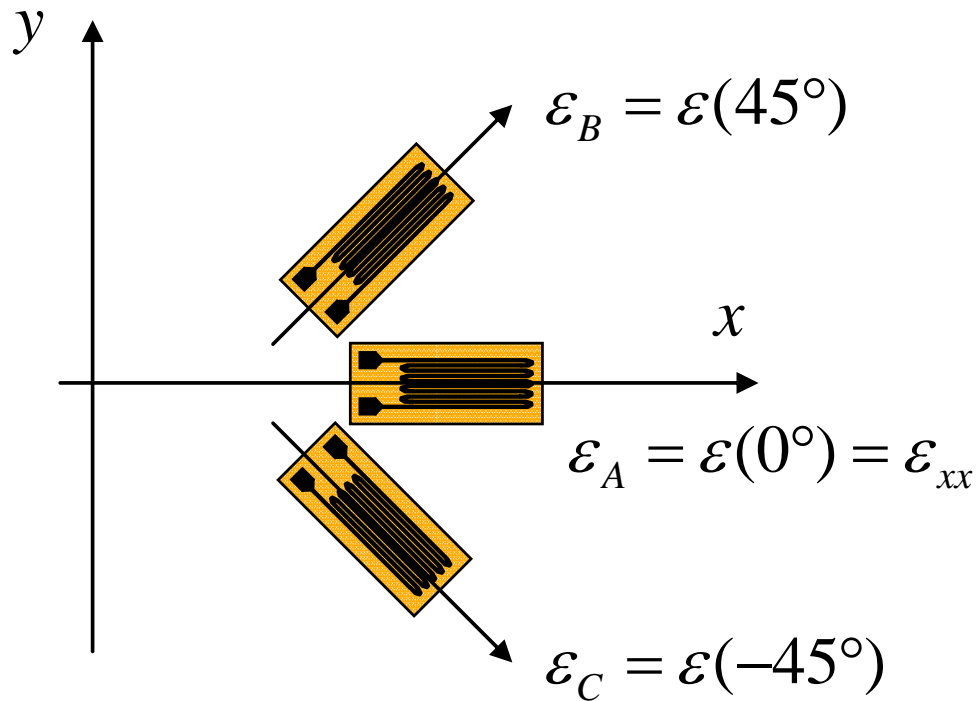
Extensômetros triaxiais (rosetas extensométricas)



$$\epsilon(\theta) = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} + \frac{\epsilon_{xx} - \epsilon_{yy}}{2} \cos 2\theta + \epsilon_{xy} \sin 2\theta$$

Medida Experimental de Deformações

Roseta a 45°



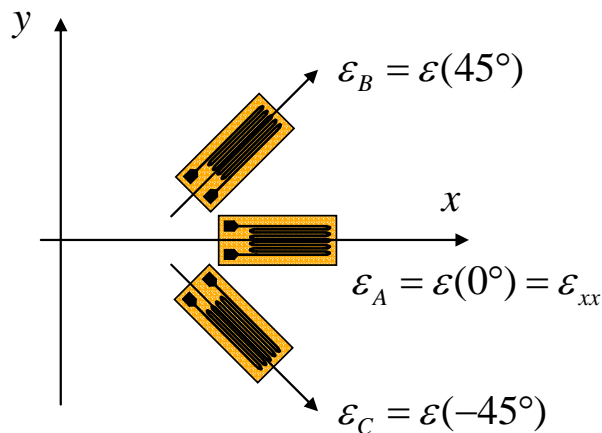
$$\varepsilon(\theta) = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \cos 2\theta + \varepsilon_{xy} \sin 2\theta$$

Medida Experimental de Deformações

Roseta a 45°

$$\varepsilon_B = \frac{\varepsilon_A + \varepsilon_{yy}}{2} + \frac{\varepsilon_A - \varepsilon_{yy}}{2} \cos(90^\circ) + \varepsilon_{xy} \sin(90^\circ) = \frac{\varepsilon_A + \varepsilon_{yy}}{2} + \varepsilon_{xy}$$

$$\varepsilon_C = \frac{\varepsilon_A + \varepsilon_{yy}}{2} + \frac{\varepsilon_A - \varepsilon_{yy}}{2} \cos(-90^\circ) + \varepsilon_{xy} \sin(-90^\circ) = \frac{\varepsilon_A + \varepsilon_{yy}}{2} - \varepsilon_{xy}$$



$$\varepsilon_{xx} = \varepsilon_A$$

$$\varepsilon_{yy} = \varepsilon_B + \varepsilon_C - \varepsilon_A$$

$$\varepsilon_{xy} = \frac{\varepsilon_B - \varepsilon_C}{2}$$

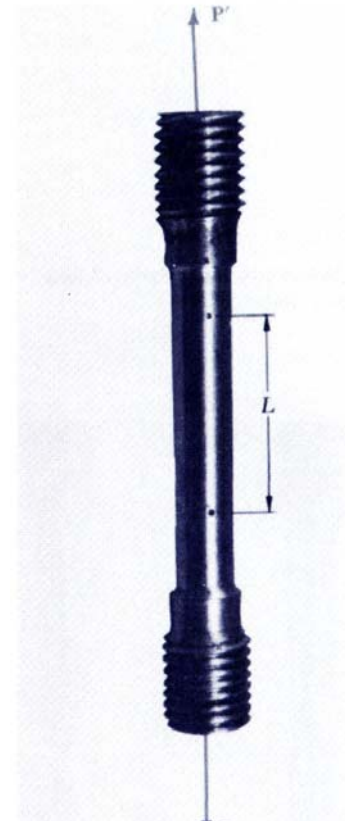
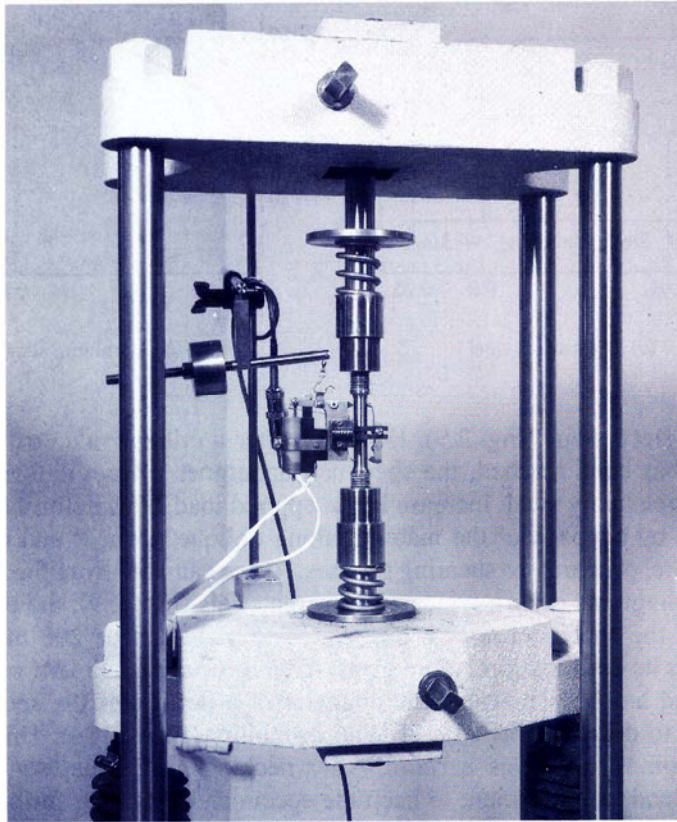
Relação Tensão *vs.* Deformação

Lei de Hooke Generalizada

- Material Isotrópico
- Material Elástico
- Material Linear
- Pequenas Deformações

Relação Tensão vs. Deformação

Ensaio de Tração

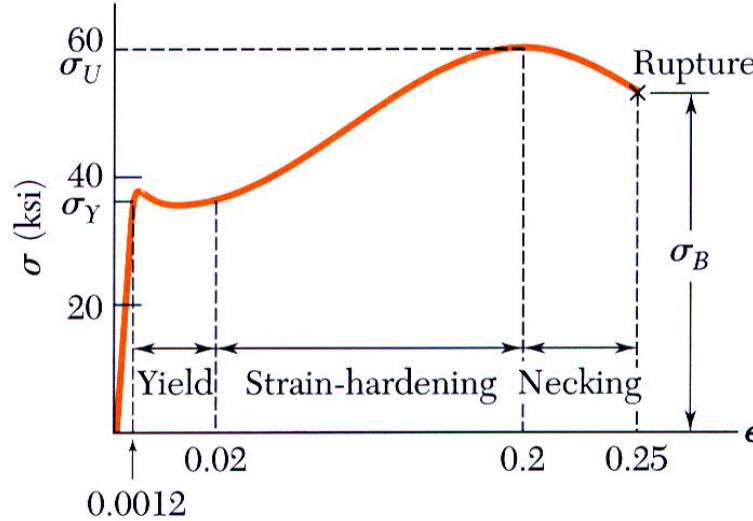
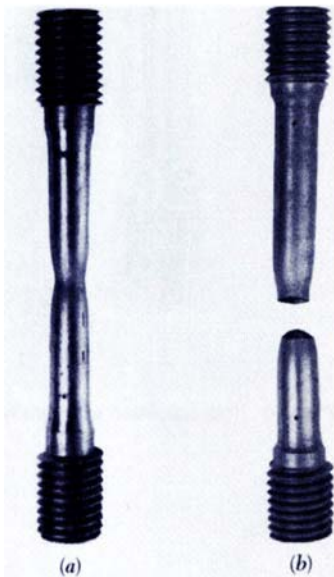


Figuras reproduzidas de:

