Fryderyk E. Gorczyca

APPLICATION OF METAL CUTTING THEORY

APPLICATION OF METAL CUTTING THEORY

Fryderyk E. Gorczyca, P.E., C.Mfg.E.
Professor of Mechanical Engineering
Southeastern Massachusetts University
North Dartmouth, Massachusetts

Industrial Press Inc.

4

Mechanics of the Cutting Process

4.1 INTRODUCTION

The mechanics of the cutting process deals primarily with the analysis of the forces acting between the tool and the work material. By analyzing forces that act in particular directions relative to the tool and work material, we can gain insight into a basic understanding of the cutting process. In this chapter, the parallelogram law for the addition and resolution of vectors serves as a means of convenient placement of component forces for purposes of analysis of specific directional factors such as friction and shear. This law states that a resultant force can be resolved into two component forces such that the resultant force is equal to the diagonal of a parallelogram and that the component forces are equal to the sides of the parallelogram. The law graphically depicts the vectorial qualities of forces that must take into account not only the magnitude but also the direction of the force. Figure 2.19 illustrates the parallelogram law.

Newton's first and third general laws of mechanics also play an important role in the force analysis of the cutting process. The first law is that of equilibrium. It states that if a body is at rest or is moving in a straight line at a constant velocity, then a vectorial summation of all the forces acting on the body is equal to zero. In other words, the force system (summation of the forces) is balanced in a state of equilibrium where the resultant of all the forces acting on the body is equal to zero.

Newton's third law is the law of mechanics dealing with action and reaction. It states that when two bodies exert forces on each other, these forces are equal in magnitude, opposite in direction, and act in the same line of action. The application of the laws of mechanics is of prime importance in the analysis of the cutting process. This is done through the drawing of free-body diagrams from which analytical relationships can be written. A free-body diagram is a drawing of a body

showing in vector form all the forces acting on the body. Figure 4.1 illustrates a free-body diagram of a tool showing the force that a workpiece exerts on the tool in balance with the reaction force that the tool-holder exerts on the tool. The tool is in equilibrium, that is, the summation of the action (workpiece force) is balanced with the reaction (tool-holder force).

Of importance in the mechanics of the cutting process is a description of the force interaction of the tool with the work material. It is through an analysis of the forces acting during the metal cutting process that this description can be made. Cutting forces reflect the effectiveness of the cutting tool as well as the resistance that the work material offers to the cutting operation. Analysis can be enhanced through the resolution of the resultant cutting force into convenient components. As an example, the component along the face of the tool is the *frictional force*, and through it the surface characteristic known as the *coefficient of friction* can be evaluated. The component of the resultant force acting in the direction of the shear plane can be used to evaluate the stress acting over the shear plane.

Isolation of a component force in a particular direction places attention on a specific segment of the cutting operation. To cite an example, a measure of the horsepower required in a cutting operation can conveniently be evaluated by isolating the component of the resultant force in the direction of the motion and multiplying it by the cutting speed.

Topics covered in this chapter take into account the application of the components of the resultant force acting on the tool and the chip. Tool forces are first considered in terms of the component in the direction of the cutting speed and the component in the direction of the feed. This is followed by an examination of the component forces along the

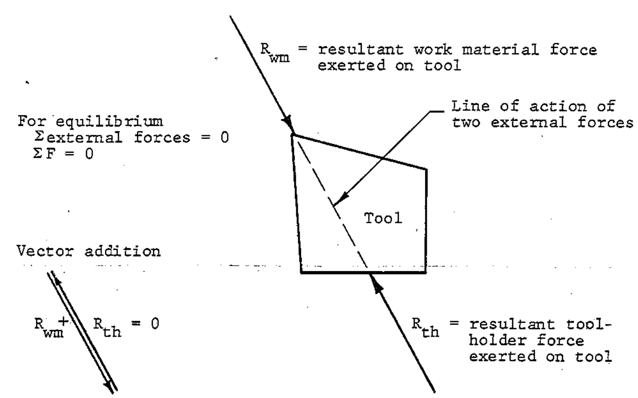


Fig. 4.1. Free-body diagram of tool.

tool–chip interface as well as an examination of the component forces acting on the chip–work material interface, that is, on the shear plane. A combination of these component forces then leads to the analysis of three sets of perpendicular force components that are all geometrically related to the resultant cutting force.

Another topic introduced in this chapter is the coefficient of friction between the tool and the chip. The results of an experiment designed to measure the coefficient of friction as a function of cutting speed are also listed. This is followed by an analysis of the velocity relationships among the tool, chip, and work material. With the availability of velocity and force relationships, energy considerations are then made that lead to an analysis of power consumption in the cutting process.

Some of the latter topics covered in this chapter deal with the effective tool clearance angles due to tool positioning as well as those due to the relative velocity of the tool to the workpiece. The final topic is a three-dimensional analysis of the cutting force, accompanied by the associated analytical expressions.

4.2 TOOL FORCE

When cutting force measurements are taken, usually force-sensing instruments (dynamometers) are used to measure directional components of the resultant force acting on the tool. Figure 4.2 illustrates a dynamometer designed to measure a vertical and a horizontal force.

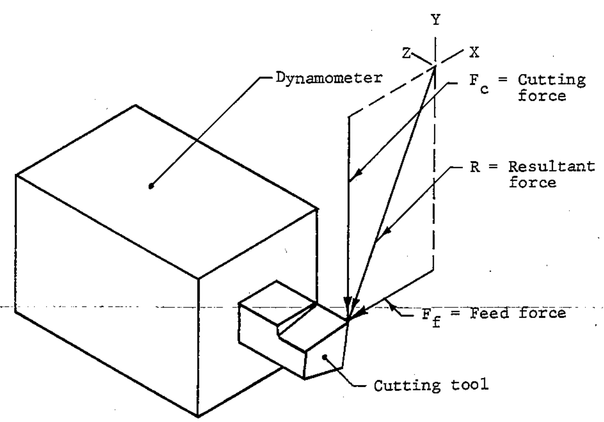


Fig. 4.2. Dynamometer with cutting and feed forces acting on tool.

In a turning operation, the vertical force is in the direction of the cutting speed and is referred to as the *cutting force* ($\mathbf{F}_c$). The horizontal force is in the direction of the feed and is referred to as the *feed force* ($\mathbf{F}_f$). The vector sum of the cutting force and the feed force is equal to the resultant force that the tool exerts on the work material. Figure 4.3 shows the resultant force resolved into two components. The action–reaction principle of Newton's third law of mechanics is illustrated in the diagram. If the force ($\mathbf{R}$) that the tool exerts on the work material is considered the action, then the reaction to this force which the work material exerts on the tool is equal to the force vector labeled $\mathbf{R}'$ in Fig. 4.3.

Of interest is the determination of the magnitude and direction of the resultant force acting between the tool and work material. If dynamometer readings are taken for a given operation that reveal the magnitude of the cutting force $\mathbf{F}_c$ and the feed force $\mathbf{F}_f$, then trigonometric relationships can be formed to determine the resultant force. From Fig. 4.3 it can be written that

$$\mathbf{F}_c = \mathbf{R}\cos\theta \quad (4.1)$$

$$\mathbf{F}_f = \mathbf{R}\sin\theta \quad (4.2)$$

$$|\mathbf{R}| = \sqrt{(\mathbf{F}_c)^2 + (\mathbf{F}_f)^2} \quad (4.3)$$

and

$$\tan\theta = \frac{\mathbf{F}_f}{\mathbf{F}_c} \quad (4.4)$$

The following example illustrates how the magnitude and direction of the resultant cutting force can be determined from dynamomter readings of the cutting force and the feed force.

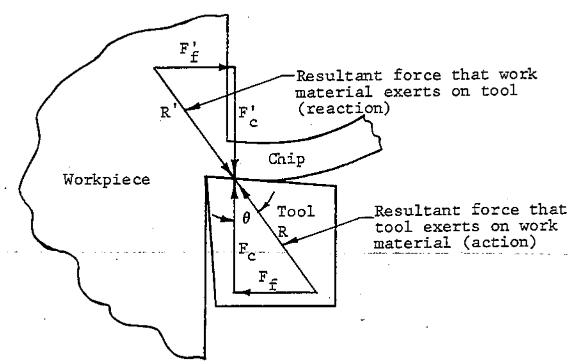


Fig. 4.3. Resultant forces acting in metal cutting process.

EXAMPLE 4.1. Given dynamometer readings where $\mathbf{F}_c = 250$ lb (1112 N) and $\mathbf{F}_f = 75$ lb (333.6 N) calculate the magnitude and direction of the resultant force that the tool exerts on the work material.

Substituting values into Eq. 4.3 yields

$$|\mathbf{R}| = \sqrt{(250)^2 + (75)^2} = 261 \text{ lb} \, (1161 \text{ N})$$

Substituting values into Eq. 4.4 yields

$$\theta = \tan^{-1}\frac{75}{250} = 16.7^\circ$$

EXAMPLE 4.2. In a cutting operation similar to that shown in Fig. 4.3, the force that the work material exerts on the tool ($\mathbf{R}'$) has a magnitude of 200 lb (890 N) and a 20° angular displacement (θ) from the vertical direction. From these data, determine the magnitude of the cutting force ($\mathbf{F}_c$) as well as the magnitude of the feed force ($\mathbf{F}_f$).

Substituting the given values into Eqs. 4.1 and 4.2 yields

$$\mathbf{F}_c = 200 \cos 20^\circ = 187.9 \text{ lb} \, (835.8 \text{ N})$$

$$\mathbf{F}_f = 200 \sin 20^\circ = 68.4 \text{ lb} \, (304.3 \text{ N})$$

4.3 TOOL–CHIP INTERFACE FORCE

The force acting between the tool and the chip during the metal cutting operation is equal to the resultant force that the tool exerts on the work material. This resultant force can be resolved into two convenient and useful components. Figure 4.4 illustrates the resolution of

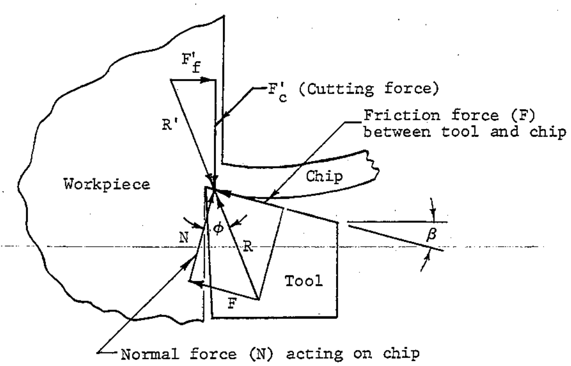


Fig. 4.4. Resolution of resultant force into a frictional and a normal component.

the resultant force into a component parallel to the face of the tool and a second component that is perpendicular to the face of the tool. The component parallel to the face of the tool gives a measure of the resistance that the surface provides to the sliding motion of the chip. It is referred to as the *frictional force* (**F**). The component perpendicular to the face of the tool is referred to as the normal force (**N**) and it provides a measure of the compressive action of the tool on the chip.

When two surfaces are in contact, as in the case of the tool with the chip (Fig. 4.4), and one surface is made to move with respect to the other surface, forces tangent to the surface develop. These forces are frictional forces and are a reflection of the quality of the surface with respect to sliding action. A frictional force is the resistance that two bodies provide with regard to sliding motion. An experiment on sliding friction reveals that there is a relationship between the normal force and the frictional force. In engineering mechanics, this relationship is called the *coefficient of friction*. It is expressed as

$$\mu = \frac{\mathbf{F}}{\mathbf{N}} = \frac{\text{Frictional force}}{\text{Normal force}} \tag{4.5}$$

An examination of Fig. 4.4 reveals that this ratio can be expressed in terms of the angle ϕ as

$$\mu = \tan\phi = \frac{\mathbf{F}}{\mathbf{N}} \tag{4.6}$$

For sliding motion, the angle ϕ is defined as the angle of kinetic friction.

As can be seen from Eq. 4.5, the coefficient of friction is defined as the ratio of the frictional force to the force acting normal to the surface. In metal cutting, this ratio is affected by the quality of the tool surface as well as by the interaction of the chip and the tool. The velocity of the chip over the tool surface has an influence on the magnitude of the coefficient of friction as does the temperature of the chip. When a comparison is made with low-speed ordinary sliding, the coefficient of friction associated with chip sliding becomes a relatively complicated term. This can be attributed to factors such as the mechanical interlocking of the surface irregularities (asperites) due to the high normal forces as well as to the tendency under conditions of elevated temperatures and high normal forces of the chip to weld itself to the tool surface. The latter condition is the cause of the built-up edge on the tool, which demands a shearing action of the metal chip as it flows over the tool. A condition of this type can cause a large increase in the value of the coefficient of friction.

Figure 4.5 is an illustration of the action–reaction relationship of the frictional and normal forces acting between the chip and the tool.

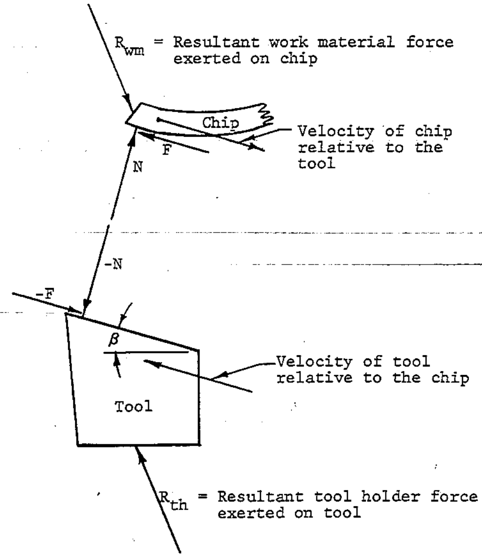


Fig. 4.5. Free-body diagram of chip and tool illustrating normal and frictional forces at the interface.

In the diagram, the chip is separated from the tool to more clearly describe the direction of the forces. Note that the frictional forces **F** and −**F** act in a direction opposite to the direction of sliding, that is, the chip is sliding over the tool in a direction opposite to the direction of the force **F**, whereas the tool is sliding with a motion relative to the chip in a direction opposite to the direction of the force −**F**. In other words, if one took a ride on the chip, it would appear, from the chip, that the tool is moving in the direction of the force **F**. From this, the conclusion can be drawn that the frictional force **F** is the force that the tool exerts on the chip in resistance to the sliding of the chip over the tool. In turn, the frictional force −**F** is the reaction to the force **F**. It is equal in magnitude, but opposite in direction, to the force **F**. The frictional force −**F** is the force that the chip exerts on the tool in an effort to resist the relative motion (sliding) of the tool to the chip.

The frictional and normal forces shown in Fig. 4.5 are internal forces that act on the tool–chip interface. They appear in Fig. 4.1 in the form of the resultant forces acting on the tool through the work material as well as through the tool holder. A vector addition of these internal forces representing the action–reaction forces acting between the tool and the chip is shown in Fig. 4.6.

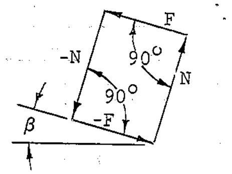


Fig. 4.6. Vector equilibrium of action–reaction forces acting between tool and chip.

This vector summation can be expressed as

$$\mathbf{N} \mathrel{+\!\!\!\rightarrow} \mathbf{F} \mathrel{+\!\!\!\rightarrow} (-\mathbf{N}) \mathrel{+\!\!\!\rightarrow} (-\mathbf{F}) = 0 \tag{4.7}$$

yielding an equilibrium condition where the sum of the vectors is equal to zero.

If the forces acting on the tool are examined, it becomes apparent, as shown in Fig. 4.5, that the summation can be written as

$$\mathbf{R}_{th} \mathrel{+\!\!\!\rightarrow} (-\mathbf{N}) \mathrel{+\!\!\!\rightarrow} (-\mathbf{F}) = 0 \tag{4.8}$$

In a similar fashion, the chip forces can be added in the form

$$\mathbf{R}_{wm} \mathrel{+\!\!\!\rightarrow} \mathbf{N} \mathrel{+\!\!\!\rightarrow} \mathbf{F} = 0 \tag{4.9}$$

Figure 4.7 illustrates a graphical representation of Eqs. 4.8 and 4.9. Additional expressions that can be written are

$$\mathbf{N} = \mathbf{R} \cos \phi \tag{4.10}$$

$$\mathbf{F} = \mathbf{R} \sin \phi \tag{4.11}$$

and

$$|\mathbf{R}| = \sqrt{\mathbf{F}^2 + \mathbf{N}^2} \tag{4.12}$$

The following numerical examples demonstrate some trigonometric manipulation dealing with the interface forces between the tool and the chip.

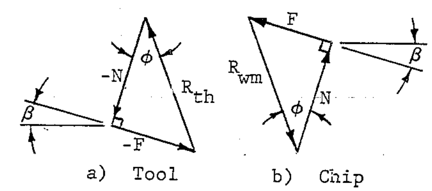
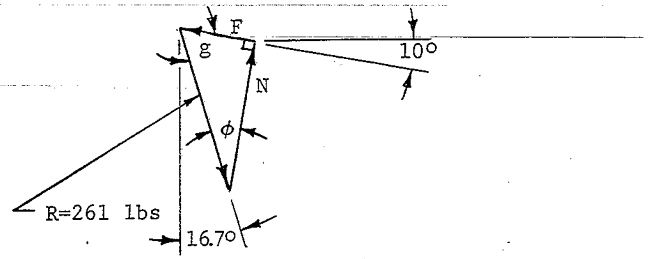


Fig. 4.7. Vector summation of external forces acting on (a) chip and (b) tool.

EXAMPLE 4.3. Given a resultant force of 261 lb (1161 N) and an angular orientation of 16.7°, as illustrated in example 4.1, determine the frictional and normal forces acting between the tool and the chip for the case where the rake angle on the tool is 10°. In addition, determine the coefficient of friction for this operation.

The following sketch can be drawn from Fig. 4.7(b):



Angle g can be determined by noting that it is part of a right angle and can be expressed as

$$90° = 16.7° + g + 10°$$
$$g = 63.3°$$

Noting that the vector polygon is a right triangle, where the sum of the interior angles is equal to 180°, the angle of kinetic friction can be evaluated as

$$180° = 90° + g + \phi$$
$$\phi = 26.7°$$

Using the angle of kinetic friction, the normal force can be written as

$$\mathbf{N} = \mathbf{R}\cos\phi$$
$$= 261(0.893) = 233.2 \text{ lb } (1037 \text{ N})$$

In a similar fashion, the friction force can be written as

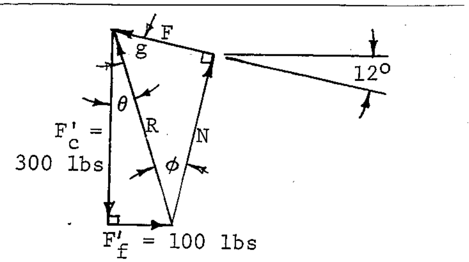
$$\mathbf{F} = \mathbf{R}\sin\phi$$
$$= 261(0.4493) = 117.27 \text{ lb } (521.6 \text{ N})$$

Solving for the coefficient of friction, it can be written that

$$\mu = \tan\phi = \frac{\mathbf{F}}{\mathbf{N}}$$
$$= \tan 26.7° = \frac{117.27}{233.2} = 0.503$$

EXAMPLE 4.4. For a given metal cutting operation, dynamometer readings reveal values of 300 lb (1334 N) for the cutting force $\mathbf{F}_c$ and 100 lb (444.8 N) for the feed force $\mathbf{F}_f$. The cutting tool has a positive rake angle of 12°. From these data, calculate the value of the frictional force, the normal force, and the coefficient of friction.

A diagram of the tool forces involved in the calculation is of the form



Solving for θ and $\mathbf{R}$ yields

$$\tan\theta = \frac{\mathbf{F}'_f}{\mathbf{F}'_c} = \frac{100}{300} = 0.333$$
$$\theta = 18.4°$$

from which

$$\mathbf{R} = \frac{\mathbf{F}'_c}{\cos\theta} = \frac{300}{0.9487} = 316.2 \text{ lb } (1406.5 \text{ N})$$

Noting that a common hypotenuse is shared by the two right triangles listed above, it can be concluded that the solution to this problem lies in evaluating angles g and ϕ:

$$g = 90° - \theta - \beta = 90° - 18.4° - 12°$$
$$= 59.6°$$

and

$$\phi = 90° - g = 90° - 59.6°$$
$$= 30.4°$$

The coefficient of friction can now be written as

$$\mu = \tan\phi = \frac{\mathbf{F}}{\mathbf{N}}$$
$$= 0.587$$

The frictional force can be evaluated as

$$\begin{aligned} F &= R \sin \phi \\ &= 316.2 \sin 30.4^\circ \\ &= 160 \text{ lb } (711.7 \text{ N}) \end{aligned}$$

and the normal force can be written as

$$\begin{aligned} N &= R \cos \phi \\ &= 316.2 \cos 30.4^\circ \\ &= 272.7 \text{ lb } (1213 \text{ N}) \end{aligned}$$

4.4 SHEAR PLANE FORCES

The resultant force that the work material exerts on the tool during the metal cutting operation is transmitted through the shear zone between the tool and the chip. For purposes of analysis, this resultant force can be resolved into two convenient components that act on the shear zone. As shown in Fig. 4.8, the component parallel to the shear zone can be considered as the force acting in the direction of the shear. It is called the *shear force* ($\mathbf{F}_s$). The second component, acting normal to the shear plane, provides a measure of the compressive force acting on the shear plane. It is called the *normal force* ($\mathbf{F}_n$) acting on the shear plane. In Fig. 4.8, the chip is separated from the work material along the shear zone in order to demonstrate the internal shear and normal forces. A vector addition of the chip–work-material interface forces is shown in Fig. 4.9. This diagram verifies the equilibrium condition of the internal forces, where the summation of the vectors is

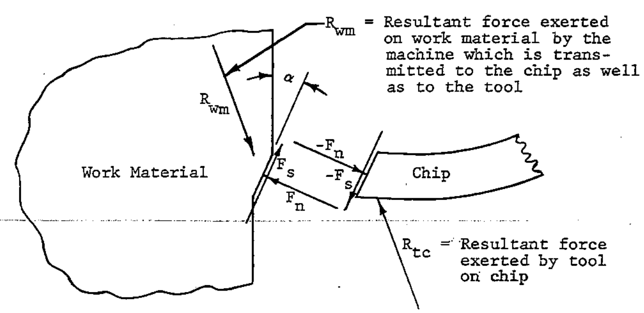


Fig. 4.8. Free-body diagrams of work material and chip, illustrating shear force and force acting normal to shear plane.

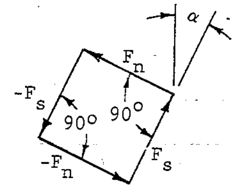


Fig. 4.9. Vector equilibrium diagram of action–reaction forces acting between the chip and the work material at the shear plane.

equal to zero. A vector summation of Fig. 4.9 can be expressed as

$$\mathbf{F}_s \nrightarrow \mathbf{F}_n \nrightarrow -\mathbf{F}_s \nrightarrow -\mathbf{F}_n = 0 \tag{4.13}$$

In Fig. 4.8, the work material and the chip are shown as free bodies. For the sake of analysis, the forces acting on each body can be considered as external, and an equilibrium force balance can be expressed in vector form. This is illustrated in Fig. 4.10.

A vector summation of the graphical display of Fig. 4.10 can be expressed as

$$-\mathbf{F}_s \nrightarrow -\mathbf{F}_n \nrightarrow \mathbf{R}_{tc} = 0 \tag{4.14}$$

and

$$\mathbf{F}_s \nrightarrow \mathbf{F}_n \nrightarrow \mathbf{R}_{wm} = 0 \tag{4.15}$$

In terms of the resultant force, the shear force and the force normal to the shear plane can be written as

$$\mathbf{F}_s = \mathbf{R} \cos h \tag{4.16}$$

$$\mathbf{F}_n = \mathbf{R} \sin h \tag{4.17}$$

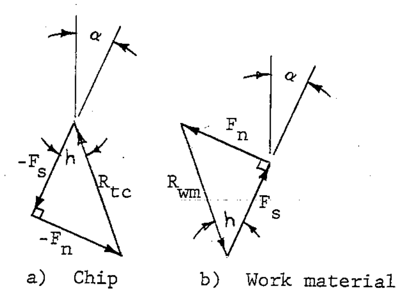
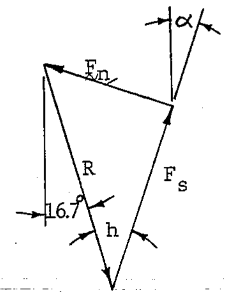


Fig. 4.10. Vector summation of the equilibrium external forces acting on (a) the chip and (b) the work material.

The magnitude of the resultant force can be expressed in terms of the shear force and the force normal to the shear plane as

$$|\mathbf{R}| = \sqrt{(\mathbf{F}_s)^2 + (\mathbf{F}_n)^2} \tag{4.18}$$

EXAMPLE 4.5. Given the resultant force of 261 lb (1161 N) with an angular orientation of 16.7°, as illustrated in examples 4.1 and 4.3, determine the shear force and the force normal to the shear plane for the metal cutting operation. To measure the shear angle α, the chip thickness was measured. It was ~~found to be 0.045 in. (1.143 mm) for a~~ corresponding feed of 0.015 in./rev (0.381 mm). The tool rake angle is 10°.



Solving for the shear angle from Eq. 2.22 yields

$$\tan\alpha = \frac{r_a\cos\beta}{1 - r_a\sin\beta} = \frac{0.33\cos 10°}{1 - 0.33\sin 10°}$$

$$= 0.3448$$

$$\alpha = 19.02°$$

From the diagram shown above, the angle h can be evaluated as

$$h = 16.7° + \alpha$$
$$= 35.72°$$

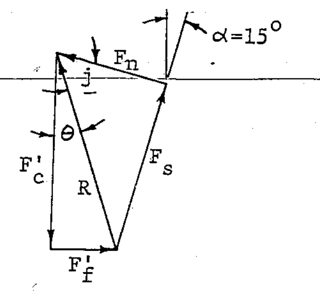
The shear force can now be written as

$$\mathbf{F}_s = \mathbf{R}\cos h$$
$$= 261\cos 35.72°$$
$$= 211.9 \text{ lb } (942.6 \text{ N})$$

The force normal to the shear plane is then equal to

$$\mathbf{F}_n = \mathbf{R}\sin h$$
$$= 261\sin 35.72°$$
$$= 152.4 \text{ lb } (677.9 \text{ N})$$

EXAMPLE 4.6. For the same metal cutting operation as described in example 4.4, it is observed that the shear plane angle α is equal to 15°. With dynamometer readings of 300 lb (1334 N) for the cutting force $\mathbf{F}_c$ and 100 lb (444.8 N) for the feed force $\mathbf{F}_f$, determine the shear force and the force normal to the shear plane.



From the diagram listed above, it can be written that

$$j = 90° - \theta - \alpha$$

where $\theta = 18.4°$ from example 4.4. As a result,

$$j = 56.6°$$

In terms of the resultant force that was calculated in example 4.4, it can be written that

$$\mathbf{F}_s = \mathbf{R}\sin j$$
$$= 316.2\sin 56.6°$$
$$= 264 \text{ lb } (1174 \text{ N})$$

and that

$$\mathbf{F}_n = \mathbf{R}\cos j$$
$$= 316.2\cos 56.6°$$
$$= 174 \text{ lb } (774 \text{ N})$$

A point of interest is that example 4.6 can be solved directly from the given cutting and feed forces by adding vector components in the x and y directions. As an example, from the diagram listed above it can be written that

$$\mathbf{F}_s \nrightarrow \mathbf{F}_n \nrightarrow \mathbf{F}'_c \nrightarrow \mathbf{F}'_f = 0 \tag{4.19}$$

These four forces are in equilibrium, that is, their vector summation is equal to zero. The conclusion can be reached that the summation of the components of these vectors in any direction must also be equal to zero. Therefore,

$$F_{sx} \leftrightarrow F_{nx} \leftrightarrow F'_{cx} \leftrightarrow F'_{fx} = 0 \qquad (4.20)$$

and

$$F_{sy} \leftrightarrow F_{ny} \leftrightarrow F'_{cy} \leftrightarrow F'_{fy} = 0 \qquad (4.21)$$

Figure 4.11 illustrates the resolution of vectors into convenient components in the x and y directions.

An illustrated example of the application of the x and y components in solving a force problem is listed below as a confirmation of the results of example 4.6. Using force components as shown in Fig. 4.11(b), the shear force and the force normal to the shear plane can easily be evaluated. It is noted from example 4.6 that $\mathbf{F}'_c = 300$ lb, $\mathbf{F}'_f = 100$ lb, and $\alpha = 15°$. This is symbolically illustrated in Fig. 4.11(a). Taking components in the x direction yields

$$\mathbf{F}'_f + F_{sx} - F_{nx} = 0 \qquad (4.22)$$

Substituting gives

$$100 + \mathbf{F}_s \sin 15° - \mathbf{F}_n \cos 15° = 0 \qquad (4.22a)$$

In a similar fashion, taking components in the y direction yields

$$-\mathbf{F}'_c + F_{sy} + F_{ny} = 0 \qquad (4.23)$$

Substituting gives

$$-300 + \mathbf{F}_s \cos 15° + \mathbf{F}_n \sin 15° = 0 \qquad (4.23a)$$

As can be seen, the result of the substitution is a set of two simultaneous

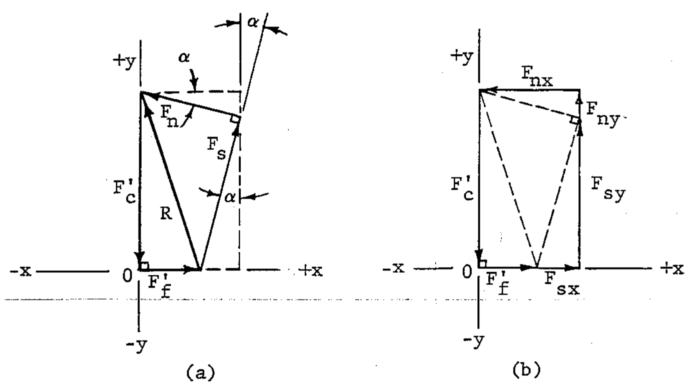


Fig. 4.11. Resolution of vectors into convenient components along the x and y axes.

equations containing the two unknown values of $\mathbf{F}_s$ and $\mathbf{F}_n$. An evaluation can be performed in the following manner:

Rewriting Eqs. 4.22a and 4.23a gives

$$100 + 0.2588\mathbf{F}_s - 0.9659\mathbf{F}_n = 0 \qquad (4.22b)$$

$$-300 + 0.9659\mathbf{F}_s + 0.2588\mathbf{F}_n = 0 \qquad (4.23b)$$

Multiplying Eq. 4.22b by the ratio 0.2588/0.9659 and adding the equations eliminates the $\mathbf{F}_n$ term and yields

$$-273.21 + 1.0352\mathbf{F}_s = 0$$

from which

$$\mathbf{F}_s = 264 \text{ lb } (1174 \text{ N})$$

Substituting 264 lb for $\mathbf{F}_s$ in either Eq. 4.22b or Eq. 4.23b enables the evaluation of $\mathbf{F}_n$ as

$$\mathbf{F}_n = 174 \text{ lb } (774 \text{ N})$$

4.5 THREE SETS OF PERPENDICULAR FORCE COMPONENTS

The resultant force that the tool exerts on the work material can be conveniently resolved into three sets of useful perpendicular components. When analyzed individually, each of these components represents the projection of the resultant force in a particular direction. For example, a projection of the resultant force onto a direction parallel to the face of the tool gives a measure of the frictional force (**F**) between the tool and the chip. The projection of the resultant force onto the shear plane gives a measure of the force acting in the direction of the shearing action ($\mathbf{F}_s$) between the chip and the work material. In a similar fashion, the cutting force ($\mathbf{F}_c$) can be determined by projecting the resultant force onto a plane parallel to the direction of the cutting velocity.

Figure 4.12 illustrates the resultant force (**R**) that the tool exerts on the work material. The resultant force (**R**) is shown in Fig. 4.12 as the diameter of a circle. If straight lines are extended from the ends of the diameter (from points O and B) in such a way as to intersect on the circle (point A), then a right triangle will be formed. The diameter serves as the hypotenuse of the triangle and in every case the angle between the lines drawn from points O and B will form a 90° angle.

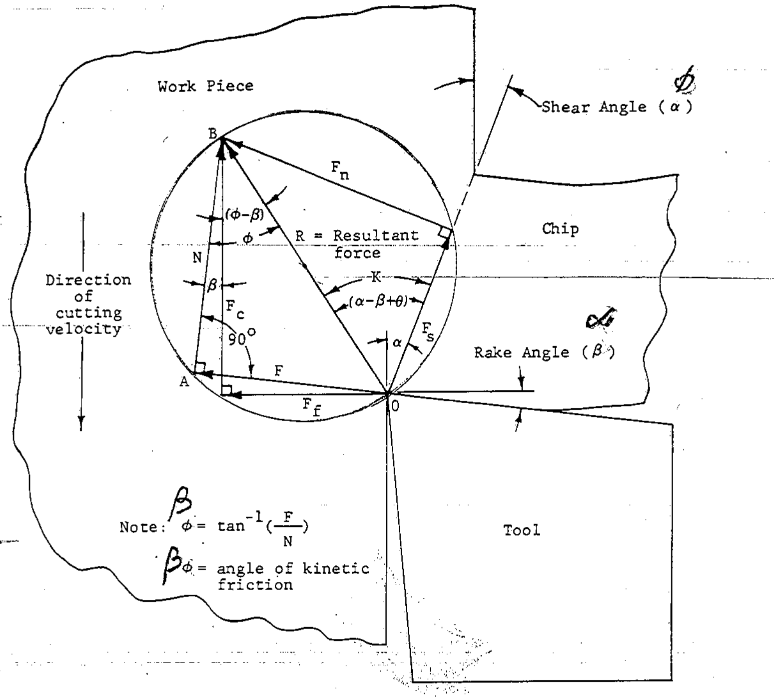


Fig. 4.12. Resolution of resultant force into three sets of perpendicular components.

An examination of Fig. 4.12 reveals that the resultant force serving as the diameter of the circle also serves as the common hypotenuse of three sets of perpendicular components. By identifying an interior angle of each of the three right triangles, one can evaluate the individual components as a function of the resultant force (**R**). The three interior angles are given in Fig. 4.12 in terms of the angle of kinetic friction (ϕ), the shear angle (α), and the rake angle (β). In order to determine the resultant force, usually dynamometer readings in terms of the cutting force ($\mathbf{F}_c$) and the feed force ($\mathbf{F}_f$) are taken. Figure 4.13 illustrates a tool with the resultant force resolved into two components, the cutting force ($\mathbf{F}_c$) and the feed force ($\mathbf{F}_f$).