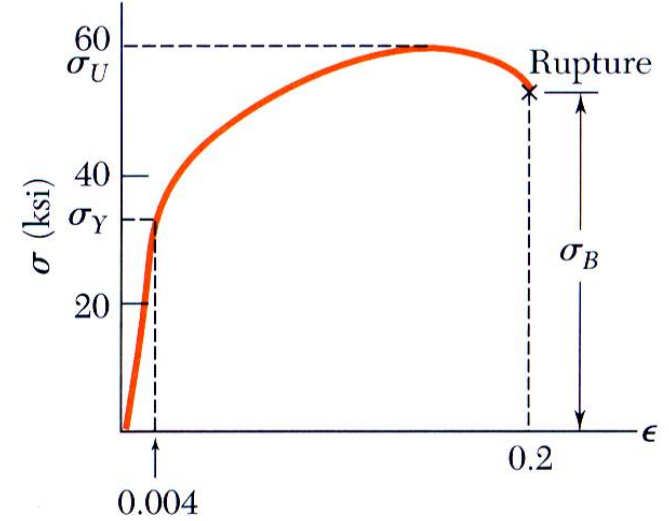
Beer, Johnston & DeWolf, *Mechanics of Materials*, 4th ed., McGraw-Hill, 2002

Relação Tensão vs. Deformação

Ensaio de Tração



(a) Low-carbon steel



(b) Aluminum alloy

Figuras reproduzidas de:
Beer, Johnston & DeWolf, *Mechanics of
Materials*, 4th ed., McGraw-Hill, 2002

Relação Tensão vs. Deformação

Ensaio de Tração

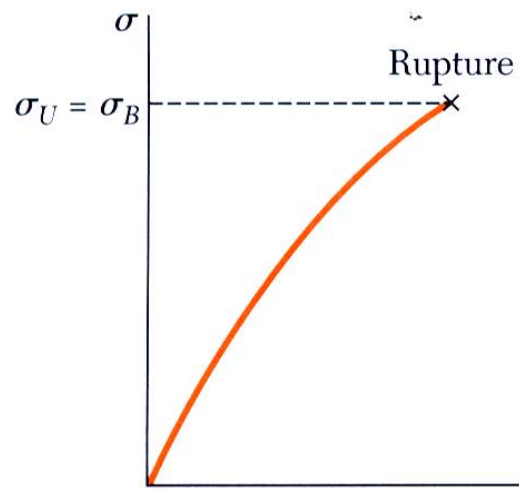
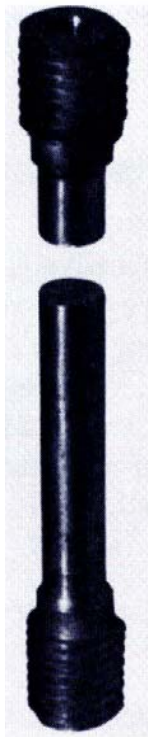
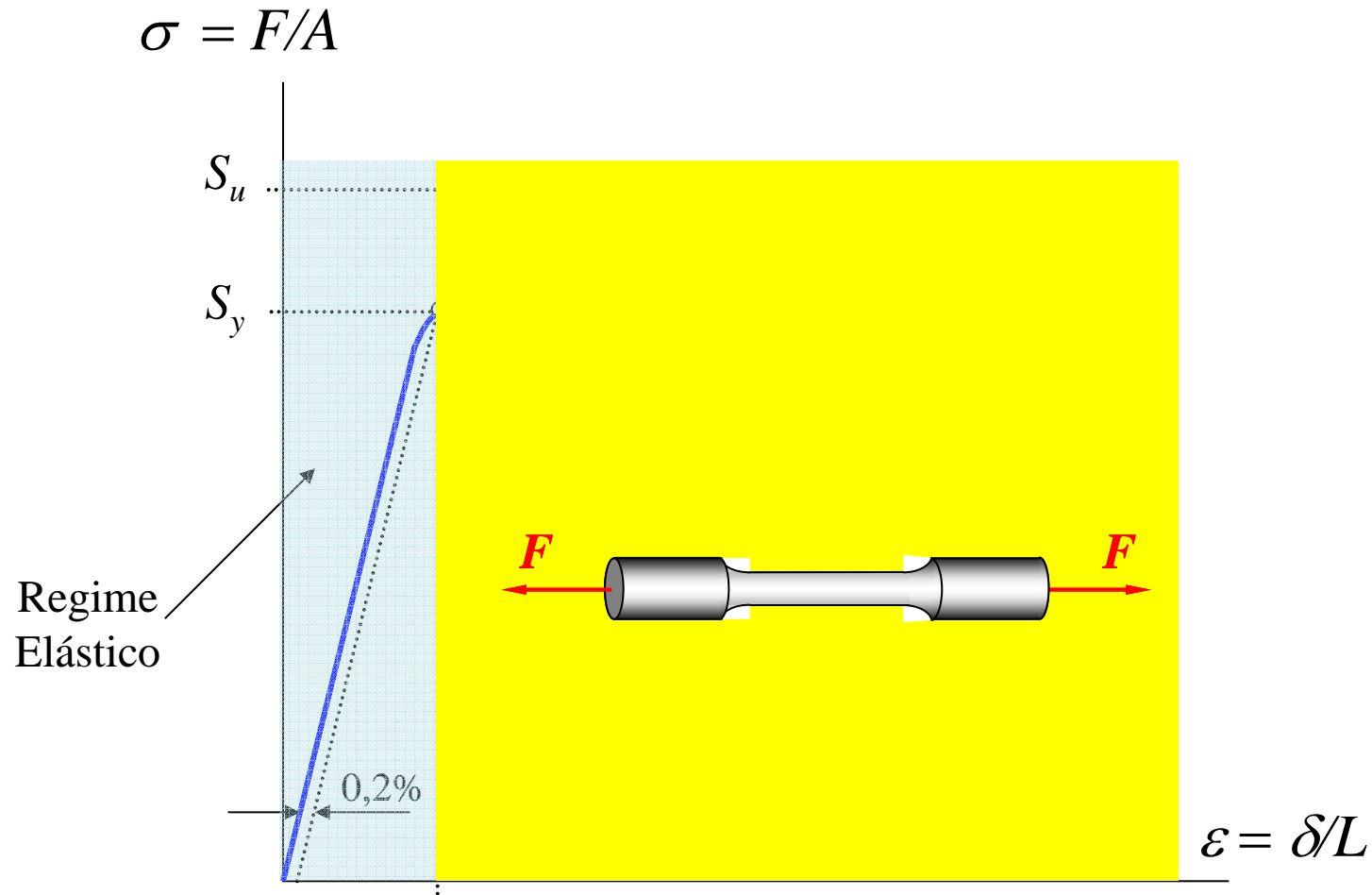


Fig. 2.11 Stress-strain diagram for a typical brittle material.

Figuras reproduzidas de:
Beer, Johnston & DeWolf, *Mechanics of
Materials*, 4th ed., McGraw-Hill, 2002