The magnitude of the resultant force can be written as

$$|\mathbf{R}| = \sqrt{(\mathbf{F}_c)^2 + (\mathbf{F}_f)^2} \tag{4.24}$$

Once the resultant force is determined, trigonometric relationships enable us to determine the other force components. Since the rake angle (β) is a known quantity for a particular cutting operation, then

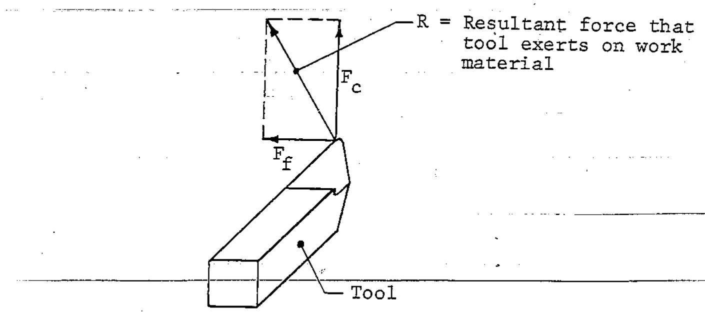


Fig. 4.13. Resultant force resolved into two components.

the angle $\phi - \beta$ shown in Fig. 4.12 can be determined by writing

$$\tan(\phi - \beta) = \frac{\mathbf{F}_f}{\mathbf{F}_c} \tag{4.25}$$

Isolating the angle of kinetic friction (ϕ) leads to

$$\phi = \tan^{-1}\left(\frac{\mathbf{F}_f}{\mathbf{F}_c}\right) + \beta \tag{4.26}$$

With the angle of kinetic friction (ϕ) computed from Eq. 4.26, the frictional force (**F**) and the normal force (**N**) can be expressed as

$$\mathbf{F} = \mathbf{R} \sin \phi \tag{4.27}$$

$$\mathbf{N} = \mathbf{R} \cos \phi \tag{4.28}$$

An examination of Fig. 4.12 indicates that the shear force ($\mathbf{F}_s$) and the normal force ($\mathbf{F}_n$) are trigonometrically related to the resultant force (**R**) by the interior angle ($\alpha - \beta + \phi$). In evaluating this angle, it is noted that the rake angle (β) is known for a given tool, the angle of kinetic friction (ϕ) can be computed from Eq. 4.26, and the shear angle (α) can be determined from Eq. 2.22 if the cutting ratio ($R_a = t_1/t_2$) is known. For convenience, the following substitution can be made:

$$K = \alpha - \beta + \phi \tag{4.29}$$

where

$$\alpha = \tan^{-1}\left(\frac{r_a \cos \beta}{1 - r_a \sin \beta}\right)$$

and

$$\phi = \tan^{-1}\left(\frac{\mathbf{F}_f}{\mathbf{F}_c}\right) + \beta$$

Equation 4.29 can then be written as

$$K = \tan^{-1}\left(\frac{r_a \cos\beta}{1 - r_a \sin\beta}\right) + \tan^{-1}\left(\frac{\mathbf{F}_f}{\mathbf{F}_c}\right) \tag{4.30}$$

With the angle K available in the form of Eq. 4.30, the shear force ($\mathbf{F}_s$) and the force normal to the shear plane ($\mathbf{F}_n$) can be expressed in terms of the resultant force as

$$\mathbf{F}_s = \mathbf{R}\cos K \tag{4.31}$$

$$\mathbf{F}_n = \mathbf{R}\sin K \tag{4.32}$$

The determination of the value of the individual component forces illustrated in Fig. 4.12 is presented with the following example.

EXAMPLE 4.7. A metal cutting operation is in the process of being evaluated with respect to the component forces shown in Fig. 4.12. Dynamometer readings indicate that the cutting force ($\mathbf{F}_c$) is equal to 300 lb (1334 N) and the feed force ($\mathbf{F}_f$) is equal to 125 lb (556 N). The rake angle on the tool (β) is 10°. A measurement of the chip thickness (t_2) indicates it is equal to 0.0343 in. (0.871 mm). The feed setting (t_1) of the machine is 0.015 in. (0.381 mm).

From these data it is desired to determine:

(a) The resultant force ($\mathbf{R}$).
(b) The shear angle (α).
(c) The angle of kinetic friction (ϕ).
(d) The frictional force ($\mathbf{F}$).
(e) The normal force ($\mathbf{N}$).
(f) The angle K.
(g) The shear force ($\mathbf{F}_s$).
(h) The force normal to the shear plane ($\mathbf{F}_n$).

Solving for the resultant force by substituting into Eq. 4.24 yields

$$\mathbf{R} = \sqrt{(\mathbf{F}_c)^2 + (\mathbf{F}_f)^2}$$
$$= \sqrt{(300)^2 + (125)^2}$$
$$= 325 \text{ lb } (1445.7 \text{ N})$$

Solving for the shear angle by substituting into Eq. 2.22 yields

$$\tan\alpha = \frac{r_a\cos\beta}{1 - r_a\sin\beta}, \quad \text{where } r_a = t_1/t_2$$
$$= \frac{0.4373(0.9848)}{1 - 0.4373(0.1736)}$$
$$\alpha = 25°$$

Solving for the angle of kinetic friction by substituting into Eq. 4.25 yields

$$\phi = \tan^{-1}\left(\frac{\mathbf{F}_f}{\mathbf{F}_c}\right) + \beta$$
$$= \tan^{-1}\left(\frac{125}{300}\right) + 10°$$
$$= 32.62°$$

Solving for the frictional force and the normal force by substituting into Eq. 4.27 and 4.28 yields

$$F = \mathbf{R}\sin\phi$$
$$= 325 \sin 32.62°$$
$$= 175 \text{ lb } (778.4 \text{ N})$$

and

$$N = \mathbf{R}\cos\phi$$
$$= 325\cos 32.62°$$
$$= 274 \text{ lb } (1219 \text{ N})$$

Solving for the angle K by substituting into Eq. 4.30 yields

$$K = \tan^{-1}\left(\frac{r_a\cos\beta}{1 - r_a\sin\beta}\right) + \tan^{-1}\left(\frac{\mathbf{F}_f}{\mathbf{F}_c}\right)$$
$$= \tan^{-1}\left(\frac{0.4373(0.9848)}{1 - 0.4373(0.1736)}\right) + \tan^{-1}\left(\frac{125}{300}\right)$$
$$= \tan^{-1}0.466 + \tan^{-1}0.4167$$
$$= 24.99° + 22.62°$$
$$= 47.61°$$

Solving for the shear force and the force normal to the shear plane by substituting into Eqs. 4.31 and 4.32 yields

$$\mathbf{F}_s = \mathbf{R}\cos K$$
$$= 325\cos 47.61°$$
$$= 219 \text{ lb } (974 \text{ N})$$

and

$$\mathbf{F}_n = 325\sin 47.61°$$
$$= 240 \text{ lb } (1068 \text{ N})$$

Table 4.1. Listing of Example of Calculations

Given Data					Calculated Values							
F_c	F_f	β	t_1	t_2	R	α	ϕ	F	N	K	F_s	F_n
300 lb	125 lb	10°	0.015 in.	0.0343 in	325 lb	25°	32.62°	175 lb	274 lb	47.61°	219 lb	240 lb

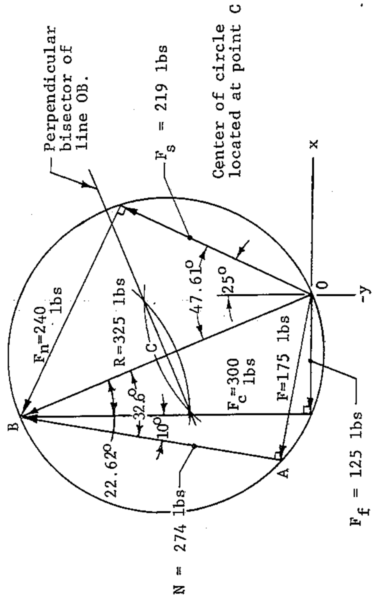


Fig. 4.14. Graphical layout of example 4.7.

The results of example 4.7 are tabulated in Table 4.1. A second method by which example 4.7 can be solved involves a graphical layout. By applying an appropriate scale to the cutting force ($\mathbf{F}_c$) and the feed force ($\mathbf{F}_f$), they can conveniently be drawn at right angles to each other. The hypotenuse of a right triangle having $\mathbf{F}_s$ and $\mathbf{F}_f$ as sides is a measure of the resultant force. By measuring the length of **R** and multiplying the length by the force scale, the value of **R** can be determined. Figure 4.14 represents a scaled graphical layout solution of example 4.7.

If one were inclined to graphically solve example 4.7, a convenient scale to use would be:

Force scale: 1 in. = 100 lb

That is, 1 in. on the drawing represents a 100-lb force.

The midpoint of the resultant force **R** is obviously the center of the circle that encompasses the perpendicular force components. A perpendicular bisector of the line representing the resultant force **R** will conveniently locate the center of the circle. This can be drawn by setting a compass at a distance greater than half of the length of **R** and by swinging arcs from points *O* and *B* as shown in Fig. 4.14. Once this is accomplished, the circular arc can be drawn with its center located at point *C*.

Once the circle is drawn, the rake angle (β) set off at 10° enables the friction force (**F**) and the normal force (**N**) to be drawn. Finally, by setting off the shear angle (α), one can proceed to draw the shear force ($\mathbf{F}_s$) and the force normal to the shear plane ($\mathbf{F}_n$).

By measuring the length of each force vector and multiplying it by the force scale, it becomes possible to evaluate the individual force components. The angles between the force vectors can be measured directly from the diagram. This graphical technique provides not only a confirmation of the solutions calculated by formula as in example 4.7 but also clears the path to the solution of the same problem by a second method. For those who intuitively feel more comfortable with a graphical solution, laying out the force vectors to scale may prove less cumbersome than the analytical approach.

4.6 SHEAR STRESS

A measure of the shear stress acting on the shear plane of a metal cutting operation can be obtained in terms of the cutting force ($\mathbf{F}_c$), the feed force ($\mathbf{F}_f$), the feed (f), the depth of cut (d), and the shear angle (α). Figure 4.15 illustrates the shear force ($\mathbf{F}_s$) acting on the shear plane. The shear stress can be written as

$$S_s = \frac{\mathbf{F}_s}{A_s} \tag{4.33}$$

where S_s is the shear stress, $\mathbf{F}_s$ is the shear force, L_s is the shear length $= f/\sin\alpha$, A_s is the shear area $= L_s \times d$, and $A_s = f \times d/\sin\alpha$. By substituting, the shear stress can be expressed as

$$S_s = \frac{\mathbf{F}_s \sin\alpha}{f \times d} \tag{4.34}$$

Since dynamometer readings are given in terms of the cutting force ($\mathbf{F}_c$) and the feed force ($\mathbf{F}_f$), it is convenient to express the shear stress in these terms. Figure 4.16 illustrates the graphical relationship of the shear force ($\mathbf{F}_s$) with the cutting force ($\mathbf{F}_c$) and the feed force ($\mathbf{F}_f$). By projecting the cutting force ($\mathbf{F}_c$) and the feed force ($\mathbf{F}_f$) onto a plane parallel to the shear force ($\mathbf{F}_s$), it can be written that

$$\mathbf{F}_s = \mathbf{F}_c \cos\alpha - \mathbf{F}_f \sin\alpha \tag{4.35}$$

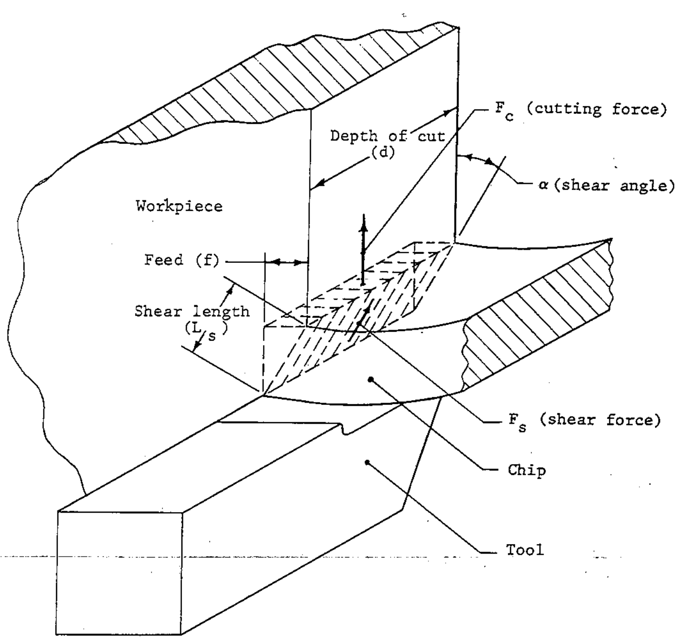


Fig. 4.15. Shear force acting on shear area, and cutting force acting on chip pressure area.

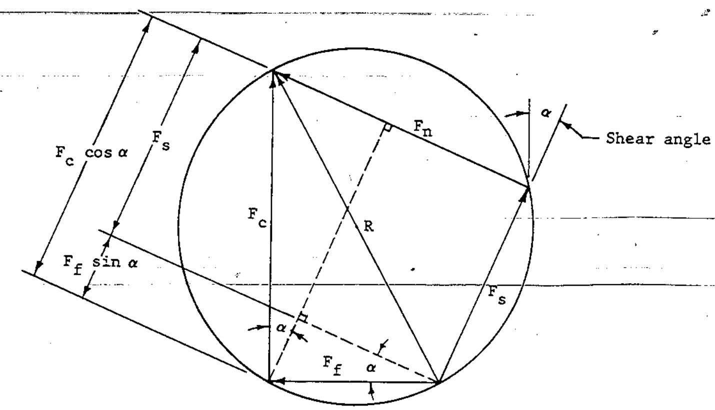


Fig. 4.16. Relationship between the shear force and the cutting and feed forces.

Substituting Eq. 4.35 into Eq. 4.34 yields

$$S_s = \frac{\mathbf{F}_c \cos\alpha \sin\alpha - \mathbf{F}_f \sin^2\alpha}{f \times d} \tag{4.36}$$

The following example illustrates how the shear stress can be calculated for a metal cutting operation by using dynamometer readings.

EXAMPLE 4.8. Using the data from example 4.7, where $\mathbf{F}_c = 300$ lb, $\mathbf{F}_f = 125$ lb, and $\alpha = 25°$, determine the shear stress for a turning operation. The feed (f) was set at 0.015 in./rev and the depth of cut (d) was set at 0.250 in.

Substituting into Eq. 4.34 yields

$$S_s = \frac{\mathbf{F}_c \cos\alpha \sin\alpha - \mathbf{F}_f \sin^2\alpha}{f \times d}$$

$$= \frac{300 \cos 25° \sin 25° - 125 \sin^2(25°)}{0.015 \times 0.250}$$

$$= 24{,}676 \text{ lb/in.}^2 \ (170 \times 10^6 \text{ Pa})$$

Another point of interest in analyzing stress is an appraisal of the chip pressure resistance. This is the ratio of the cutting force to the chip area perpendicular to the cutting force. This is illustrated in Fig. 4.17 as a segment removed from Fig. 4.15.

The chip pressure can be written as

$$P_c = \frac{\mathbf{F}_c}{f \times d} \tag{4.37}$$

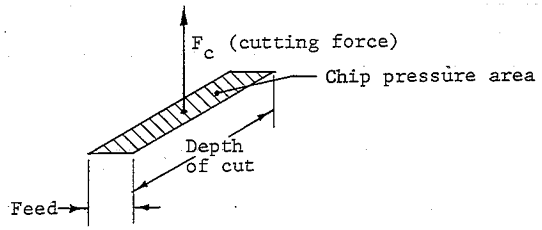


Fig. 4.17. Cutting force acting on chip pressure area.

where P_c is the chip pressure, $f \times d$ is the chip pressure area, and $\mathbf{F}_c$ is the cutting force. A calculation of the chip pressure is given with the following example, where a comparison is made between the chip pressure resistance and the shear stress for a metal cutting operation.

EXAMPLE 4.9. From the data collected in example 4.8, it is desired to calculate the chip pressure resistance and to compare it with the shear stress.

Substituting values into Eq. 4.37 yields

$$P_c = \frac{\mathbf{F}_c}{f \times d}$$

$$= \frac{300}{0.015 \times 0.250}$$

$$= 80{,}000 \text{ psi } (551.6 \times 10^6 \text{ Pa})$$

Solving for the ratio of the shear stress to the chip pressure resistance yields

$$R_{sc} = \frac{S_s}{P_c}$$

$$= \frac{24{,}676}{80{,}000}$$

$$= 0.308$$

4.7 INFLUENCE OF SHEAR ANGLE ON CUTTING FORCE

The cutting force ($\mathbf{F}_c$) that the tool exerts on the workpiece can usually be reduced in a metal cutting operation by increasing the rake angle (β) on the tool. This is a consequence of the general effect that

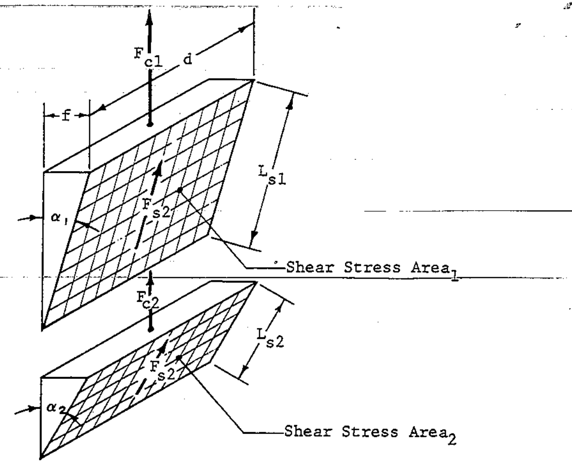


Fig. 4.18. Shear stress area reduction due to increase in shear angle.

an increase of the tool rake angle (β) has on the shear angle (α). Usually, a rake angle increase on the cutting tool will increase the shear angle on the work material. The result is a thinner chip and a reduction in the expenditure of energy. Figure 4.18 illustrates the effect of an increase in the shear angle on the shear area.

As can be seen in Fig. 4.18, the area ($f \times d$) over which the cutting force acts is equal to the feed (f) multiplied by the depth of cut (d). In the two cases shown, this area has the same value. However, an increase in the shear angle from α_1 to α_2, as shown, has a dramatic influence on the shear area over which the shear force acts. If the assumption is made that the shear strength of the material being cut is a true constant, that is, its value does not vary as the shear angle changes, then an interesting analysis can be conducted noting the influence of the shear angle on the cutting force.

Figure 4.19 illustrates the resultant force (**R**) resolved into two sets of perpendicular components. By projecting the shear force ($\mathbf{F}_s$) and the force normal to the shear plane ($\mathbf{F}_n$) onto the cutting force ($\mathbf{F}_c$), the following geometric relationship can be written:

$$\mathbf{F}_c = \mathbf{F}_s \cos\alpha + \mathbf{F}_n \sin\alpha \qquad (4.38)$$

An examination of Fig. 4.19 reveals that the two sets of perpendicular components share a common hypotenuse (the resultant cutting force, **R**), which serves two right triangles. Taking the two right triangles, into account, it can be expressed that

$$\mathbf{R} = \frac{\mathbf{F}_c}{\cos(\phi - \beta)} = \frac{\mathbf{F}_s}{\cos(\alpha - \beta + \phi)} \qquad (4.39)$$

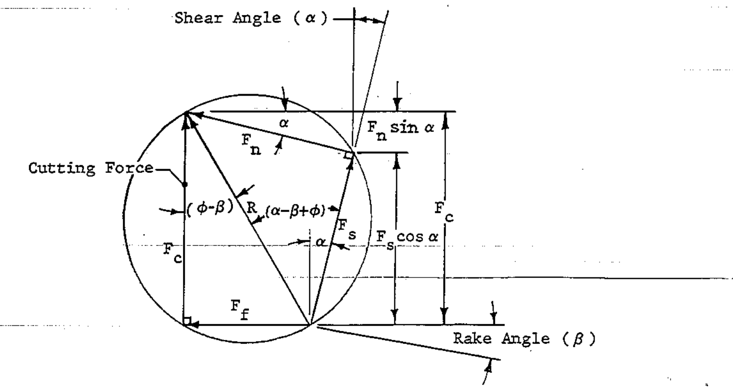


Fig. 4.19. Relationship between the cutting force and the shear force.

where α is the shear angle, β is the rake angle, ϕ is the angle of kinetic friction, ϕ is the $\tan^{-1}$ (**F**/**N**) $= \tan^{-1}\mu$, and μ is the coefficient of friction. From Eq. 4.39 the cutting force ($\mathbf{F}_c$) can be expressed as

$$\mathbf{F}_c = \frac{\mathbf{F}_s \cos(\phi - \beta)}{\cos(\alpha - \beta + \phi)} \tag{4.40}$$

A simplified analysis of the influence of the shear angle on the forces acting between the tool and the work material will now be presented with the following assumptions imposed on the cutting process:

1. The shear strength of the material being cut remains constant.
2. The ratio between the shear force ($\mathbf{F}_s$) and the force normal to the shear plane ($\mathbf{F}_n$) remains constant.

In other words, the shear stress and the ratio $\mathbf{F}_n/\mathbf{F}_s$ are assumed not to vary during the chip forming operation as the shear angle changes due to different values being set on the rake angle of the tool. With the imposition of these two assumptions, the following example is scrutinized.

EXAMPLE 4.10. A turning operation is to be analyzed for the relationship between the shear angle and the cutting force. Experimental data reveal that the yield shear stress of the work material is 30,000 psi (206×10^6 Pa). In addition, the following measurements were taken:

Test	Rake Angle (β)	Shear Angle (α)
1	0°	9°
2	10°	13.5°
3	20°	18.0°
4	25°	20.0°

The feed was set at 0.005 in./rev (0.127 mm/rev) and the depth of cut was set at 0.250 in. (6.35 mm).

From these data and with the assumptions that the shear stress remains constant and that the ratio $\mathbf{F}_n/\mathbf{F}_s = 1$, it is desired to find the influence of the shear angle on the cutting force.

The shear area can be expressed as

$$A_s = \frac{f \times d}{\sin\alpha} \tag{1}$$

the shear force can be expressed as

$$\mathbf{F}_s = S_s \times A_s \tag{2}$$

and the cutting force can be expressed as

$$\mathbf{F}_c = \mathbf{F}_s \cos\alpha + \mathbf{F}_n \sin\alpha \tag{3}$$

where the assumption is made that $\mathbf{F}_n = \mathbf{F}_s$. With this substitution, Eq. 3 can be written as

$$\mathbf{F}_c = \mathbf{F}_s(\cos\alpha + \sin\alpha)$$

Substituting the given numerical values into Eqs. 1–3 gives the following results for the four tests:

Test	Shear Force, $\mathbf{F}_s$ (lb)	Shear Force, $\mathbf{F}_s$ (N)	Cutting Force, $\mathbf{F}_c$ (lb)	Cutting Force, $\mathbf{F}_c$ (N)
1	239.7	1066	274.25	1220
2	160.5	714	193.53	861
3	121.35	540	152.91	680
4	109.64	488	140.53	625

As can be seen in example 4.10, the shear angle, which has an affiliation with the tool rake angle, has, in turn, a strong influence on the cutting force. A graphically scaled diagram of the results of example 4.10 is shown in Fig. 4.20. A comparison of the cutting force in Test 1, 274.25 lb (1220 N), with the cutting force in Test 4, 140.53 lb (625 N), reveals that a 25° change in the rake angle ($\beta_4 - \beta_1$) results in a 48.8% reduction in the cutting force.

A point of interest in analyzing Fig. 4.20 is to see what effect the assumption that $\mathbf{F}_n/\mathbf{F}_s = 1$ had on the angle $(\phi - \beta)$. It is obvious that if $\mathbf{F}_n/\mathbf{F}_s = 1$, then the angle $(\alpha - \beta + \phi)$ must be equal to 45°. Since α and β are known angles, the angle of kinetic friction (ϕ) can be extracted in example 4.10 by writing that

$$\alpha - \beta + \phi = 45°$$

or

$$\phi = 45° - \alpha + \beta$$

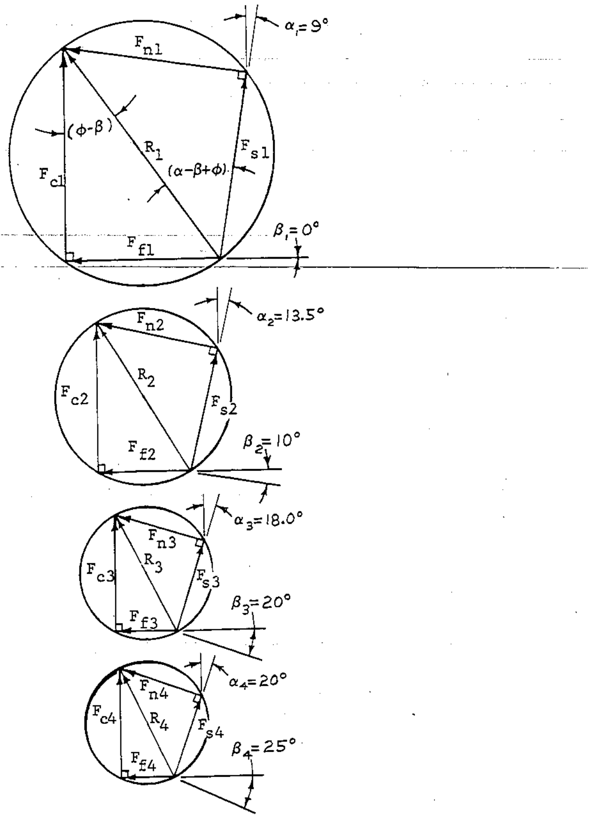


Fig. 4.20. Graphically scaled diagram of results of example 4.10.

Using the data for α and β from example 4.10 yields

$$\phi_1 = 45° - 9° + 0° = 36°$$

$$\phi_2 = 45° - 13.5° + 10° = 41.5°$$

$$\phi_3 = 45° - 18° + 20° = 47°$$

$$\phi_4 = 45° - 20° + 25° = 52°$$

The coefficient of friction can be extracted through the use of the angle ϕ by writing

$$\mu = \tan\phi = \mathbf{F}/\mathbf{N} \tag{4.6}$$

As a result,

$$\mu_1 = 0.727$$

$$\mu_2 = 0.885$$

$$\mu_3 = 1.072$$

$$\mu_4 = 1.280$$

As can be seen, to satisfy the equilibrium conditions as dictated by Fig. 4.20 with the imposition of the assumption that $\mathbf{F}_n/\mathbf{F}_s = 1$ and that the shear stress remains constant results in a wide fluctuation of the coefficient of friction. This leads to the questioning of the degree of reliability of the assumptions made. Although we are depending on pressure and temperature conditions that can lead up to a welding action between the tool and the chip resulting in a built-up edge on the tool, a large change in the coefficient of friction is possible. In a similar fashion, it can also be pointed out that the shear stress may vary as a function of the temperature of the work material, the rate of the shearing action, and the stress acting normal to the shear plane.

EXAMPLE 4.11. Calculate the expected cutting force for an operation where the shear stress is 35,000 psi (241 × 10^6 Pa) and the shear angle is 25°. The feed is set at 0.015 in. (0.381 mm) and the depth of cut is 0.125 in. (3.175 mm). Assume that the $\mathbf{F}_n/\mathbf{F}_s$ ratio is equal to 1.

Solving for shear area we get

$$A_s = \frac{f \times d}{\sin\alpha} = \frac{0.015 \times 0.125}{\sin 25°}$$

$$= 0.0044 \text{ in.}^2 \ (0.1118 \text{ mm}^2)$$

The shear force is equal to

$$\mathbf{F}_s = S_s \times A_s = 35{,}000 \times 0.0044$$

$$= 154 \text{ lb } (685 \text{ N})$$

The cutting force is equal to

$$\mathbf{F}_c = \mathbf{F}_s \cos\alpha + \mathbf{F}_n \sin\alpha$$

If $\mathbf{F}_n = \mathbf{F}_s$, then

$$\mathbf{F}_c = \mathbf{F}_s(\cos\alpha + \sin\alpha) = 154(\cos 25° + \sin 25°)$$

$$= 204.7 \text{ lb } (910.3 \text{ N})$$

4.8 COEFFICIENT OF FRICTION

When two surfaces are in contact and one surface is moving relative to the other surface, as in the case of the chip and the tool in a metal cutting operation, a force resistant to the motion is generated and acts in a direction opposite to the relative velocity. This force is called the *frictional force*. It is a measure of the resisting force that the relative motion encounters. It can also be interpreted as a measure of the quality of the surface of a cutting tool. A tool that provides a low frictional force while the chip slides over it can be expected to expend less energy and to generate lower temperatures in the cutting operation.

Figure 4.21 illustrates the frictional force acting in a direction opposite to the relative sliding velocity. In Fig. 4.21(a) the velocity of the chip is viewed from the tool, that is, when viewed from the tool it is observed that the chip is moving in the direction shown. As can be seen, the frictional force that the tool exerts on the chip acts in a direction opposite to the chip motion. On the other hand, when viewed from the chip, it is observed that the tool is moving in the direction indicated in Fig. 4.21(b). In this case, the frictional force that the chip exerts on the tool has a tendency to try to move the tool along with the chip.

Newton's third law of mechanics states that when two bodies exert forces on each other, these forces are equal in magnitude, opposite in direction, and act in the same line of action. As a result, it can be written that

$$\begin{pmatrix}\text{Frictional force that}\\ \text{tool exerts on chip}\end{pmatrix} = -\begin{pmatrix}\text{Frictional force that}\\ \text{chip exerts on tool}\end{pmatrix}$$

or

$$\mathbf{F}_{t/c} = -\mathbf{F}_{c/t} \tag{4.41}$$

where the subscript t/c indicates tool relative to chip, and the subscript c/t indicates chip relative to tool.

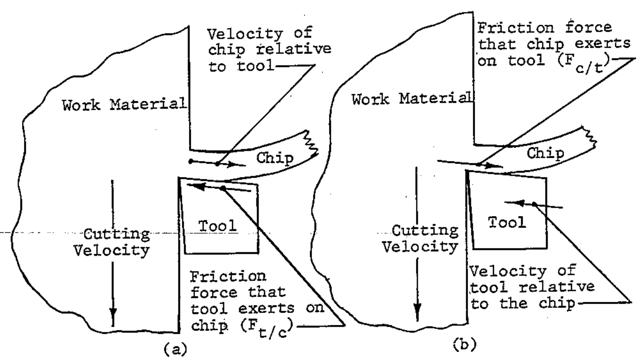


Fig. 4.21. Frictional force and relative chip velocity.

A measurement of the frictional-force-generating characteristic of surfaces in contact during sliding motion is known as the *coefficient of friction*. This is equal to the ratio of the friction force (**F**) to the force acting normal to the sliding surface (**N**). It can be expressed as

$$\mu = \frac{\mathbf{F}}{\mathbf{N}} \tag{4.5}$$

An examination of Fig. 4.4 reveals that trigonometrically this ratio can be expressed in terms of the angle ϕ as

$$\mu = \tan\phi = \frac{\mathbf{F}}{\mathbf{N}} \tag{4.6}$$

where ϕ is the angle of kinetic friction for sliding motion.

A test can be conducted to determine the value of the coefficient of friction in a metal cutting operation. However, using Eq. 4.5 is not convenient insofar as the frictional force (**F**) and the normal force (**N**) are difficult to measure directly. Usually in conducting a test of this type, a dynamometer is used, from which a measurement of the cutting force ($\mathbf{F}_c$) and the feed force ($\mathbf{F}_f$) can be determined. Figure 4.22 illustrates the action–reaction relationship between the work material and the cutting tool. **R**′ represents the resultant force (action) that the work material exerts on the tool, whereas **R** represents the resultant force (reaction) that the tool exerts on the work material. As can be seen, the resultant is resolved into convenient components relating the cutting force ($\mathbf{F}'_c$) and the feed force ($\mathbf{F}'_f$) with the frictional force (**F**) and the normal force (**N**).

In order to evaluate the coefficient of friction directly from a dynamometer measurement of the cutting force ($\mathbf{F}_c$) and the feed force ($\mathbf{F}_f$),

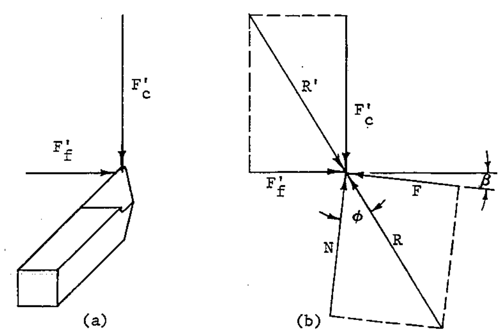


Fig. 4.22. Cutting and feed forces resolved into frictional and normal components.

expressions for the friction force (**F**) and the normal force (**N**) as functions of $\mathbf{F}_c$ and $\mathbf{F}_f$ are necessary. These can be conveniently attained from geometric projections, as shown in Fig. 4.23. The rake angle (β) is the trigonometric relationship that enables the projections to take place. By this technique, expressions can be derived that enable the coefficient of friction to be written in terms of the cutting force ($\mathbf{F}_c$), the feed force ($\mathbf{F}_f$), and the rake angle (β) of the tool.

To illustrate, by projecting the feed force ($\mathbf{F}_f$) and the cutting force ($\mathbf{F}_c$) onto the friction force (**F**) we can write the frictional force as

$$\mathbf{F} = \mathbf{F}_c \sin\beta + \mathbf{F}_f \cos\beta \tag{4.42}$$

In a similar fashion, the normal force can be written as

$$\mathbf{N} = \mathbf{F}_c \cos\beta - \mathbf{F}_f \sin\beta \tag{4.43}$$

Substituting into Eq. 4.6 yields

$$\mu = \tan\phi = \frac{\mathbf{F}_c \sin\beta + \mathbf{F}_f \cos\beta}{\mathbf{F}_c \cos\beta - \mathbf{F}_f \sin\beta}\left(\frac{\cos\beta}{\cos\beta}\right)$$

Multiplying by unity in the form of $\cos\beta/\cos\beta$ and noting that $\sin\beta/\cos\beta = \tan\beta$ simplifies the equation to

$$\mu = \tan\phi = \frac{\mathbf{F}_f + \mathbf{F}_c \tan\beta}{\mathbf{F}_c - \mathbf{F}_f \tan\beta} \tag{4.44}$$

The following numerical example demonstrates how the value of the coefficient of friction for a cutting operation can be extracted from dynamometer force measurements when the tool rake angle is known.

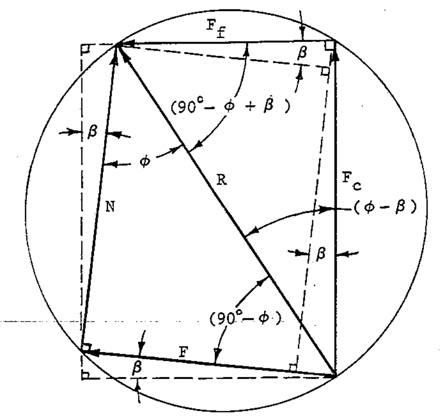


Fig. 4.23. Projection of cutting and feed forces in direction of frictional and normal forces.

EXAMPLE 4.12. Calculate the coefficient of friction for a metal cutting operation that has a measured cutting force of 310 lb (1379 N) and a feed force of 215 lb (956 N). The tool rake angle is 5°.

Substituting into Eq. 4.44 yields

$$\mu = \frac{215 + 310\tan 5°}{310 - 215\tan 5°}$$
$$= 0.8315$$

Another method by which the coefficient of friction can be determined from dynamometer readings and the value of the tool rake angle is through the use of the angle of kinetic friction (ϕ). An examination of Fig. 4.23 reveals that

$$\tan(\phi - \beta) = \frac{\mathbf{F}_f}{\mathbf{F}_c} \tag{4.45}$$

where

$$\phi = \left(\tan^{-1}\frac{\mathbf{F}_f}{\mathbf{F}_c}\right) + \beta \tag{4.46}$$

Substituting into Eq. 4.6 yields

$$\mu = \tan\phi = \tan\left[\left(\tan^{-1}\frac{\mathbf{F}_f}{\mathbf{F}_c}\right) + \beta\right] \tag{4.47}$$

EXAMPLE 4.13. Confirm the results of example 4.13 by using Eq. 4.47. Substituting numerical values into Eq. 4.46 results in

$$\mu = \tan[(\tan^{-1}\tfrac{215}{310}) + 5]$$
$$= \tan 39.74° = 0.8315$$

As can be seen in example 4.13, the angle of kinetic friction is equal to 39.74°.

Of special interest in examining Fig. 4.23 in conjunction with equation 4.44 is the special case when the rake angle (β) is equal to zero. With $\beta = 0$, the angle $\phi - \beta$ in Eq. 4.45 becomes equal to ϕ and thus

$$\tan\phi = \frac{\mathbf{F}_f}{\mathbf{F}_c} = \mu \tag{4.48}$$

with $\beta = 0°$. In a fashion similar with $\beta = 0°$ in Eq. 4.44, the coefficient of friction can be written as

$$\mu = \tan\phi = \frac{\mathbf{F}_f}{\mathbf{F}_c} \tag{4.49}$$

where $\tan\beta = 0$. An appraisal of the effects of the condition $\beta = 0°$ in Fig. 4.23 indicates that the perpendicular components of **R** would form a rectangle where

$$\mathbf{N} = \mathbf{F}_c$$

and

$$\mathbf{F} = \mathbf{F}_f$$

As a result, with $\beta = 0°$,

$$\mu = \tan\phi = \frac{\mathbf{F}_f}{\mathbf{F}_c} = \frac{\mathbf{F}}{\mathbf{N}} \tag{4.50}$$

An important fact to note is that the coefficient of friction for metals in contact does not remain constant under conditions where the normal force, the contact temperature, and the relative velocity vary. In the metal cutting process, where high pressures exist between the surfaces, large deformations take place along the shear plane of the chip. This condition complicates the frictional relationship beyond a simple surface effect analysis. Physical properties of the metals in contact also have a major influence on the frictional force. At the points of contact between the chip and the tool, local welding adhesion and plastic flow add to the frictional resistance by requiring the shearing of the metallic junctions that are formed.