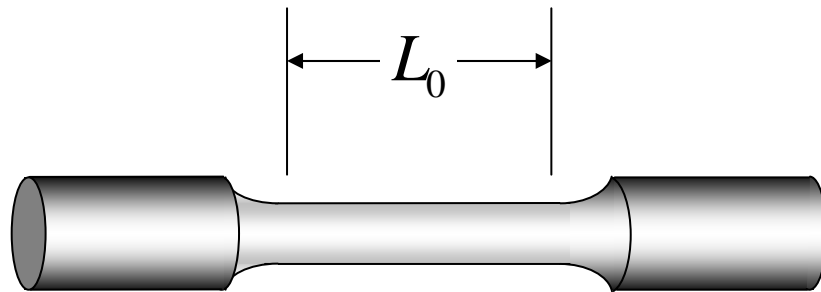
Relação Tensão vs. Deformação

Relação entre Tensão e Deformação



Relação Tensão vs. Deformação

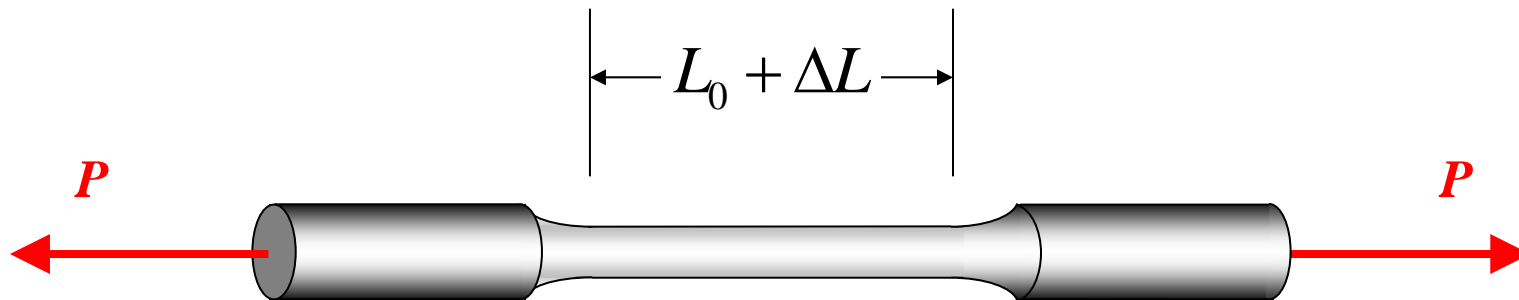
Deformação longitudinal



$$\varepsilon_L = \Delta L / L_0$$

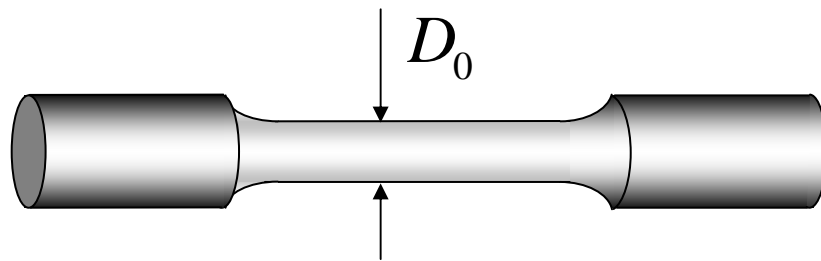
$$\sigma = P / A$$

$$\varepsilon_L = \sigma / E$$



Relação Tensão vs. Deformação

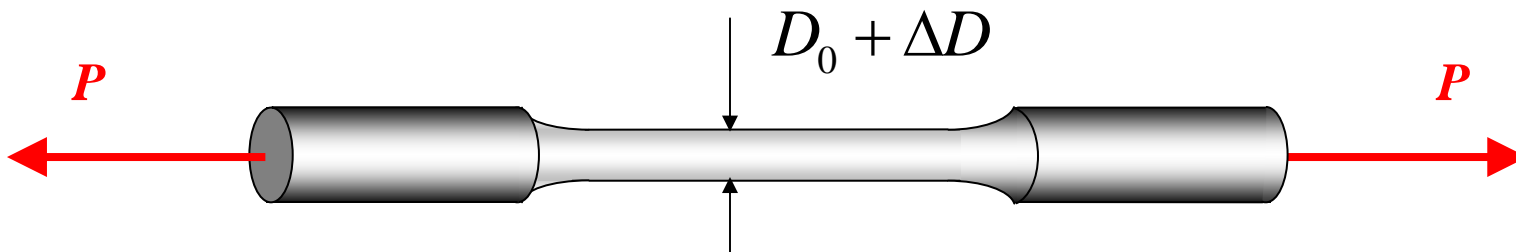
Deformação transversal



$$\varepsilon_T = \Delta D / D_0$$

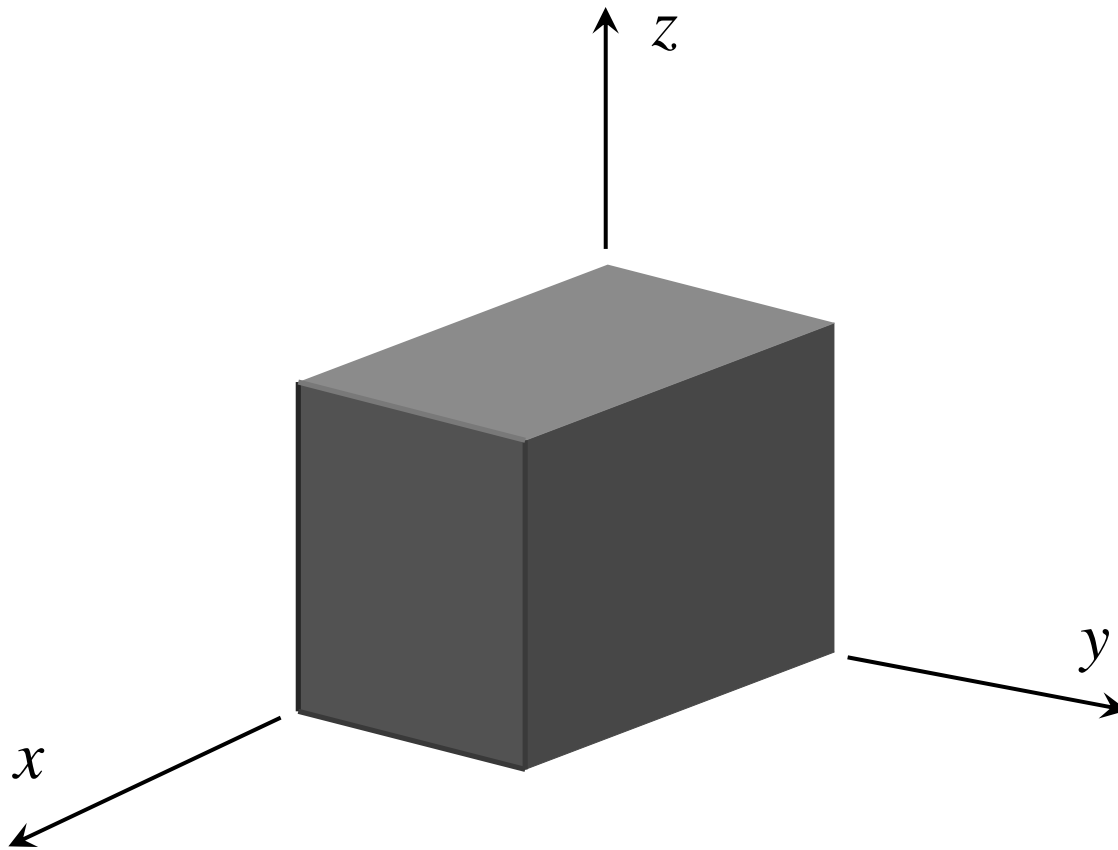
$$\Delta D / D_0 = -\nu(\Delta L / L_0)$$

$$\varepsilon_T = -\nu\varepsilon_L = -\nu(\sigma / E)$$



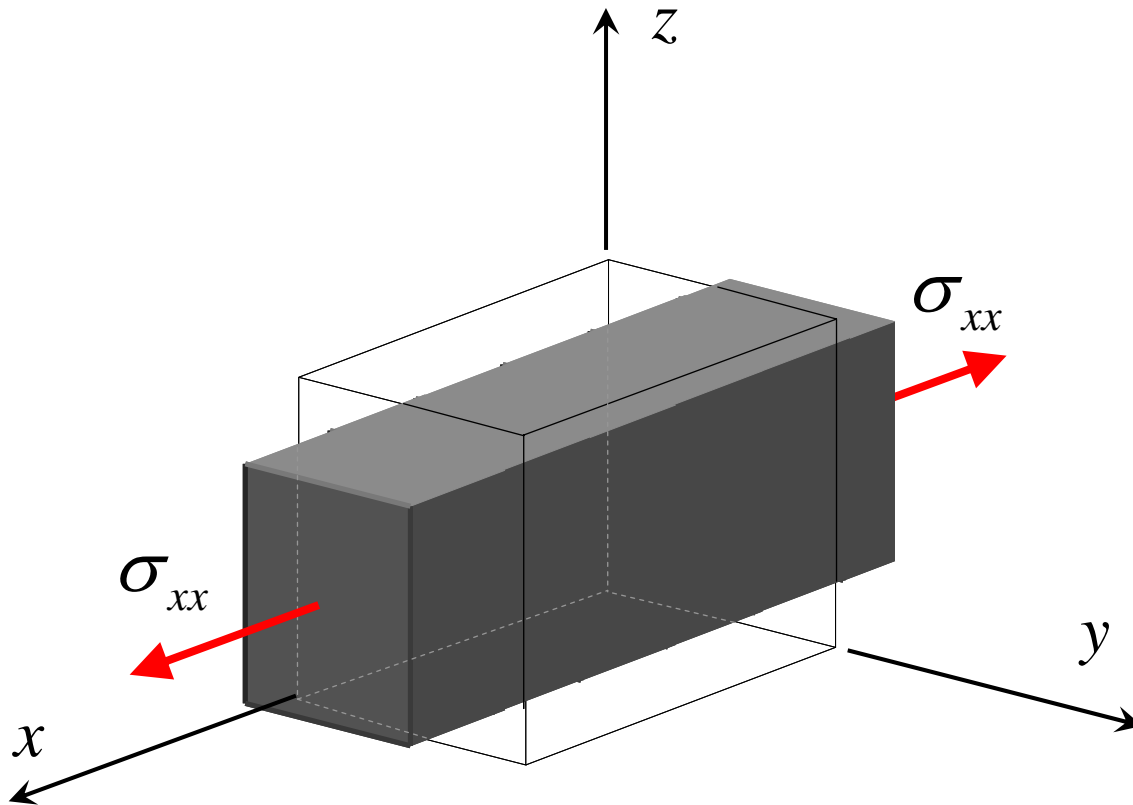
Relação Tensão vs. Deformação

Estado uniaxial de tensão (direção x)



Relação Tensão vs. Deformação

Estado uniaxial de tensão (direção x)



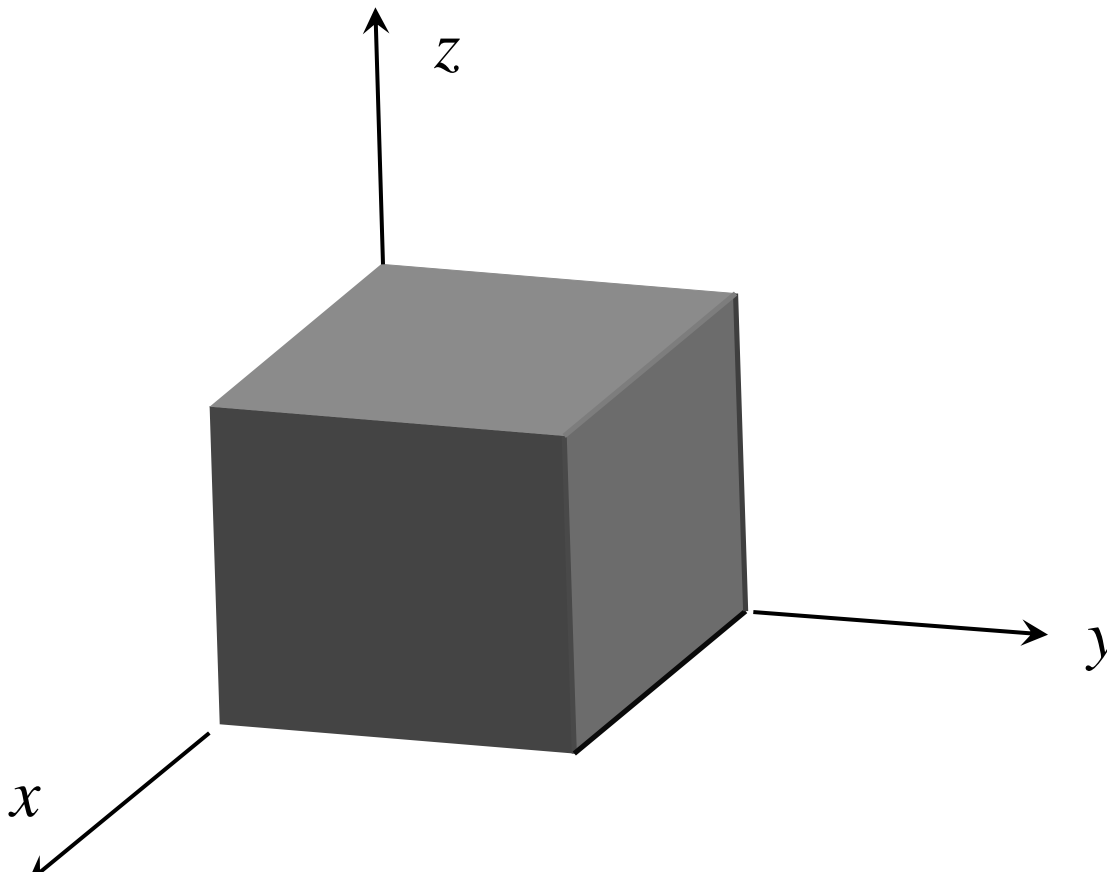
$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E}$$

$$\varepsilon_{yy} = -\nu \frac{\sigma_{xx}}{E}$$

$$\varepsilon_{zz} = -\nu \frac{\sigma_{xx}}{E}$$

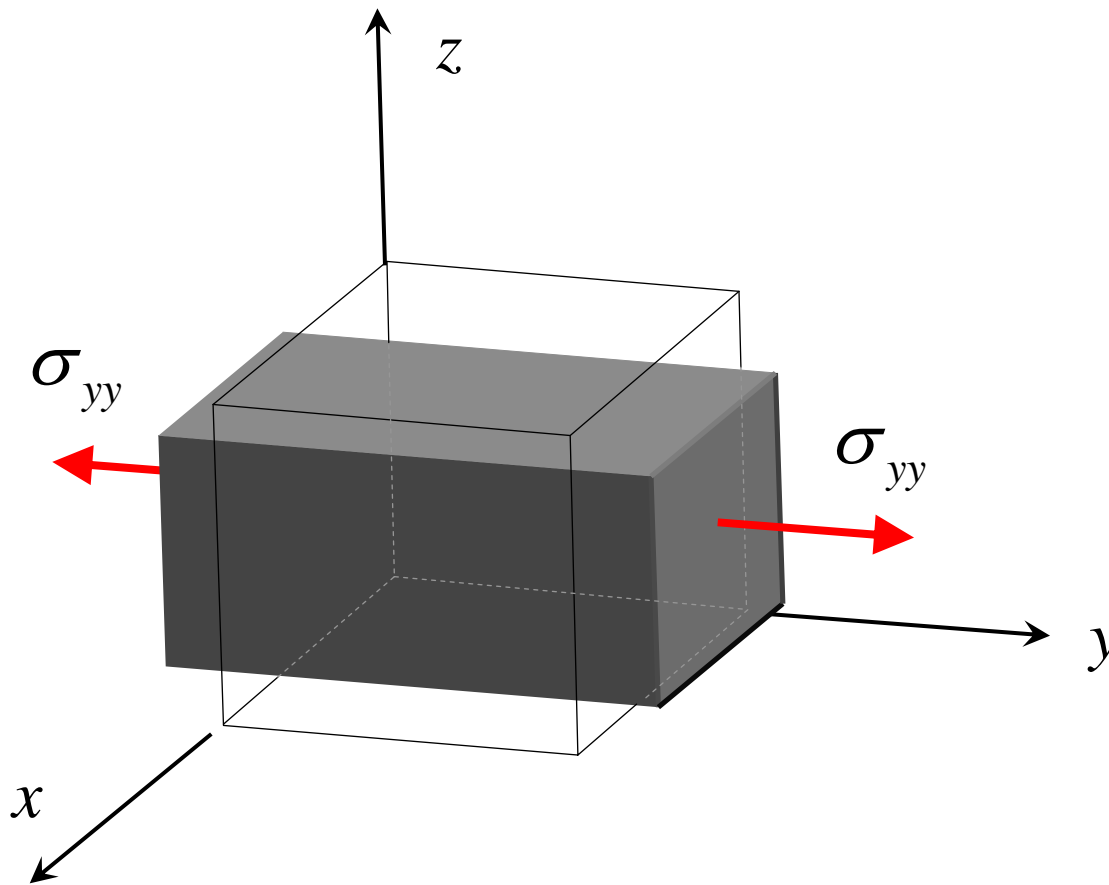
Relação Tensão vs. Deformação

Estado uniaxial de tensão (direção y)



Relação Tensão vs. Deformação

Estado uniaxial de tensão (direção y)



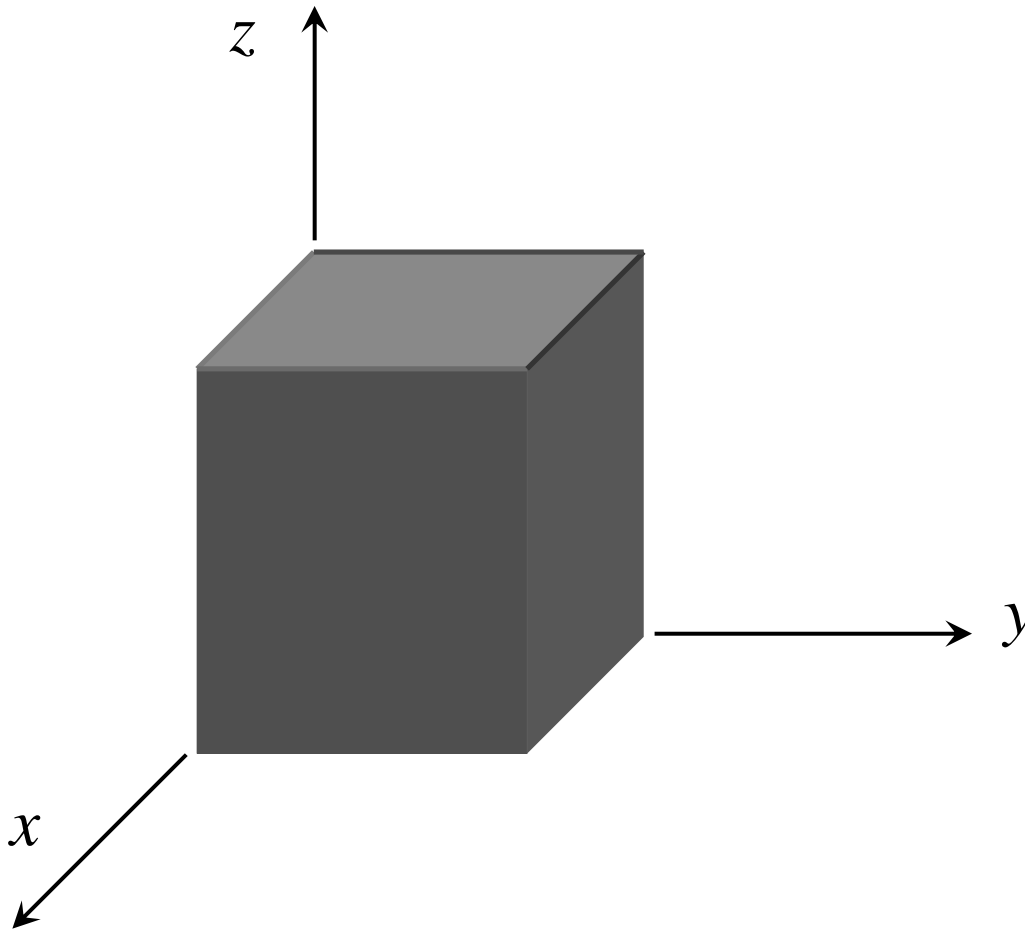
$$\varepsilon_{xx} = -\nu \frac{\sigma_{yy}}{E}$$