Test results of an experiment[1] carried out to measure the coefficient of friction applying Eq. 4.44 are shown in Table 4.2. A high-speed tool with a rake angle of 0° was used to machine a bar of cold-rolled steel. The depth of cut was set to 0.050 in. (1.27 mm) and the feed was set at 0.0041 in./rev (0.104 mm/rev). The cutting speed varied from 40 to 550 ft/min (12 to 167.6 m/min). The experimental data listed in Table 4.2 are plotted in Fig. 4.24.

An examination of the test results reveal that the coefficient of friction increased dramatically as a function of cutting speed for values below 350 ft/min (10.67 m/min). Above 400 ft/min (122 m/min), the experimental data indicate a constant coefficient of friction equivalent to 0.806. An intuitive evaluation of this response of coefficient of friction as a function of cutting speed could be justified by taking into account the complexity of the interaction of the chip flow over the tool. Work hardening of the chip as a result of plastic deformation, combined with high pressure and temperature conditions, can cause adhesion and

[1] Leslie F. Costa, Jr., "A Measurement of the Coefficient of Friction as a Function of Cutting Speed," Senior Design Project, Southeastern Massachusetts University, North Dartmouth, Massachusetts, 1977.

Table 4.2. Listing of Results of Coefficient of Friction Test

Cutting Speed		Dynamometer Readings F_c		Dynamometer Readings F_f		Coefficient of Friction
(ft/min)	(m/min)	(lb)	(N)	(lb)	(N)	
40	12.2	74	329	34	151	0.459
60	18.3	74	329	39	173	0.527
80	24.4	74	329	39	173	0.527
100	30.5	67	298	35	156	0.522
120	36.6	64	285	39	173	0.609
140	42.7	67	298	39	173	0.582
160	48.8	74	329	43	191	0.581
180	54.9	67	298	43	191	0.624
200	61.0	67	298	49	218	0.731
280	85.3	67	298	68	302	1.015
300	91.4	67	298	68	302	1.015
350	106.7	67	298	88	391	1.313
400	122.0	67	298	54	240	0.806
450	137.2	67	298	54	240	0.806
500	152.4	67	298	54	240	0.806
550	167.6	67	298	54	240	0.806

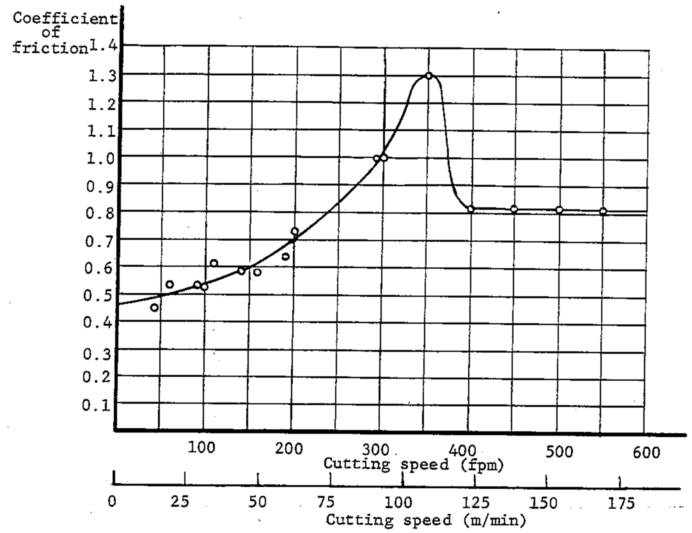


Fig. 4.24. Plot of results of coefficient of friction test.

welding of the points of contact between the chip and the tool. This state can lead to a built-up edge of the chip material on the tool. The result causes the chip formation to be formed by a ploughing of the softer chip material over the deposited harder chip material. What appears to be a high frictional force can be detected. This is, in reality, the force required to shear the chip material over the deposited built-up edge. The end product is a measured high coefficient of friction due to complications of internal chip behavior.

The drop in the coefficient of friction above the 350-ft/min (107-m/min) level can be rationalized by a removal of the built-up edge and a return to a simple sliding action between the chip and tool. This is contrasted with a complicated high-coefficient-of-friction metal shearing condition that may possibly exist in the 350-ft/min (107-m/min) range.

A point of interest would be the examination of a simple sliding friction case. Experimental results of sliding friction on dry surfaces of steel on steel reveal that, in general, the coefficient of friction declines as the sliding velocity between the surfaces increases. If this simple sliding friction case were imposed on a metal cutting operation, typical experimental data would be of the form indicated in Table 4.3. The data from Table 4.3 is graphically illustrated in Fig. 4.25.

A comparison of Fig. 4.24 with Fig. 4.25 indicates that there is a substantial difference in the shape of the curves. In reference sources, the value of the sliding coefficient of friction of hard steel on hard steel is usually given as about 0.42, whereas for mild steel on mild steel it is usually about 0.57. Focusing attention on the high values of the sliding coefficient of friction in metal cutting emphasizes the fact that there is a disparity when comparing these values with the results from the general laws of friction for clean, dry, and smooth surfaces.

In practice, the coefficient of friction associated with chip sliding has higher values than for ordinary sliding. The difference can be explained through the welding action that takes place between the chip and the tool. Many times, there is a welding of the chip to the tool and a mechanical interlocking of the surface asperites. This results in a deposit of a harder built-up edge on the tool requiring the

Table 4.3. Coefficient-of-Friction Sliding-Velocity Data

Sliding velocity						
(ft/min):	100	200	300	400	500	600
(m/min):	30.5	61.0	91.4	122	152	183
Coefficient of friction:	1.10	1.04	0.98	0.92	0.86	0.80

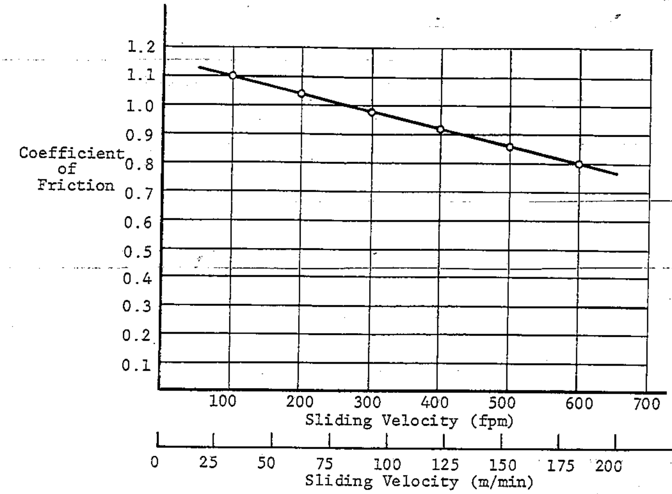


Fig. 4.25. Coefficient of friction versus sliding velocity.

ploughing of the deposited chip through the softer chip material. The result is dramatically unlike expectations from simple sliding friction analysis. In metal cutting, the coefficient of friction has a relationship with temperature, magnitude of sliding velocity, area of contact, and the applied normal load.

EXAMPLE 4.14. For a metal cutting operation with a tool rake angle of 0°, where the coefficient of friction is equal to 0.8 and the cutting force is measured to be 250 lb, calculate the magnitude of the feed force.

From Eq. 4.50,

$$\mathbf{F}_f = \mu \mathbf{F}_c \qquad \text{when } \beta = 0°$$

Substituting yields

$$\mathbf{F}_f = 0.8(250) = 200 \text{ lb } (889.6 \text{ N})$$

4.9 VELOCITY RELATIONSHIPS

An examination of the metal cutting process reveals that there are three velocities that are of special significance. These are the cutting velocity ($\mathbf{V}_c$), the chip velocity ($\mathbf{V}_p$), and the shear velocity ($\mathbf{V}_s$). Figure 4.26 illustrates the different directions of these three velocities. The cutting velocity ($\mathbf{V}_c$) is the velocity of the work material relative to the

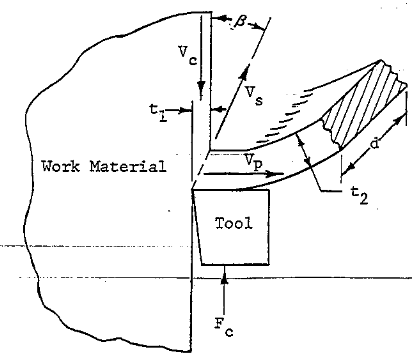


Fig. 4.26. Metal cutting velocities for stationary tool.

tool, and it has a direction parallel to the cutting force ($\mathbf{F}_c$). The chip velocity ($\mathbf{V}_p$) is the velocity of the chip relative to the tool, and it has a direction parallel to the tool face. The shear velocity ($\mathbf{V}_s$) is the velocity of the chip relative to the work material, and it has a direction parallel to the shear plane. If one were to observe the chip from the work material, it would appear as if the chip were moving in the direction of the shear velocity. As a result, it can be written that

$$\mathbf{V}_s = \mathbf{V}_p - \mathbf{V}_c \tag{4.51}$$

where $\mathbf{V}_s$ is the velocity of chip relation to work material, $\mathbf{V}_p$ is the velocity of chip relative to the tool, $\mathbf{V}_c$ is the cutting velocity or velocity of work material relative to the tool, and $\mathbf{V}_c = \mathbf{V}_{w/t} = -\mathbf{V}_{t/w}$ (w/t indicates work material relative to tool, and t/w indicates tool relative to work material).

Figure 4.27 is a graphical layout of Eq. 4.51, indicating that the shear velocity ($\mathbf{V}_s$) is vectorially equal to the chip velocity ($\mathbf{V}_p$) plus the velocity of the tool relative to the work material ($\mathbf{V}_{t/w}$).

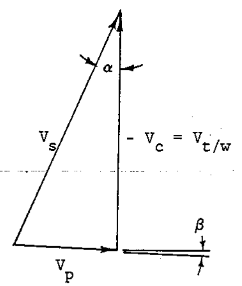


Fig. 4.27. Graphical layout of Eq. 4.51.

The relationship between the magnitudes of the cutting velocity and the chip velocity can be derived by noting that the volumetric quantity of material being cut per unit time must be accumulated in the chip. This can be expressed as

$$\text{Volume material} = \text{Volume chip}$$

or

$$\text{Feed} \times \text{Depth of cut} \times \text{Cutting velocity} = \text{Chip thickness} \times \text{Depth of cut} \times \text{Chip velocity}$$

or

$$t_1 \times d \times \mathbf{V}_c = t_2 \times d \times \mathbf{V}_p$$

from which

$$\mathbf{V}_p = \frac{t_1}{t_2}\mathbf{V}_c \tag{4.52}$$

EXAMPLE 4.15. Given a feed of 0.0041 in./rev (0.104 mm/rev), a cutting velocity of 300 ft/min (91.44 m/min), and a measured chip thickness of 0.016 in. (0.4064 mm), determine the chip velocity.

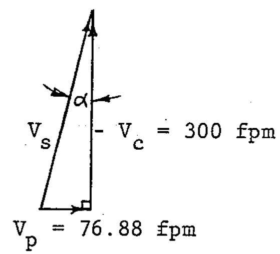
Substituting into Eq. 4.49 yields

$$\mathbf{V}_p = \frac{0.0041}{0.016}(300)$$

$$= 76.88 \text{ ft/min } (23.43 \text{ m/min})$$

Of interest in evaluating chip velocity ($\mathbf{V}_p$) is the observation that with the given cutting velocity ($\mathbf{V}_c$) two of the three vectors constituting Fig. 4.27 are available. If the angular orientation of the vectors is known, then the shear velocity as well as the shear angle (α) can be evaluated. The following example illustrates this technique.

EXAMPLE 4.16. Given the data from example 4.15, where the tool rake angle is 0°, solve for the shear velocity as well as the shear angle.



With $\beta = 0°$, the vector polygon constitutes a right triangle. Therefore, the magnitude of the shear velocity can be written as

$$\mathbf{V}_s = \sqrt{(\mathbf{V}_p)^2 + (-\mathbf{V}_c)^2}$$
$$= 309.7 \text{ ft/min } (94.4 \text{ m/min})$$

The shear angle (α) can be written as

$$\tan\alpha = \frac{\mathbf{V}_p}{\mathbf{V}_c} = \frac{76.88}{300} = 0.25627$$

$$\alpha = 14.37°$$

With the availability of Eq. 4.52, a chip thickness measurement can lead to the evaluation of the chip velocity for given machine settings of the feed (t_1) and the cutting velocity ($\mathbf{V}_c$). Table 4.4 lists values for the chip thickness as well as for the chip velocity for the coefficient of friction test specified in Table 4.2. The test involved turning cold-rolled steel with a high-speed tool that had a 0° rake angle. The feed was set at 0.0041 in./rev (0.104 mm/rev) and the depth of cut was 0.050 in. (1.27 mm). A plot of the chip thickness versus cutting velocity is given in Fig. 4.28.

As can be seen, the chip thickness (t_2) varies in conjunction with the changes in the coefficient of friction, which, in turn, responds to

Table 4.4. Chip Thickness and Sliding-Chip Velocity Values for Coefficient of Friction Test with Feed Setting of 0.0041 in./rev

Cutting Velocity, V_c		Chip Thickness, t_2		Sliding Velocity, V_p	
(ft/min)	(m/min)	(in.)	(mm)	(ft/min)	(m/min)
40	12.2	0.011	0.279	14.91	4.54
60	18.3	0.011	0.279	22.36	6.82
80	24.4	0.012	0.305	27.33	8.33
100	30.5	0.01125	0.286	36.44	11.11
120	36.6	0.012	0.305	41.00	12.50
140	42.7	0.012	0.305	47.83	14.58
160	48.8	0.011	0.279	59.64	18.18
180	54.9	0.012	0.305	61.50	18.75
200	61.0	0.013	0.330	63.08	19.23
280	85.3	0.0155	0.394	74.06	22.57
300	91.4	0.016	0.406	76.88	23.43
350	106.7	0.015	0.381	95.67	29.16
400	122.0	0.014	0.356	117.14	35.70
450	137.2	0.013	0.330	141.92	43.26
500	152.4	0.013	0.330	157.69	48.06
550	167.4	0.0125	0.318	180.40	54.99

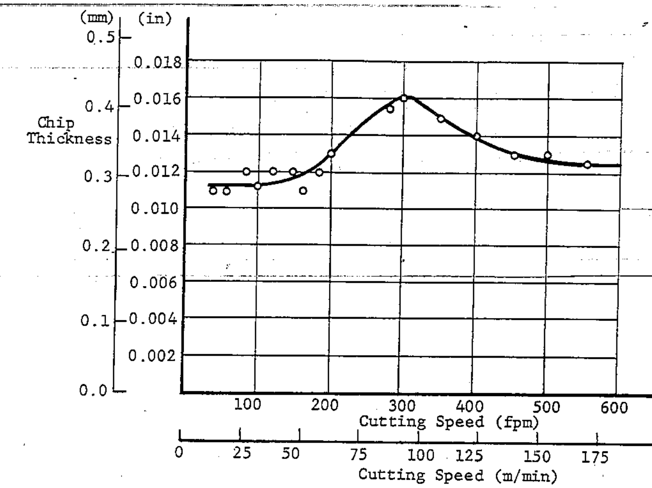


Fig. 4.28. Chip thickness versus cutting velocity.

the different settings of the cutting velocity. Of interest also is the relationship of the chip velocity to the cutting velocity. This is illustrated in Fig. 4.29.

The fluctuation in the slope of the chip-velocity–cutting-velocity curve indicates that the chip flow velocity as compared to the cutting velocity does not have a fixed linear relationship over the range of cutting velocity. The change is most evident between 200 and

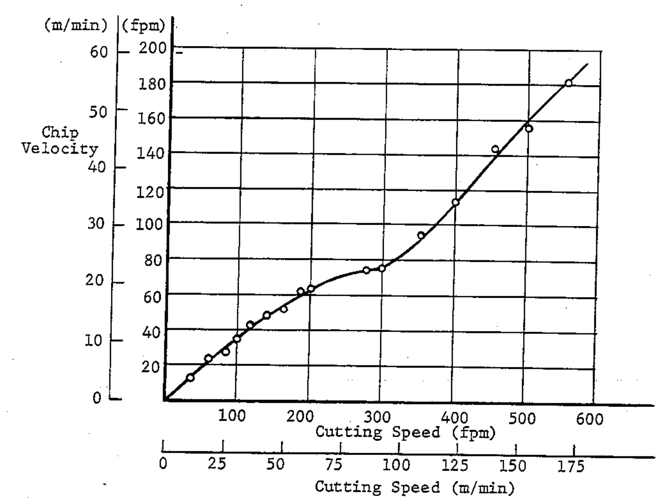


Fig. 4.29. Chip velocity versus cutting velocity.

300 ft/min, where the slope drops off significantly; this indicates an accumulating effect in the chip, which is confirmed by the corresponding chip thickness increase shown in Fig. 4.28.

The chip velocity ($\mathbf{V}_p$) and the shear velocity ($\mathbf{V}_s$) are related to the shear angle (α) and the rake angle (β), as well as with the cutting velocity ($\mathbf{V}_c$). Figure 4.30 shows these three velocities, along with the associated angles, for the case where the tool is considered stationary. This example is similar to the turning operation, where there is a tool feed velocity, but for the sake of the analysis it is so small on a relative scale that it can be neglected.

If the tool is considered to be fixed relative to the earth, and velocities relative to the earth are considered absolute, then the cutting velocity and the chip velocity can be considered to be absolute since they are both relative to the tool. An examination of the shear velocity indicates that relative to the tool it is zero since the shear plane remains in the same position when viewed from the tool. Note that the shear plane is the interface between the work material and the chip. If one places oneself on the work material and observes the motion of the chip, the relative velocity of the chip to the work material can be written in vectorial form as

$$\mathbf{V}_{p/c} = \mathbf{V}_p - \mathbf{V}_c = \mathbf{V}_s \tag{4.53}$$

If the cutting velocity ($\mathbf{V}_c$) is subtracted from the chip velocity ($\mathbf{V}_p$), the result is the velocity of the chip relative to the work material ($\mathbf{V}_{p/c}$). Figure 4.31 illustrates the vector polygon of Eq. 4.53, where $-\mathbf{V}_c$ is in the direction opposite to $\mathbf{V}_c$.