$$\varepsilon_{yy} = \frac{\sigma_{yy}}{E}$$

$$\varepsilon_{zz} = -\nu \frac{\sigma_{yy}}{E}$$

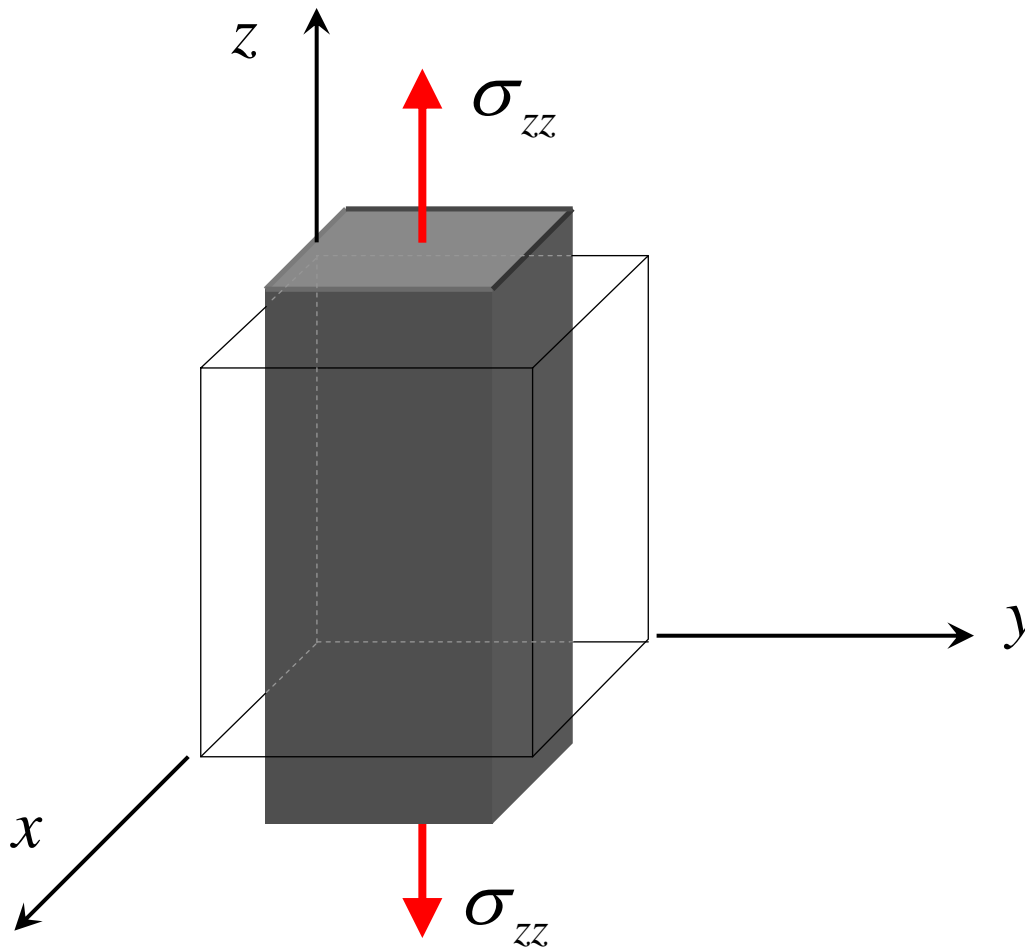
Relação Tensão vs. Deformação

Estado uniaxial de tensão (direção z)



Relação Tensão vs. Deformação

Estado uniaxial de tensão (direção y)



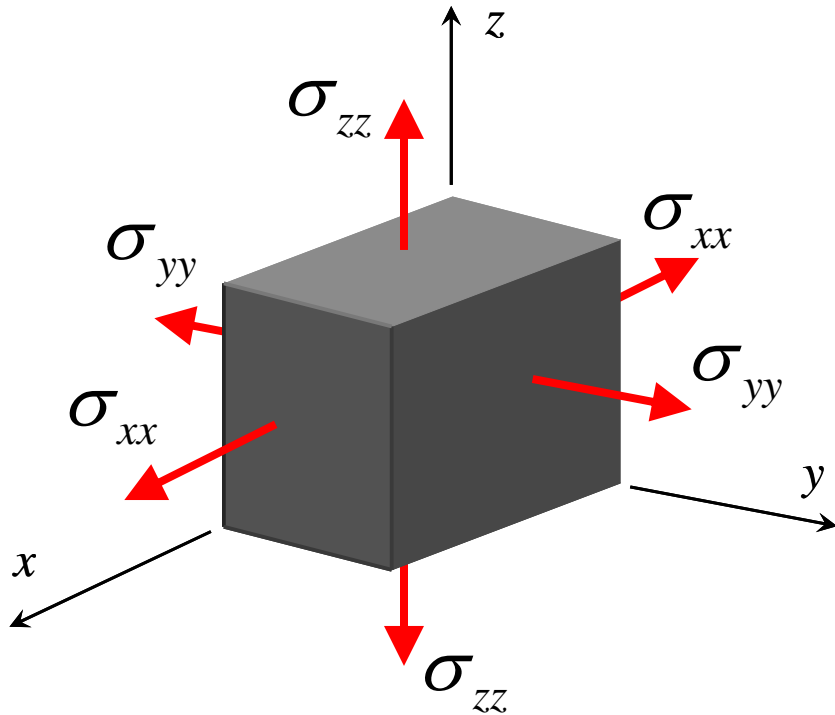
$$\varepsilon_{xx} = -\nu \frac{\sigma_{zz}}{E}$$

$$\varepsilon_{yy} = -\nu \frac{\sigma_{zz}}{E}$$

$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E}$$

Relação Tensão vs. Deformação

Estado triaxial de tensão (emprega-se o princípio da superposição):



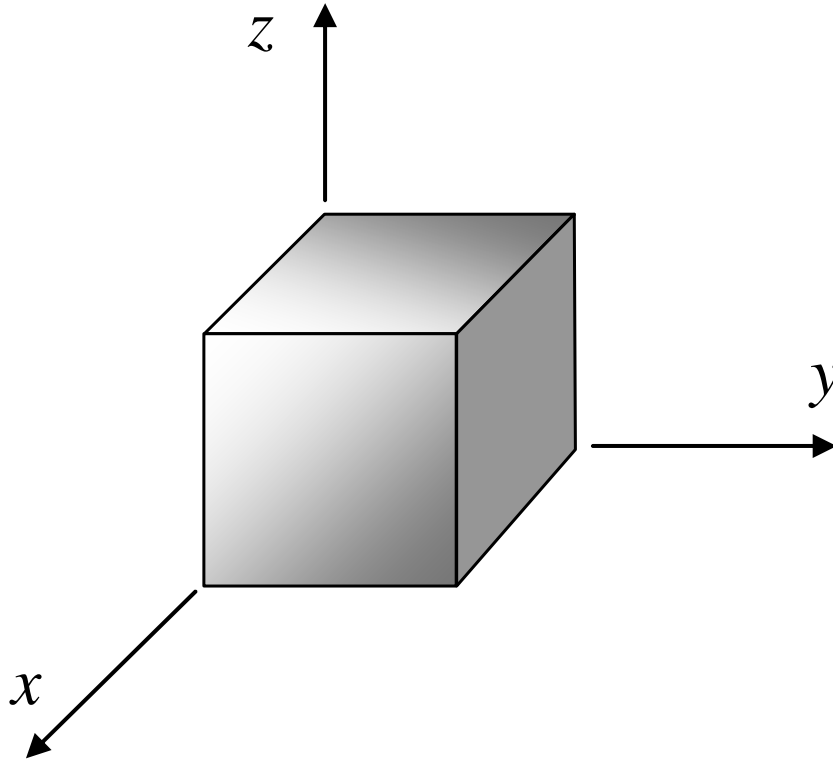
$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \nu \frac{\sigma_{yy}}{E} - \nu \frac{\sigma_{zz}}{E}$$

$$\varepsilon_{yy} = -\nu \frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \nu \frac{\sigma_{zz}}{E}$$

$$\varepsilon_{zz} = -\nu \frac{\sigma_{xx}}{E} - \nu \frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}$$

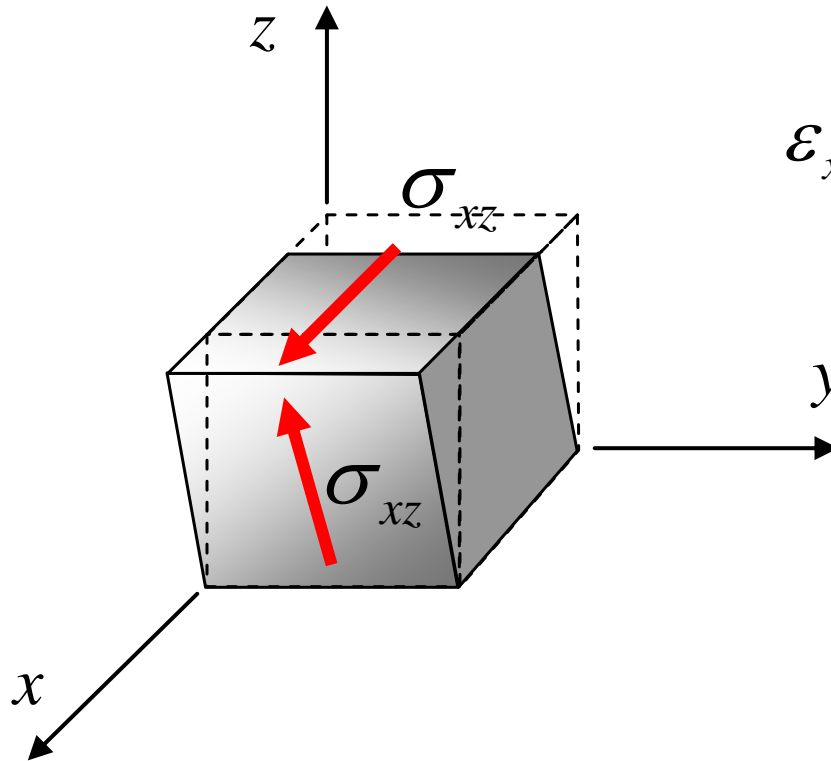
Relação Tensão vs. Deformação

Cisalhamento puro (plano xz)