The relationships among the magnitudes of the velocities can be established by applying the law of sines for triangles. From Fig. 4.31,

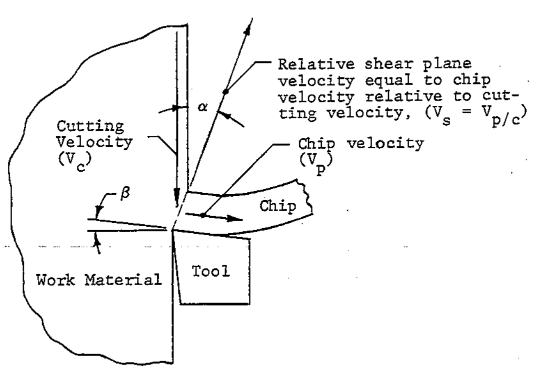


Fig. 4.30. Angular orientation of velocities.

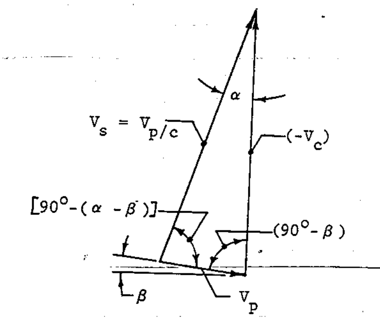


Fig. 4.31. Vector polygon representation of Eq. 4.53.

it can be written that

$$\frac{\sin\alpha}{\mathbf{V}_p} = \frac{\sin[90° - (\alpha - \beta)]}{\mathbf{V}_c} = \frac{\sin(90° - \beta)}{\mathbf{V}_{p/c}} \tag{4.54}$$

Using trigonometric identities where

$$\sin(90° - \beta) = \cos\beta$$

and

$$\sin[90° - (\alpha - \beta)] = \cos(\alpha - \beta)$$

and substituting into Eq. 4.54 yields

$$\frac{\sin\alpha}{\mathbf{V}_p} = \frac{\cos(\alpha - \beta)}{\mathbf{V}_c} = \frac{\cos\beta}{\mathbf{V}_s}, \qquad \text{where } \mathbf{V}_s = \mathbf{V}_{p/c} \tag{4.55}$$

Since the cutting velocity ($\mathbf{V}_c$) is set in a machining operation and can be readily determined, it is convenient to write the chip velocity ($\mathbf{V}_p$) and the shear velocity ($\mathbf{V}_s$) in terms of the cutting velocity ($\mathbf{V}_c$). Therefore, from Eq. 4.55,

$$\mathbf{V}_p = \frac{\sin\alpha}{\cos(\alpha - \beta)}\mathbf{V}_c \tag{4.56}$$

and

$$\mathbf{V}_s = \frac{\cos\beta}{\cos(\alpha - \beta)}\mathbf{V}_c \tag{4.57}$$

An examination of Eqs. 4.56 and 4.57 indicates that the chip velocity ($\mathbf{V}_p$) and the shear velocity ($\mathbf{V}_s$) are functions of the cutting velocity ($\mathbf{V}_c$), the shear angle (α), and the rake angle (β). In other words, if one is given $\mathbf{V}_c$, α, and β, then $\mathbf{V}_p$ and $\mathbf{V}_s$ can be evaluated. The following example demonstrates the application.

EXAMPLE 4.17. Confirm the results of examples 4.15 and 4.16 by using Eqs. 4.56 and 4.57, given the following data: cutting velocity, 300 ft/min (91.44 m/min); shear angle, 14.37°; and tool rake angle, 0°.

Substituting into Eq. 4.56 yields

$$\mathbf{V}_p = \frac{\sin 14.37°}{\cos 14.37°}(300) = 76.86 \text{ ft/min } (23.43 \text{ m/min})$$

Substituting into Eq. 4.57 yields

$$\mathbf{V}_s = \frac{\cos 0°}{\cos 14.37°}(300) = 309.7 \text{ ft/min } (94.4 \text{ m/min})$$

EXAMPLE 4.18. A test is being conducted to measure the chip velocity and the shear velocity for a metal cuttting operation. The cutting velocity is set at 300 ft/min (91.44 m/min) and the rake angle on the tool is 10°. A measurement of the chip thickness reveals that it is equal to 0.02286 in. (0.581 mm). What is the chip velocity and the shear velocity when the feed is set at 0.010 in. (0.254 mm)?

Solving for the shear angle by substituting into Eq. 2.22 yields

$$\tan\alpha = \frac{0.434 \cos 10°}{1 - 0.4374 \sin 10°} = 0.4663$$

$$\alpha = 25°$$

Substituting into Eq. 4.56 yields

$$\mathbf{V}_p = \frac{\sin 25°}{\cos 15°}(300)$$

$$= 131.3 \text{ ft/min } (40.02 \text{ m/min})$$

Substituting into Eq. 4.57 yields

$$\mathbf{V}_s = \frac{\cos 10°}{\cos 15°}(300)$$

$$= 305.9 \text{ ft/min } (993.23 \text{ m/min})$$

Another method by which the shear velocity and the chip velocity can be determined is by means of a graphical layout. If the cutting velocity ($\mathbf{V}_c$) is drawn to scale and if the angles of α and β are known, then a vector polygon can be completed, yielding the magnitudes of $\mathbf{V}_p$ and $\mathbf{V}_s$. An illustration of this technique is given for the data from example 4.18. Figure 4.32 constitutes the graphical layout. The cutting velocity ($\mathbf{V}_c$) is drawn to scale and the known angles α and β are drawn. As can be seen in Fig. 4.32, the intersection of the known directions

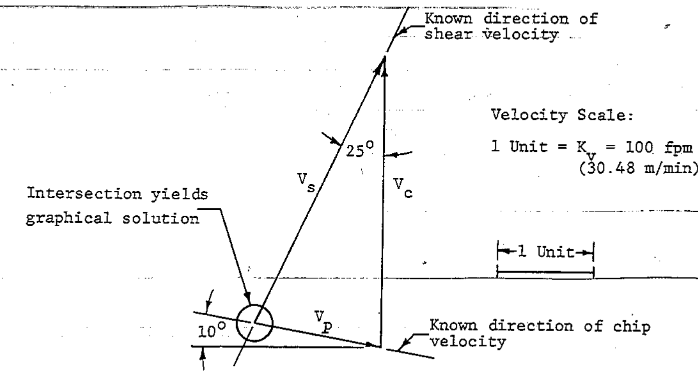


Fig. 4.32. Graphical solution of chip and shear velocities.

signified by α and β provide for a simultaneous graphical solution that yields the magnitudes of $\mathbf{V}_p$ and $\mathbf{V}_s$. Using the scale factor as shown, the velocities are confirmed to be equal to

$$\mathbf{V}_p = 131.3 \text{ ft/min } (40.02 \text{ m/min})$$

and

$$\mathbf{V}_s = 305.9 \text{ ft/min } (93.23 \text{ m/min})$$

Ordinarily, it is difficult to get exact accuracy when using a graphical layout. This is usually due to the precision required of the angular layout as well as the scaling measurement. However, with careful attention, an accurate range within a few percentage points can ordinarily be achieved with a graphical layout.

4.10 ENERGY ANALYSIS OF THE METAL CUTTING PROCESS

It is the objective of this section to show that in evaluating the energy expended in a metal cutting operation, the cutting speed and the cutting force are factors of prime importance. Work can be defined as the transference of energy by a process involving the motion of the point of application of a force. When there is movement against a resisting force, energy, a measure of work performed, is said to be used in providing the motion. Energy and work have the same units, given in terms of the product of the force and the displacement in the direction

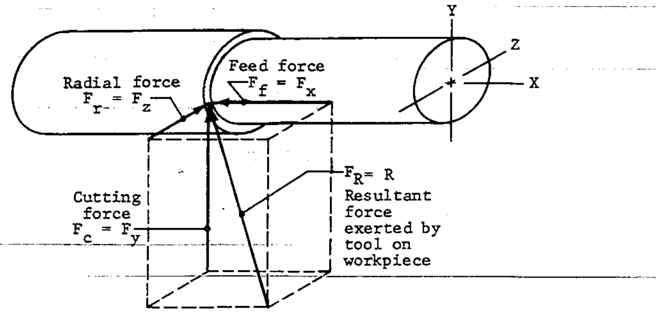


Fig. 4.33. Rectangular components of tool force acting on workpiece in turning operation.

of the force. In U.S. customary units, work is expressed in foot-pounds or inch-pounds, whereas in SI units work is expressed in newton-meters. The newton-meter, also known as a *joule*, is a unit of work:

$$\text{Energy} = \text{Work} = \text{Force} \times \text{Displacement}$$

$$E = W = \mathbf{F} \cdot \mathbf{D} \tag{4.58}$$

Figure 4.33 illustrates the rectangular components of the resultant tool force acting on the workpiece for a turning operation. Corresponding velocities are shown in Fig. 4.34, where the cutting velocity of the workpiece and the feed velocity of the tool are represented. The forces that the workpiece exerts on the tool are in the direction opposite to those that the tool exerts on the workpiece.

When an energy analysis (amount of work performed) of the three rectangular components of the cutting force ($\mathbf{F}_c$, $\mathbf{F}_f$, $\mathbf{F}_t$) is made, it becomes apparent that most of the energy of cutting is used by the cutting force ($\mathbf{F}_c$). This occurs not only because the cutting force is the largest of the component forces, but especially because the displacement in the direction of the cutting force is very large when compared

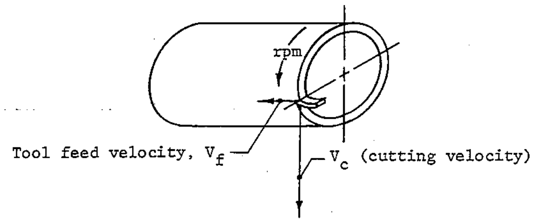


Fig. 4.34. Tool feed and cutting velocity.

to the displacement of the other component forces. As an example, the displacement for one revolution of the workpiece in the direction of the feed force is equal to the feed per revolution, whereas the displacement in the direction of the radial force is zero when turning a cylinder. When the displacement associated with the cutting force for one revolution of the workpiece is compared to the displacement of the feed force and the radial force, it is found to be relatively large. It is equivalent to the circumference of the workpiece.

EXAMPLE 4.19. It is desired to find the amount of work performed by each of the three components of the resultant force for one revolution of a turning operation. The cutting force measurements are $\mathbf{F}_c =$ 210 lb (934 N), $\mathbf{F}_f = 75$ lb (333.6 N), and $\mathbf{F}_r = 20$ lb (89 N). The diameter of the workpiece is 2 in. (50.8 mm), and it is being machined with a cutting velocity of 150 ft/min (45.72 m/min), a feed setting of 0.015 in./rev (0.381 mm/rev), and a depth of cut of 0.200 in. (5.08 mm).

Since the depth of cut is set at a fixed value, the displacement in the direction of the radial force is equal to zero. As a result, the work performed by the radial force can be written as

$$W_r = \mathbf{F}_r \cdot \mathbf{D}_r = 20 \times 0 = 0 \text{ in.-lb } (0 \text{ N-m})$$

The displacement for one revolution in the direction of the feed force is equal to the feed or 0.015 in. (0.381 mm). As a result, the work performed by the feed force can be written as

$$W_f = \mathbf{F}_f \cdot \mathbf{D}_f = 75 \times 0.015$$
$$= 1.125 \text{ in.-lb } (0.127 \text{ N-m})$$

The displacement for one revolution in the direction of the cutting force can be written in terms of the circumference as

$$\mathbf{D}_c = \pi D = 3.14 \times 2 = 6.28 \text{ in. } (159.5 \text{ mm})$$

The work performed per revolution by the cutting force can now be written as

$$W_c = \mathbf{F}_c \cdot \mathbf{D}_c = 210 \times 6.28$$
$$= 1318.8 \text{ in.-lb } (148.8 \text{ N-m})$$

A comparison of the magnitude of the work done by the component forces in example 4.19 indicates that practically all of the work was performed by the cutting force as a result of the affiliated large displacement in the direction of the cutting force. The percentage of work done by the feed force constitutes a very small portion of the total work performed. As a result, for practical purposes, the work performed by the feed force can be neglected. Consequently, the conclusion can be reached that the work done by the cutting force provides an accurate appraisal of the energy required for the cutting process. The following example delineates the percentage of energy used by each of the component forces in example 4.19.

EXAMPLE 4.20. Calculate the percentage of work performed by each of the three components of the resultant force in example 4.19.

Percentage of total work performed can be written as

$$\%\ W_{\text{comp}} = \frac{W_{\text{comp}}}{W_{\text{total}}} \times 100$$

where W_{comp} is the work performed by component force and W_{total} is the total work performed. Solving for the percentage of total work performed by the radial force yields

$$\%\ W_r = \frac{W_r}{W_c + W_f + W_r} \times 100 = \frac{0}{1318.8 + 1.125 + 0} \times 100$$
$$= 0$$

The percentage of total work performed by the feed force is written as

$$\%\ W_f = \frac{W_f}{W_c + W_f + W_r} \times 100 = \frac{1.125}{1319.93} \times 100$$
$$= 0.085\%$$

The percentage of total work performed by the cutting force is written as

$$\%\ W_c = \frac{W_c}{W_c + W_f + W_r} \times 100 = \frac{1318.8}{1319.925} \times 100$$
$$= 99.915\%$$

As can be seen from the results of example 4.20, 99.915% of the energy used in the cutting process described was expended by the cutting force, 0.085% was expended by the feed force, and 0.0% was expended by the radial force. From this analysis, the conclusion can be reached that for practical calculations, the energy expended to feed the tool into the workpiece and the energy expended by the radial force can be neglected.

The energy expended in the metal cutting process by multiple-edge cutting tools such as drills, end mills, milling cutters, hobs, and reamers can be analyzed in a fashion similar to that used for a single-edge turning tool. The key to the analysis is based on the reduction

of the logical scheme to elementary fundamental principles. Figure 4.35 illustrates the resultant cutting force acting on one of the two cutting edges of a standard drill. Although in reality the cutting force is distributed over the length of the cutting edge, it can be represented in terms of a single concentrated force (**R**) acting at a distance (r_c) from the centerline of the drill. In Fig. 4.35, the resultant force is resolved into three convenient components, which are the cutting force ($\mathbf{F}_c$), the feed force ($\mathbf{F}_f$), and the radial force ($\mathbf{F}_r$). These forces are similar to those shown in Fig. 4.33 for the turning operation.

In analyzing the work performed by the radial component ($\mathbf{F}_r$) for the drilling operation, it can be concluded that this force travels in a circular path and that there is no displacement in the direction of this force, and, as a result, it does no work. The feed force ($\mathbf{F}_f$) in one revolution of the drill moves down a distance equivalent to the feed per revolution. The work it performs in one revolution is equal to the product of the feed force multiplied by the feed per revolution displacement. In analyzing the work performed by the cutting force ($\mathbf{F}_c$) in one revolution of the drill, the distance traveled by the cutting force

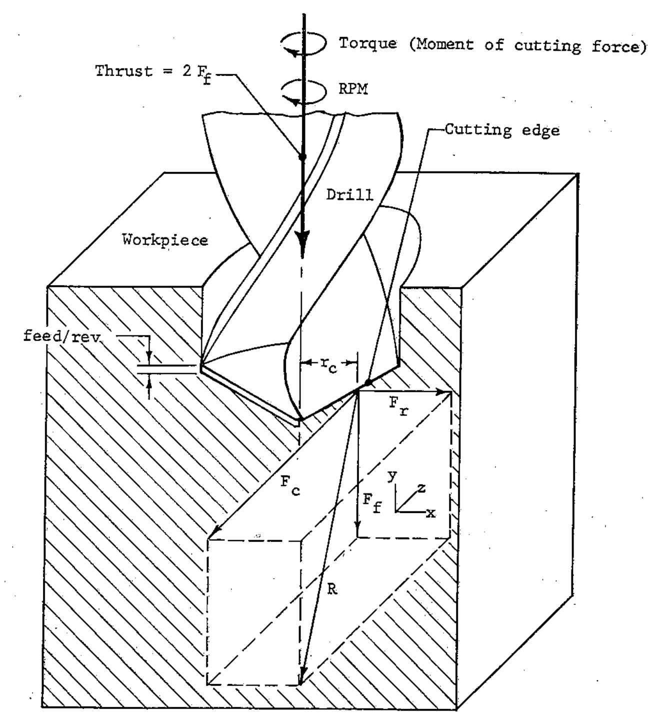


Fig. 4.35. Components of resultant force acting on cutting edge of drill.

must be taken into account. This displacement can be written as the circumference of the circle inscribed by the cutting force, or

$$\mathbf{D}_c = 2\pi r_c \tag{4.59}$$

The work of the cutting force for one revolution can now be written as

$$W_c = \mathbf{F}_c \cdot \mathbf{D}_c$$

or

$$W_c = \mathbf{F}_c(2\pi r_c) \tag{4.60}$$

The point described by r_c, at which the cutting force is concentrated, is an unknown quantity. Therefore, for the sake of convenience, it is more expedient to write Eq. 4.60 as

$$W_c = T(2\pi) \tag{4.61}$$

where $T = \mathbf{F}_c \times r_c$, which represents (1) the moment of the cutting force acting on the cutting edge and (2) the torque acting on drill; and 2π is the angle in radians through which the torque acts for one revolution of the drill.

As can be seen in the description of Eq. 4.61, the moment of the cutting force is equal to the product of the cutting force multiplied by the perpendicular distance extended from the cutting force to the center of rotation of the drill. In the example shown in Fig. 4.35, this distance is equal to r_c. It is noted that the moment of the cutting force is also referred to as the *torque*.

A more general expression of Eq. 4.61 is in the form

$$W = T \times \theta \tag{4.62}$$

where W is the work, T is the torque, and θ is the angle (in radians) through which the torque acts. The unit for work in Eq. 4.62 is the inch-pound or the foot-pound in the U.S. customary units and is the newton-meter in S.I. units. The unit of work, the newton-meter (N-m), is called a *joule*. It is noted that the torque has the same units as the work. The units are balanced on both sides of Eq. 4.62 because the unit for the angular displacement is given in radians, which, for this application, is not considered in the unit balance.