Relação Tensão vs. Deformação

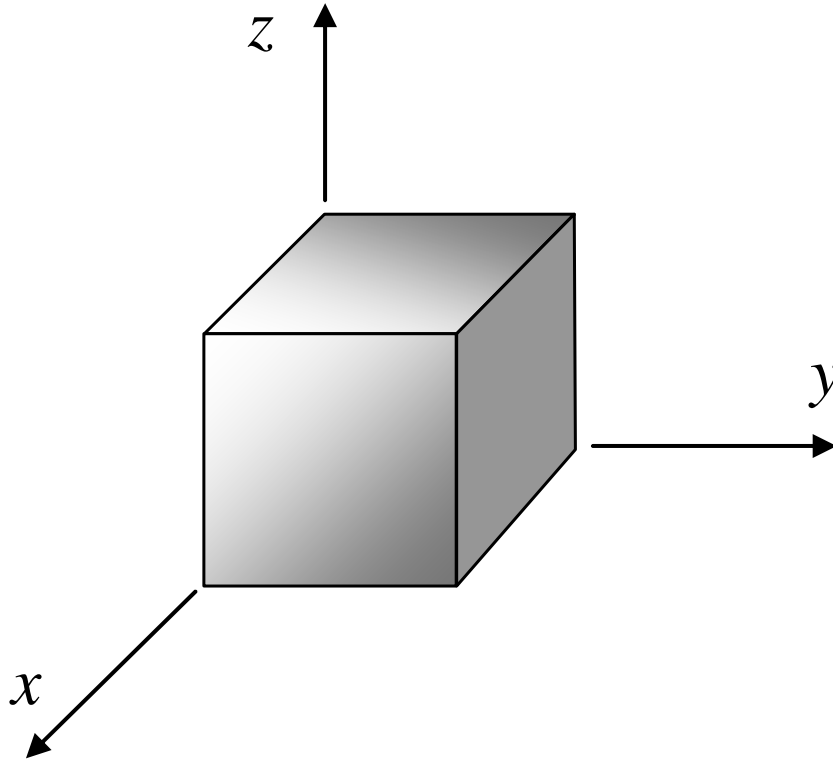
Cisalhamento puro (plano xz)



$$\varepsilon_{xz} = \frac{1}{2} \gamma_{xz} = \frac{\sigma_{xz}}{2G}$$

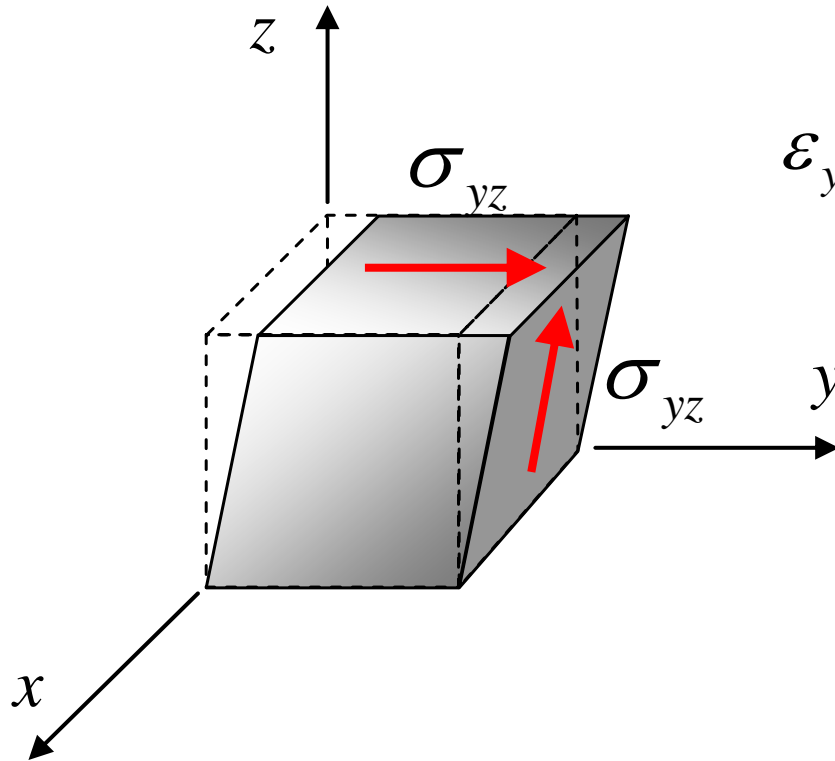
Relação Tensão vs. Deformação

Cisalhamento puro (plano yz)



Relação Tensão vs. Deformação

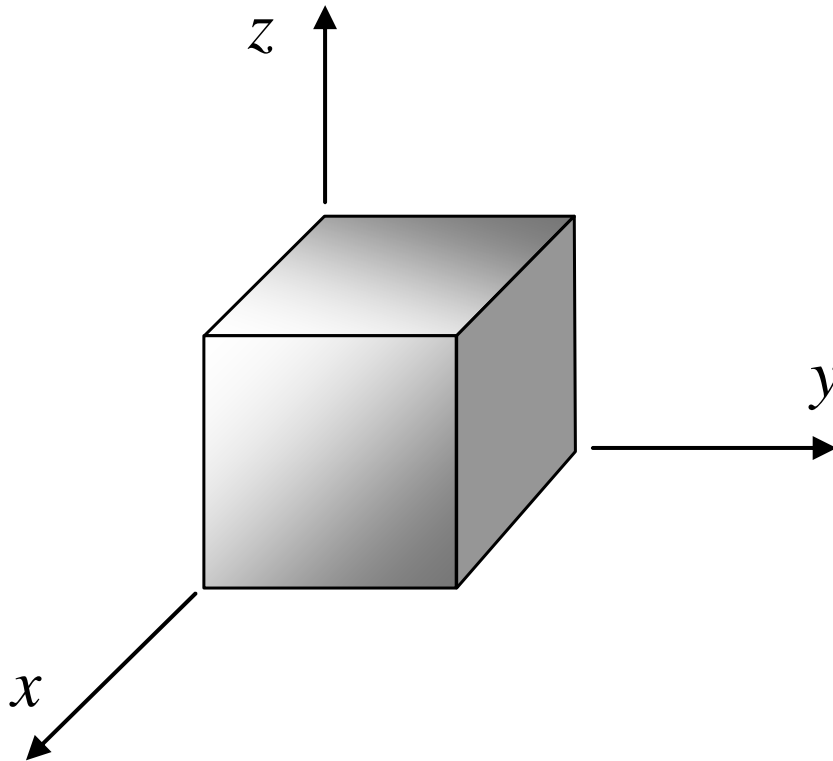
Cisalhamento puro (plano yz)



$$\varepsilon_{yz} = \frac{1}{2} \gamma_{yz} = \frac{\sigma_{yz}}{2G}$$

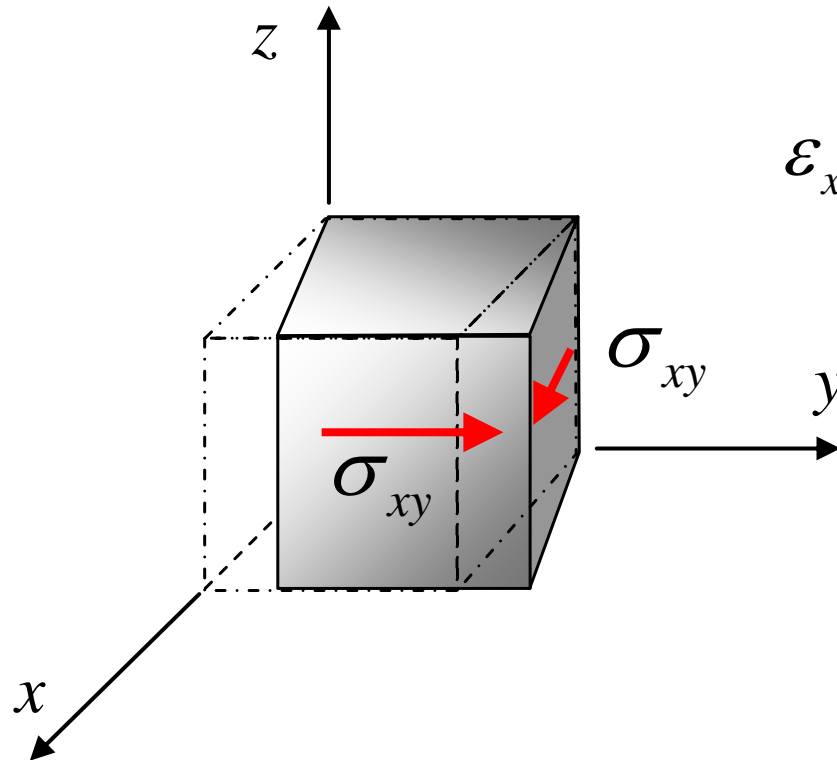
Relação Tensão vs. Deformação

Cisalhamento puro (plano xy)



Relação Tensão vs. Deformação

Cisalhamento puro (plano xy)



$$\varepsilon_{xy} = \frac{1}{2} \gamma_{xy} = \frac{\sigma_{xy}}{2G}$$

Relação Tensão vs. Deformação

Empregando o princípio da superposição:

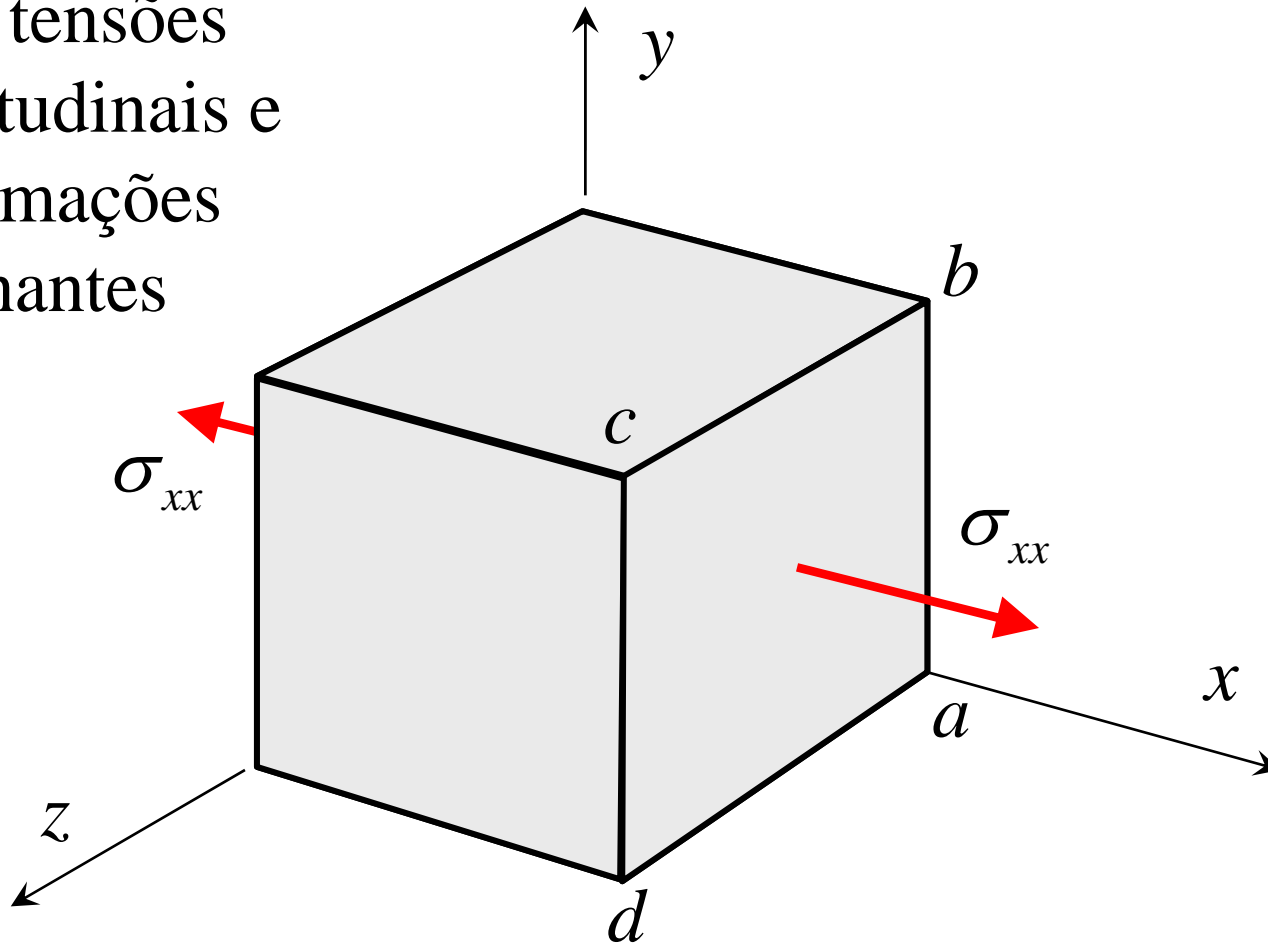
$$\varepsilon_{xy} = \frac{1}{2} \gamma_{xy} = \frac{\sigma_{xy}}{2G}$$

$$\varepsilon_{xz} = \frac{1}{2} \gamma_{xz} = \frac{\sigma_{xz}}{2G}$$

$$\varepsilon_{yz} = \frac{1}{2} \gamma_{yz} = \frac{\sigma_{yz}}{2G}$$

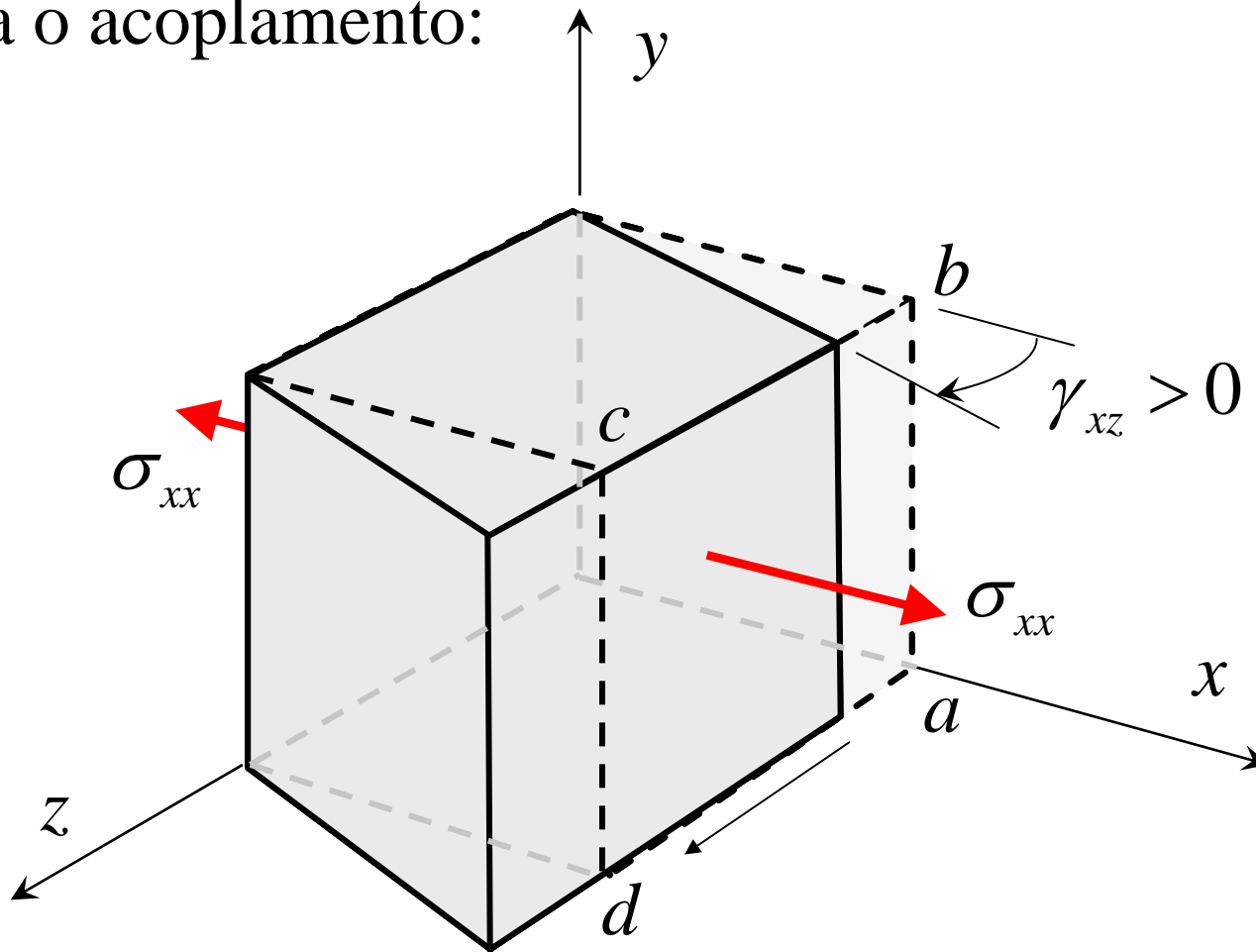
Relação Tensão vs. Deformação

Acoplamento
entre tensões
longitudinais e
deformações
cisalhantes



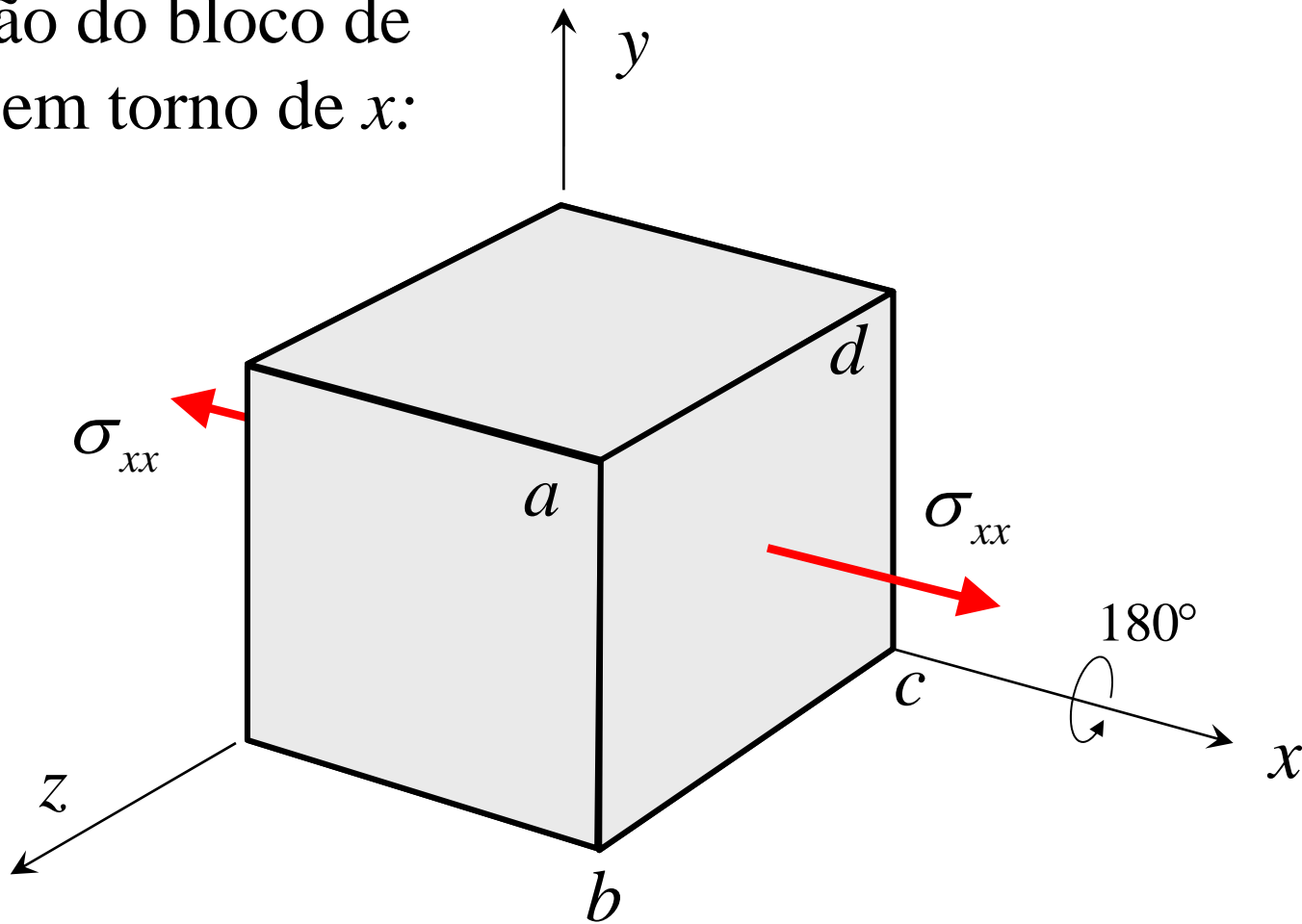
Relação Tensão vs. Deformação

Supondo-se que
exista o acoplamento:



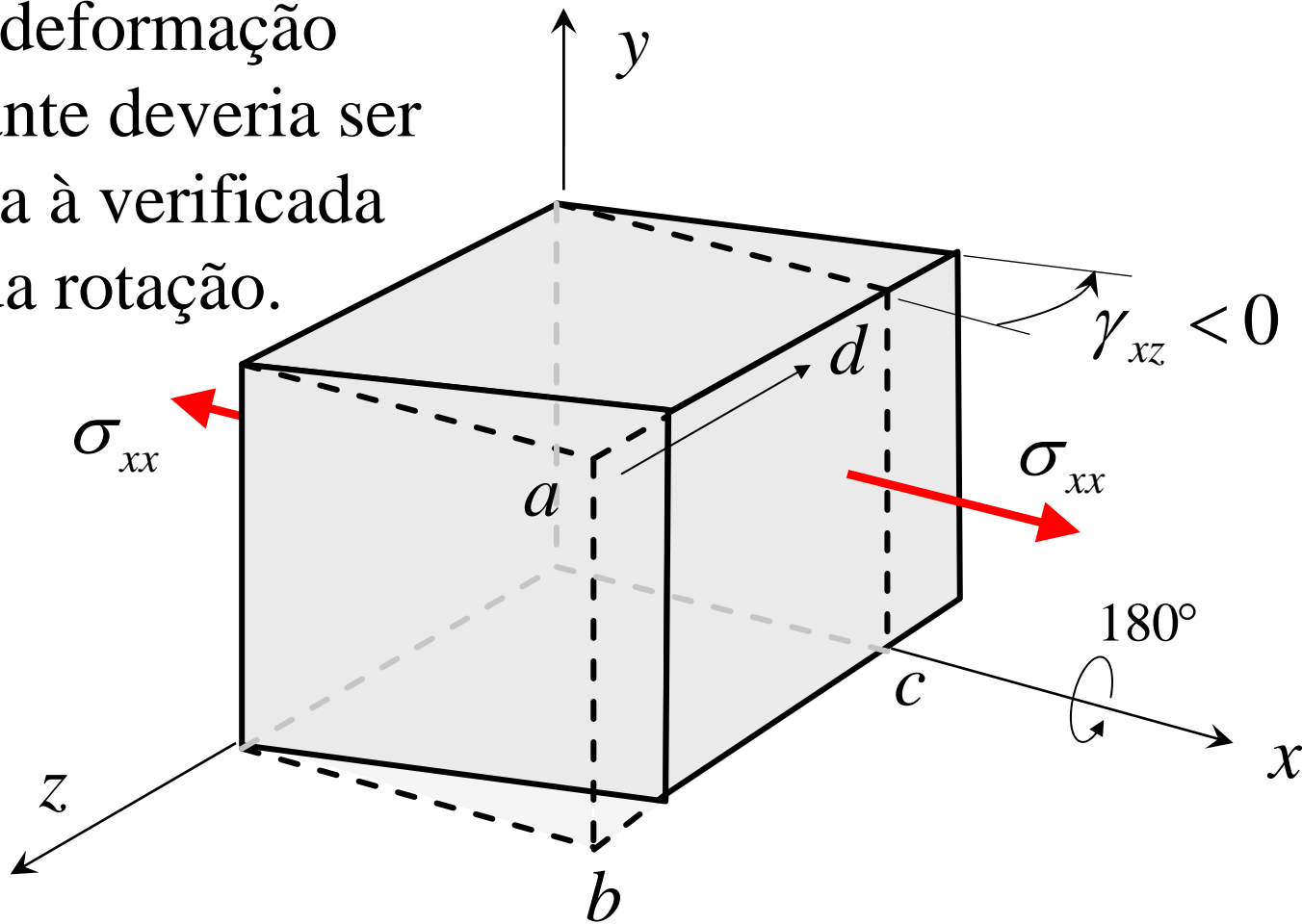
Relação Tensão vs. Deformação

Considerando uma rotação do bloco de 180° em torno de x :



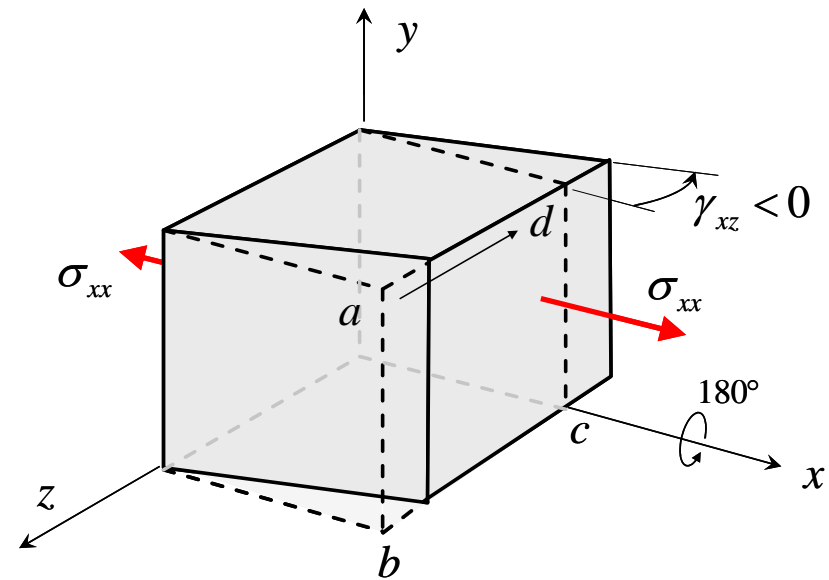
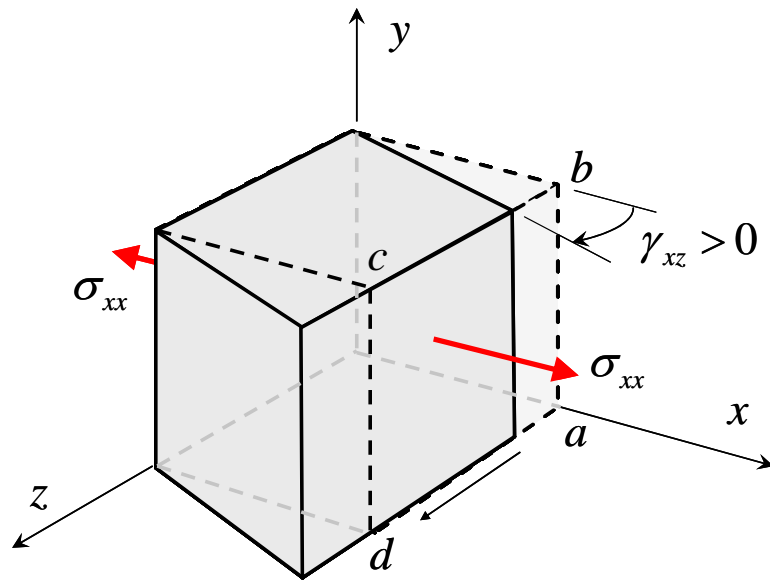
Relação Tensão vs. Deformação

Material é isotrópico,
logo a deformação
cisalhante deveria ser
idêntica à verificada
antes da rotação.



Relação Tensão vs. Deformação

Para materiais isotrópicos, tensões normais *não* produzem deformações cisalhantes. Pode-se mostrar que tensões cisalhantes também *não* produzem deformações longitudinais.



Relação Tensão vs. Deformação

Equações constitutivas para material isotrópico, linear (pequenas deformações) e elástico

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \nu \frac{\sigma_{yy}}{E} - \nu \frac{\sigma_{zz}}{E}$$

$$\varepsilon_{xy} = \frac{1}{2} \gamma_{xy} = \frac{\sigma_{xy}}{2G}$$

$$\varepsilon_{yy} = -\nu \frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \nu \frac{\sigma_{zz}}{E}$$

$$\varepsilon_{xz} = \frac{1}{2} \gamma_{xz} = \frac{\sigma_{xz}}{2G}$$

$$\varepsilon_{zz} = -\nu \frac{\sigma_{xx}}{E} - \nu \frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}$$

$$\varepsilon_{yz} = \frac{1}{2} \gamma_{yz} = \frac{\sigma_{yz}}{2G}$$

Relação Tensão vs. Deformação

Considerando variações de temperatura:

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \nu \frac{\sigma_{yy}}{E} - \nu \frac{\sigma_{zz}}{E} + \alpha \Delta T$$

$$\varepsilon_{xy} = \frac{1}{2} \gamma_{xy} = \frac{\sigma_{xy}}{2G}$$

$$\varepsilon_{yy} = -\nu \frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \nu \frac{\sigma_{zz}}{E} + \alpha \Delta T$$

$$\varepsilon_{xz} = \frac{1}{2} \gamma_{xz} = \frac{\sigma_{xz}}{2G}$$

$$\varepsilon_{zz} = -\nu \frac{\sigma_{xx}}{E} - \nu \frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E} + \alpha \Delta T$$

$$\varepsilon_{yz} = \frac{1}{2} \gamma_{yz} = \frac{\sigma_{yz}}{2G}$$