Figure 4.36 illustrates the measure of one radian, which, by definition, is a measure of a plane angle with its vertex at the center of a circle and subtended by an arc equal in length to the radius. Since the ratio between the radius and circumference of a circle is 2π, it can be stated that there are 2π radians in a circle. In degrees, one radian is equal to

$$1\ \text{rad} = \frac{360^\circ\ \text{(degrees per circle)}}{2\pi\text{(radians per circle)}} = 57.3^\circ$$

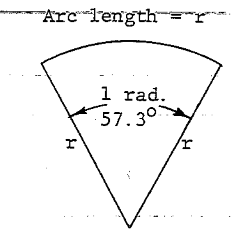


Fig. 4.36. The measure of one radian.

Equations for determining the torque and thrust of a standard drill, that is, one with a chisel length-to-diameter ratio of 0.18, have been derived by Shaw and Oxford.[2] The experiments that were conducted used SAE 3245 steel of 196–207 Bhn, with no cutting fluid, running at 20 ft/min. The equations are of the form

$$T = 123.54\ \text{Bhn}\ (f^{0.8} d^{1.8}) \tag{4.63}$$

and

$$T_{st} = 276.9\ \text{Bhn}\ (f^{0.8} d^{0.8}) + 3.124\ \text{Bhn}\ (d^2) \tag{4.64}$$

where T is the torque (in.-lb), T_{st} is the thrust (lb), Bhn stands for Brinell hardness number, f is the feed (in./rev), and d is the drill diameter (in.). An application of Eqs. 4.61 and 4.62 is illustrated in the following example.

EXAMPLE 4.21. Determine the torque and thrust for a $\frac{1}{2}$-in.-diameter drilling operation. The work material is steel with a Bhn of 200. The feed is set at 0.0097 in./rev and the cutting speed is 20 ft/min.

Solving for the torque by substituting into Eq. 4.63 yields

$$T = 123.54(200)(0.0097)^{0.8}(0.5)^{1.8}$$
$$= 173.7\ \text{in.-lb (19.6 N-m)}$$

Solving for the thrust by substituting into Eq. 4.64 yields

$$T_{st} = 276.9(200)(0.0097)^{0.8}(0.5)^{0.8} + 3.124(200)(0.5)^2$$
$$= 935\ \text{lb (4159 N)}$$

With the availability of a means of determining the torque and the thrust, an analysis can now be made as to the distribution of energy between the cutting force and the feed force. It is noted that the cutting force is affiliated with the torque, whereas the feed force is affiliated with the thrust. The following example illustrates the analysis.

[2] M. C. Shaw, and C. J. Oxford, Jr., "On the Drilling of Metals 2—The Torque and Thrust in Drilling," *Trans. ASME*, Vol. 79, 1957, p. 147.

EXAMPLE 4.22. It is desired to find the amount of work (energy) performed in one revolution for the torque and the thrust for the drilling operation illustrated in example 4.21.

For one revolution, the torque acts through an angle of 360° or 2π rad. The thrust acts through a displacement equivalent to the feed, in this case 0.0097 in. in one revolution. From Eq. 4.62 the work performed by the torque acting through one revolution can be expressed as

$$W_c = T \times \theta$$

Substituting yields

$$W_c = 173.7 \times 6.28$$
$$= 1090.9 \text{ in.-lb } (123.06 \text{ N-m})$$

With work being defined as the product of the force multiplied by the distance through which the force acts, the work performed by the thrust force per revolution of the drill can be written as

$$W_f = T_{st} \times f$$
$$= 935 \times 0.0097$$
$$= 9.07 \text{ in.-lb } (1.023 \text{ N-m})$$

As can be seen from the results of example 4.22, most of the energy of the drilling operation is expended by the cutting force. Similar to the turning example, less than 1% of the energy used to drill the parts is expended by the feed force, whereas more than 99% of the energy is used by the cutting force.

A point of interest would be a technique for extracting the magnitude of the cutting force acting on each of the cutting edges of the drill in examples 4.21 and 4.22. In order to do this, an assumption as to the value of r_c in Fig. 4.35 is necessary. If r_c is assumed to be equal to one-quarter of the diameter of the drill, then Fig. 4.37 represents the couple (two parallel forces of equal magnitude which act in opposite directions and are not collinear) that is generated by the cutting force acting on each cutting edge of the drill.

A couple consists of two forces having the same magnitude, parallel lines of action, and an opposite sense. It should be obvious that the summation of the forces of a couple is equal to zero. However, it is noted that the summation of the moments (force × distance) of a couple about any point is not equal to zero. Rather, the moment is equal to one of the forces of the couple multiplied by the perpendicular distance between the forces. As a result, it can be stated that the influence of

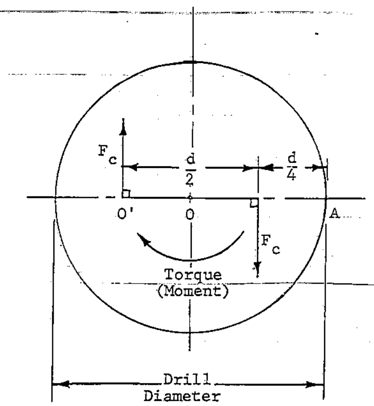


Fig. 4.37. Torque-generating couple exerted by drill on work material, with cutting force assumed to be concentrated at distance $r_c = \frac{1}{4}d$ from centerline.

a couple on a body is not zero. The two forces of a couple cause a torque (moment) to act on the body, providing a turning action.

An examination of Fig. 4.37 reveals that a summation of moments about point O is equal to

$$\circlearrowleft + \sum M_O = (\mathbf{F}_c \times \tfrac{1}{4}d) + (\mathbf{F}_c \times \tfrac{1}{4}d)$$
$$\circlearrowleft + \sum M_O = \mathbf{F}_c \times \tfrac{1}{2}d$$

Of interest is that a summation of moments about point 0′ or any other point (A) will yield the same results for a couple. To illustrate from Fig. 4.37,

$$\circlearrowleft + \sum M_{O'} = \mathbf{F}_c \times \tfrac{1}{2}d$$
$$\circlearrowleft + \sum M_A = (-\mathbf{F}_c \times \tfrac{1}{4}d) + (\mathbf{F}_c \times \tfrac{3}{4}d)$$
$$\circlearrowleft + \sum M_A = \mathbf{F}_c \times \tfrac{1}{2}d$$

EXAMPLE 4.23. With the assumption that the cutting force ($\mathbf{F}_c$) acting on a cutting edge of a drill is concentrated at a distance of $\frac{1}{4}d$ from the center of the drill, that is, $r_c = \frac{1}{4}d$, where d is the drill diameter, determine the cutting force from the data given in example 4.21.

Taking into account the moment of the cutting force couple yields

$$T = \mathbf{F}_c \times \tfrac{1}{2}d$$

$$\mathbf{F}_c = \frac{2T}{d} = \frac{2 \times 173.7}{0.5}$$

$$= 694.8 \text{ lb } (3090.6 \text{ N})$$

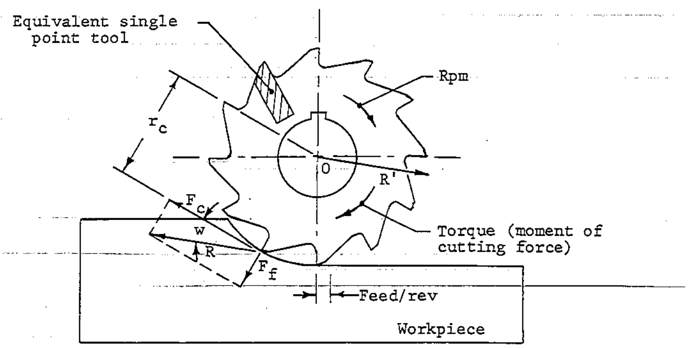


Fig. 4.38. Components of resultant force acting on cutting edge of milling cutter.

Figure 4.38 illustrates the cutting force exerted by a milling cutter on a workpiece. It is noted that the torque generated by the cutter is equal to the cutting force multiplied by the radius of the cutter. By taking a summation of moments about point O, it can be written that

$$\circlearrowleft + \sum M_O = T = \mathbf{F}_c \times r_c \tag{4.65}$$

The resultant force (**R**), the vector summation of the cutting force ($\mathbf{F}_c$) and the feed force ($\mathbf{F}_f$), is also shown in Fig. 4.38. The reaction to the resultant force (**R**) is labeled (**R**′). This is the force that the cutter exerts on the arbor to which it is attached. Note that **R** and **R**′ constitute a couple, and the moment of this couple is the same as that expressed in Eq. 4.65. It can be written as

$$T = \mathbf{R} \times r_c \cos w \tag{4.66}$$

where

$$\mathbf{R} \cos w = \mathbf{F}_c$$

4.11 POWER CONSUMPTION

Power is a convenient term that is used to measure the energy required to perform a metal cutting operation during a specific time period. By definition, power is equal to the energy (work) consumed per unit time or the rate at which work is performed. It can be

expressed as

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

$$P = \frac{W_c}{t} \tag{4.67}$$

Since work is equal to the product of a force multiplied by the displacement in the direction of the force, Eq. 4.67 can be expressed as

$$P = \mathbf{F}_c \cdot \mathbf{V}_c \tag{4.67a}$$

where $\mathbf{F}_c$ is the cutting force; and $\mathbf{V}_c = \mathbf{D}_c/t$, which represents velocity at cutting force or cutting speed. The unit for power in U.S. customary units is foot-pound per second, whereas in SI units it is newton-meter per second or joules per second.

A measure of power consumption in a metal cutting operation is important. Machine capacities, as an example, are determined in terms of how much energy per unit time can be delivered. If a high rate of cutting is desired, then a machine with the capacity to deliver the energy must be used. If the required power to perform a given operation exceeds the capacity of a machine tool, then obviously the machine will stall. The result is that machining rates can be limited if the capacity of the machine is below that demanded by the cutting process. Under these conditions, adjustments in machine settings must be made to match the machine capacity with the energy demands of the cutting process.

Figure 4.39 depicts the cutting velocity and cutting force acting in a turning operation. With the availability of these two values, power for an operation can be evaluated. The following example shows the method.

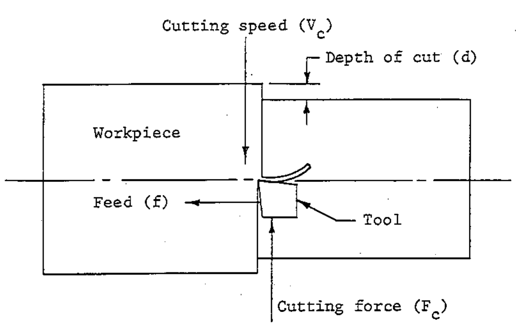


Fig. 4.39. Turning operation.

EXAMPLE 4.24. Calculate the power in foot-pounds per second (newton-meters per second) for a turning operation with a plain carbon steel workpiece that has a Brinell hardness number of 200 and a cutting velocity of 480 ft/min (146.3 m/min). A carbide tool is being used and machine settings are 0.020 in./rev (0.508 mm/rev) for the feed and 0.150 in. (3.81 mm) for the depth of cut. A measurement of the cutting force is found to be 1306 lb (5809.4 N).

Substituting numerical values into Eq. 4.67a enables the energy consumption per unit time to be calculated:

$$P = \mathbf{F}_c \cdot \mathbf{V}_c = 1306 \times 480$$
$$= 626{,}880 \text{ ft-lb/min} = 10{,}448 \text{ ft-lb/sec } (14{,}165.6 \text{ N-m/sec})$$

The units of power in example 4.24, foot-pounds per second (newton-meters per second), are affiliated with the definition of power in terms of the time rate at which work is being done. These units are obtained by dividing units of work by the unit of time. In U.S. customary units, the unit of power is known as *horsepower*, which is defined as

$$1 \text{ hp} = 550 \text{ ft-lb/sec}$$

In SI units, another measure of power is the *watt*, which is defined as

$$1 \text{ watt (1 W)} = 1 \text{ joule/sec} = 1 \text{ newton-meter/sec}$$

The relationship between watts and horsepower are given as

$$1 \text{ hp} = 746 \text{ W}$$

Using the conversions for horsepower and watts, the result of example 4.24 can now be listed as

$$P = 10{,}448 \frac{\text{ft-lb}}{\text{sec}} \times \frac{1 \text{ hp-sec}}{550 \text{ ft-lb}}$$
$$= 19 \text{ hp } (14{,}171 \text{ W})$$
$$= 14.17 \text{ kW}$$

where 1 kW = 1000 W.

For a rotating tool, it is convenient to write the horsepower in terms of torque and rpm. A direct expression using these terms is

$$\text{hp} = \frac{T \times \text{rpm}}{63{,}025} \tag{4.68}$$

where T is the torque (in.-lb) and rpm stands for revolutions per minute, and 63,025 in.-lb-rpm/hp is the units conversion factor. Figure 4.40

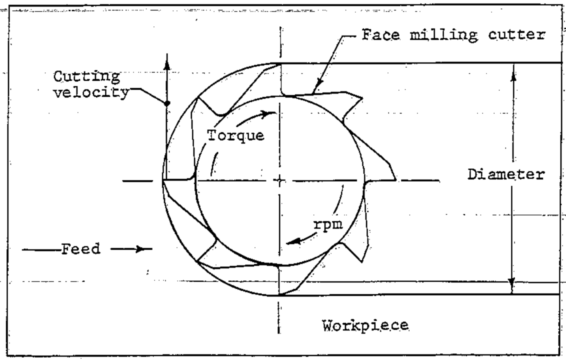


Fig. 4.40. Top view of face milling operation.

shows the torque and rpm acting on a face milling cutter. The following examples list basic calculations involving milling cutters.

EXAMPLE 4.25. A 2-in.(50.8-mm)-diameter end mill with six cutting edges (teeth) is programmed to rotate at 100 rpm. Calculate the cutting velocity and the feed per tooth for a workpiece feed of 1.75 in./min (44.45 mm/min).

From Eq. 2.1, substituting numerical values yields

$$\mathbf{V}_c = \frac{\text{rpm} \times \pi \times D}{12}$$
$$= \frac{100 \times 3.14 \times 2}{12}$$
$$= 52.3 \text{ ft/min } (15.95 \text{ m/min})$$

The advance of the end mill per revolution can be written as

$$\text{feed/rev} = \frac{\text{feed/min}}{\text{rev/min}} = \frac{1.75}{100}$$
$$= 0.0175 \text{ in./rev } (0.4445 \text{ mm/min})$$

Taking into account the six cutting edges yields

$$\text{feed/tooth} = \frac{\text{feed}}{\text{rev}} \times \frac{\text{rev}}{\text{cutting edges}}$$
$$= \frac{0.0175}{6}$$
$$= 0.0029 \text{ in. } (0.0737 \text{ mm})$$

EXAMPLE 4.26. The milling operation described in example 4.25 is set for a depth of cut of 0.500 in. (12.7 mm), and a torque measurement indicates a value of 100 lb-ft (135.6 N-m). The rpm is 100 and the work feed is 1.75 in./min (44.45 mm/min). From these data, calculate the horsepower required for the operation.

Substituting into Eq. 4.66 yields

$$\text{hp} = \frac{T \times \text{rpm}}{63{,}025} = \frac{100 \times 12 \times 100}{63{,}025}$$

$$= 1.9 \text{ horsepower } (1.42 \text{ kW})$$

Another way of expressing the horsepower required for a metal cutting operation is to write the equation in the form

$$\text{hp} = \frac{\mathbf{F}_c \cdot \mathbf{V}_c}{33{,}000} \tag{4.69}$$

where $\mathbf{F}_c$ is the cutting force (lb) and $\mathbf{V}_c$ is the cutting velocity (ft/min). Equation 4.69 can be derived directly from Eq. 4.68 by writing that the torque is equal to the average cutting force multiplied by the radius of the milling cutter. Substituting into Eq. 4.68 yields

$$\text{hp} = \frac{\mathbf{F}_c \times \frac{1}{2}D \times \text{rpm}}{63{,}025}$$

where

$$\mathbf{V}_c = \text{circumference} \times \text{rpm}$$

$$= \frac{\pi \times 2 \times \frac{1}{2}D \times \text{rpm}}{12}$$

and

$$\frac{D}{2} \times \text{rpm} = \frac{12\mathbf{V}_c}{2\pi}$$

Therefore

$$\text{hp} = \frac{\mathbf{F}_c(12\mathbf{V}_c/2\pi)}{63{,}025}$$

or

$$\text{hp} = \frac{\mathbf{F}_c \cdot \mathbf{V}_c}{33{,}000} \tag{4.69}$$

Equation 4.69 can be used for any metal cutting operation where the cutting force and cutting speed are known.

EXAMPLE 4.27. Confirm the results of example 4.26 by using Eq. 4.69.

The cutting force that generates the torque can be written as

$$\mathbf{F}_c = \frac{T}{D/2} = \frac{100 \times 12}{1}$$

$$= 1200 \text{ lb } (5337.9 \text{ N})$$

In example 4.25 the cutting velocity was determined to be equal to 52.3 ft/min (15.95 m/min). Substituting into Eq. 4.69 yields

$$\text{hp} = \frac{\mathbf{F}_c \cdot \mathbf{V}_c}{33{,}000} = \frac{1200 \times 52.3}{33{,}000}$$

$$= 1.9 \text{ horsepower } (1.42 \text{ kW})$$

The results of the turning example (example 4.24) can also be easily confirmed by Eq. 4.69, where it is noted that the constant 33,000 can be defined as

$$\frac{33{,}000 \text{ lb-ft}}{\text{min-hp}} = \frac{550 \text{ ft-lb}}{\text{sec-hp}} \times \frac{60 \text{ sec}}{\text{min}}$$

When accessibility to measurement of forces or torque is not available for the metal cutting operation, a technique by which energy and power requirements can be estimated is through the use of the term unit horsepower (unit power). This term provides for a measurement of the amount of power required to machine a metal at a specific volumetric rate. In U.S. customary units it is expressed as

$$U_p = \text{Unit horsepower (hp/in.}^3\text{/min)}$$

In SI units it is expressed as

$$U_p = \text{Unit power (kW/cm}^3\text{/sec)}$$

Since

$$1 \text{ hp} = 0.746 \text{ kW}$$

$$1 \text{ in.} = 2.54 \text{ cm}$$

$$1 \text{ in.}^3 = 16.387 \text{ cm}^3$$

and

$$1 \text{ min} = 60 \text{ sec}$$

then

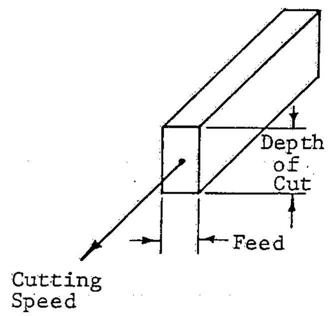
$$1 \text{ hp/in.}^3\text{/min} = 0.746 \text{ kW}/16.387 \text{ cm}^3/60 \text{ sec}$$

$$1 \text{ hp/in.}^3\text{/min} = 2.73 \text{ kW/cm}^3\text{/sec}$$

Unit horsepower for various materials are found in reference sources and can be determined experimentally. As an example, if a wattmeter is attached to a machine tool and a measurement of the power demanded by the cutting operation is made, and the volumetric rate of metal removal is evaluated, then the unit power can be calculated. The following example illustrates the technique.

EXAMPLE 4.28. A wattmeter reading indicates that 14.174 kW are drawn by the required cut of a turning operation. The cutting speed is 480 ft/min (146.3 m/min), the feed is 0.020 in./rev (0.508 mm/rev), and the depth of cut is 0.150 in. (3.81 mm). From these data, calculate the unit horsepower (unit power), that is, the amount of power required to remove material at a specific rate.

A graphical description of the volumetric rate of machining is shown below.



From Eq. 2.2 it can be written that

$$R_v = 12\mathbf{V}_c \times f \times d$$
$$= 12 \times 480 \times 0.02 \times 0.15$$
$$= 17.28 \text{ in.}^3/\text{min}$$

The power required for this operation is

$$P = 14.174 \text{ kW}$$

or

$$\text{horsepower} = 19 \text{ hp}$$

Since the unit horsepower can be written as

$$U_p = \text{hp/in.}^3/\text{min} = \text{hp}_r/R_v$$

where hp_r is the required horsepower and R_v is the volumetric rate of machining, then by substituting

$$U_p = \frac{19}{17.28} = 1.1 \text{ hp/in.}^3/\text{min } (3.003 \text{ kW/cm}^3/\text{sec})$$

For example 4.28, in order to remove the material at a rate of 1 in.^{3}/min, 1.1 hp was required. The amount of energy (work) consumed to remove 1 in.^{3} (16.4 cm^3) can be calculated by substituting for horsepower and then multiplying by unity for unit balance in the unit horsepower equation. To illustrate:

$$U_p = \frac{1.1 \text{ hp-min}}{\text{in.}^3}$$
$$= \frac{1.1 \text{ hp} \times 550 \text{ ft-lb}}{\text{sec-hp}} \times \frac{\text{min}}{\text{in.}^3} \times \frac{60 \text{ sec}}{1 \text{ min}} \times \frac{12 \text{ in.}}{\text{ft}}$$
$$= \frac{435{,}000 \text{ in.-lb}}{\text{in.}^3}$$

As can be seen, the operation described requires 435,600 in-lb (44,676 joules) of energy to remove 1 in.^{3} (16,387 cm^3) of material.

Values of unit horsepower (unit power) vary with different materials and also vary with the hardness of a given material. The condition of the cutting tool can also have an influence. A dull tool ordinarily would require more energy to remove a specific amount of material than a sharp tool. In addition, the shape of the tool can also have an influence. A high side-rake-angle tool produces, on average, a lower chip thickness than a low-rake-angle tool. Consequently, this leads to a lower unit horsepower used in machining the same material. This is a result of less energy being absorbed in forming the thinner chip than in forming the thicker chip. The same argument can be extended to the sharp tool, which provides a cleaner chip and requires less energy than a tool that is dull. Figure 4.41 illustrates the wide distribution of

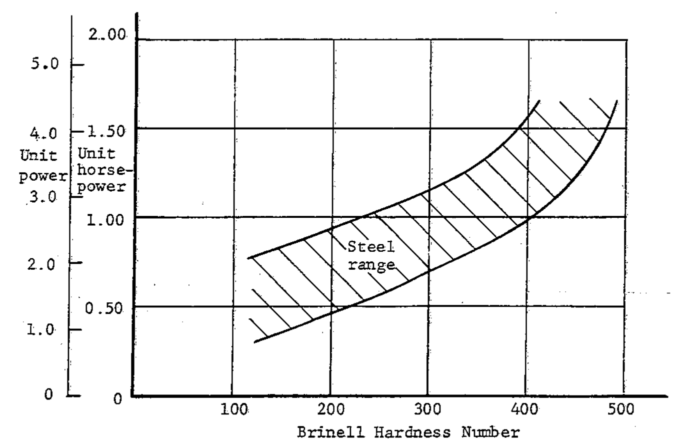


Fig. 4.41. Range of unit horsepower as a function of hardness for steel.

Table 4.5. Sample Values of Unit Horsepower

Material	Hardness (Bhn)	Unit Horsepower (hp/in.^{3}/min)	Unit Power (kW/cm^3/sec)
Steel	125	0.62	1.69
	180	0.75	2.05
	260	0.90	2.46
	430	1.50	4.10
Cast iron	150	0.4	1.09
	250	0.9	2.46
	270	1.2	3.28
Leaded brass	35	0.23	0.63
	75	0.26	0.71
	130	0.30	0.82
Copper	40	0.90	2.46
Aluminum	35	0.14	0.38
	90	0.16	0.44
	120	0.20	0.55
	150	0.24	0.66

unit horsepower (unit power) as a function of hardness for steel. The wide range for a given hardness reflects the effects of the constituents, condition, and heat treatment of the steel. Values at the lower portion of the range curve are for the free-cutting steel varieties, whereas the values at the higher portion of the range curve are for the high-carbon and alloy-steel varieties. Sample values of unit horsepower (unit power) for a variety of materials are shown in Table 4.5.

When using the unit horsepower (unit power) as a reference for calculations, the evaluation of the volumetric rate of metal removal becomes important. For the milling operation, the volumetric rate of metal removal can be written as

$$R_{vm} = f \times d \times w \tag{4.70}$$

where f is the feed (in./min), d is the depth of cut (in.), and w is the width of cut (in.). Figure 4.42 is a graphical representation of a volumetric rate of metal removal for a plain milling operation. A similar representation for the vertical milling case is shown in Fig. 4.43. In both cases, the volumetric rate of metal removal is symbolized by a right rectangular prismatic block of dimensions feed, depth of cut, and width of cut.

EXAMPLE 4.29. A 0.750-in.(19.05-mm)-diameter end mill is set to cut a 0.375-in.(9.525-mm)-deep slot in a low-carbon-steel workpiece that has a Brinell hardness number of 125. The high-speed-steel end mill is set

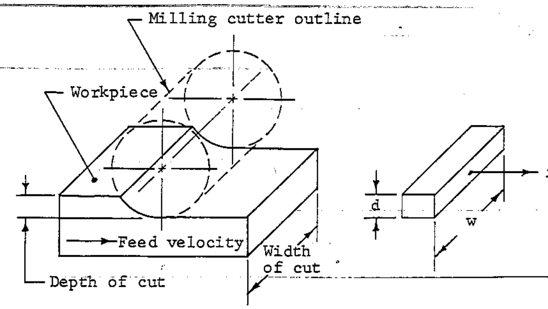


Fig. 4.42. Graphical representation of volumetric rate of metal removal for plain milling.

to run at 60 ft/min (18.29 m/min) with a recommended feed of 0.002 in. (0.0508 mm) per tooth. The end mill has four teeth (cutting edges).

From these data, calculate:

(a) The feed set on the machine tool.
(b) The volumetric rate of metal removal.
(c) The torque acting on the end mill by using a unit horsepower value from Table 4.5.
(d) The average cutting force acting on the end mill.

The feed per revolution of the end mill is

$$f_{rev} = 4 \times f_{tooth} = 0.008 \text{ in. } (0.203 \text{ mm})$$

The feed setting on the machine can be written as

$$f = f_{rev} \times \text{rpm}$$

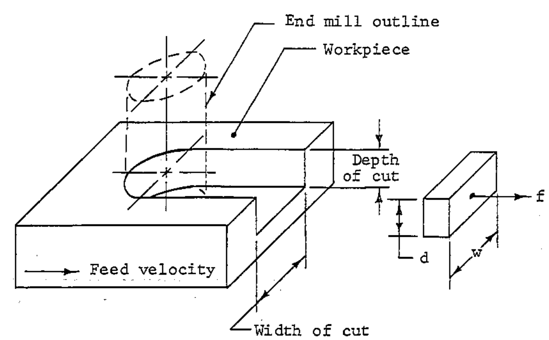


Fig. 4.43. Graphical representation of volumetric rate of metal removal for vertical milling cutter.

where

$$\text{rpm} = \frac{12\mathbf{V}_c}{\pi D} = \frac{12 \times 60}{3.14 \times 0.75} = 305.7$$

Thus,

$$f = 0.008 \times 305.7$$
$$= 2.45 \text{ in./min } (62.23 \text{ mm/min})$$

The volumetric rate of metal removal can be evaluated by substitution in Eq. 4.70:

$$R_v = f \times d \times w = 2.45 \times 0.375 \times 0.75$$
$$= 0.689 \text{ in.}^3\text{/min } (0.188 \text{ cm}^3\text{/sec})$$

From Table 4.5, for steel with a Bhn of 125 we have

$$U_p = 0.62 \text{ hp/in.}^3\text{/min}$$

Solving for the horsepower consumed in the operation, we obtain

$$\text{hp} = U_p \times R_v = 0.62 \times 0.689$$
$$= 0.427 \text{ horsepower } (0.319 \text{ kW})$$

From Eq. 4.68,

$$T = \frac{\text{hp} \times 63{,}025}{\text{rpm}} = \frac{0.427 \times 63{,}025}{305.7}$$
$$= 88 \text{ lb-in. } (9.9 \text{ N-m})$$

The average cutting force can be written as

$$\mathbf{F}_c = \frac{T}{D/2} = \frac{88}{0.75/2} = 235 \text{ lb } (1044 \text{ N})$$

Of interest is that the average cutting force can also be extracted from Eq. 4.69 as

$$\mathbf{F}_c = \frac{\text{hp} \times 33{,}000}{\mathbf{V}_c} = \frac{0.427 \times 33{,}000}{60}$$
$$= 235 \text{ lb } (1044 \text{ N})$$

Figure 4.44 illustrates the volumetric rate of metal removal for a drilling operation. The volumetric rate can be expressed as

$$R_{vd} = \text{Area of point cone} \times \text{Feed velocity}$$
$$= A_{pc} \times f$$

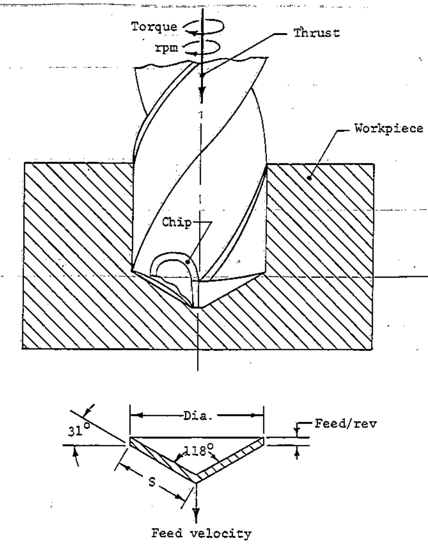


Fig. 4.44. Graphical representation of volumetric rate of metal removal for a drilling operation.

where

$$A_{pc} = \pi \frac{D}{2} \times S$$

$$S = \text{Slant length of point cone}$$
$$= \frac{\frac{1}{2}D}{\cos 31°} \text{ (for standard drill)}$$
$$= 0.5833 \times D \text{ (for standard drill)}$$

Substituting for the area of the point cone generated by a drill yields

$$R_{vd} = \pi \frac{D}{2} \times S \times f \qquad \textbf{(4.71)}$$

For a standard drill with a point angle of 118°,

$$S = S_{sd} = \frac{\frac{1}{2}D}{\cos 31°} = 0.5833 \times D$$

and Eq. 4.71 can now be written as

$$R_{vd} = 0.9165 \times D^2 \times f \qquad \textbf{(4.72)}$$

The following example demonstrates how the required torque for a drilling operation can be determined through the use of a reference unit horsepower (unit power) value.

EXAMPLE 4.30. A 0.500-in.(12.7-mm)-diameter high-speed standard drill is machining steel that has a Brinell hardness number of 260. The cutting speed is set at 40 ft/min (12.19 m/min) and the feed rate is 2 in./min (50.8 mm/min). Neglecting the effects of the chisel edge at the center of the drill point and using Table 4.5 as a reference, calculate the torque that is generated.

From Table 4.5,

$$U_p = 0.9 \text{ hp/in.}^3\text{/min } (2.46 \text{ kW/cm}^3\text{/sec})$$

The volumetric rate of metal removal can be determined from Eq. 4.72 as

$$R_{vd} = 0.9165D^2 \times f = 0.9165 \times (0.5)^2 \times 2$$
$$= 0.4583 \text{ in.}^3\text{/min}$$

Solving for the horsepower consumed in the operation, we obtain

$$\text{hp} = U_p \times R_{vd} = 0.9 \times 0.4583$$
$$= 0.4125 \text{ horsepower } (0.3075 \text{ kW})$$

The torque can now be determined by applying Eq. 4.68.

$$T = \frac{\text{hp} \times 63{,}025}{\text{rpm}}$$

where

$$\text{rpm} = \frac{12\mathbf{V}_c}{\pi \times D} = \frac{12 \times 40}{3.14 \times 0.5}$$
$$= 306 \text{ rpm}$$

Substituting yields

$$T = 84.95 \text{ lb-in. } (9.59 \text{ N-m})$$

The results of example 4.30 assume that the cutting edges of the drill act in a manner similar to that shown in Fig. 4.15. This is a close approximation when the effects of the chisel edge at the point of the drill are neglected. An example is the case where a pilot lead drill has been used and the drilling operation involves the enlargement of the hole. In this case, the chisel edge does no cutting.

However, in the ordinary case, the method of metal removal at the center of the drill is a complex one. It involves not only a cutting process but also an extrusion process. The influence of the action at the chisel edge can be significant. As a result, values of a simplified analysis as indicated in example 4.30 may need to be adjusted upward by a large factor (as large as 2 in some cases) to provide a more accurate appraisal.

For a turning operation similar to that shown in Fig. 4.39, the cutting force ($\mathbf{F}_c$) can be determined directly as a function of the feed (f), depth of cut (d), and the unit horsepower (unit power). This can be done by writing that

$$U_p = \frac{\text{hp}_c}{R_v} \tag{4.73}$$

where hp_c is the horsepower required at cutting tool,

$$\text{hp}_c = \frac{\mathbf{F}_c \cdot \mathbf{V}_c}{33{,}000}$$

and R_v is the volumetric rate,

$$R_v = 12f \times d \times \mathbf{V}_c \text{ (for turning)}$$

Substituting into Eq. 4.73 yields

$$U_p = \frac{(\mathbf{F}_c \cdot \mathbf{V}_c)/33{,}000}{12 \times f \times d \times \mathbf{V}_c}$$

or

$$U_p = \frac{\mathbf{F}_c}{396{,}000 \times f \times d}$$

Isolating the cutting force results in

$$\mathbf{F}_c = 396{,}000 \times f \times d \times U_p \tag{4.74}$$

As can be seen in Eq. 4.74, for a turning operation, if the feed, depth of cut, and unit horsepower are known, the cutting force can be directly evaluated. The constant 396,000 is for the U.S. customary unit system. The following example illustrates the application of Eq. 4.74.

EXAMPLE 4.31. It is desired to find the cutting force and power consumption for a turning operation. A medium-carbon-steel specimen with a Bhn of 180 is being machined by a C-6 carbide tool. The cutting velocity is set at 525 ft/min (160 m/min). The feed is 0.020 in./rev (0.508 mm/rev) and the depth of cut is 0.150 in. (3.81 mm). Reference sources reveal that the unit horse-power for the material is 0.75 hp/in.^{3}/min (2.048 kW/cm^3/sec).

Substituting numerical values into Eq. 4.74 yields

$$\mathbf{F}_c = 396{,}000 \times f \times d \times U_p$$
$$= 396{,}000(0.020 \times 0.150 \times 0.75)$$
$$= 891 \text{ lb } (3963 \text{ N})$$

The volumetric rate of machining for this operation is equal to

$$R_v = 12 \times f \times d \times \mathbf{V}_c = 12 \times 0.020 \times 0.150 \times 525$$
$$= 18.9 \text{ in.}^3/\text{min } (5.16 \text{ cm}^3/\text{sec})$$

The horsepower consumed by the cut can now be expressed as

$$\text{hp}_c = U_p \times R_v = 0.75 \times 18.9$$
$$= 14.175 \text{ horsepower } (10.57 \text{ kW})$$

4.12 MOTOR HORSEPOWER

In determining the power required to perform a specific metal cutting operation, two factors must be taken into consideration. The first of these is the efficiency of the power delivery system. The second is the amount of power necessary to drive a machine tool when no cutting is taking place, that is, when the machine is idling.

Efficiency is a measurement of the portion of energy received by a machine that is finally delivered to the cutting process in the form of useful work. It is expressed as a percentage. A machine that transforms into useful work 80% of the energy supplied to it is said to have an efficiency of 80%. In all machines that involve sliding motion, friction forces will do some work and the resultant output work will be smaller than the input work due to the frictional losses. A machine tool with a high efficiency has a large part of the energy supplied to it expended on the cutting process and only a small part wasted. Analytically, efficiency can be expressed as

$$\text{Eff} = \frac{\text{Output work}}{\text{Input work}} \tag{4.75}$$

Since power is the time rate of doing work expressed as

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

then Eq. 4.75 can be written as

$$\text{Eff} = \frac{\text{Output power}}{\text{Input power}} \tag{4.76}$$

The following example shows how the overall efficiency of a machine tool can be measured for a metal cutting operation.

EXAMPLE 4.32. It is desired to measure the overall efficiency of a machine tool for a turning operation. A wattmeter is set up to measure the input power and a dynamometer is used to measure the cutting force. The workpiece has a 2.5-in. (63.5-mm) diameter and the machine is set to run at 230 rpm. The depth of cut is set at 0.125 in. (3.175 mm) and the feed is 0.006 in./rev (0.076 m/rev). The cutting force is found to be equal to 450 lb (2002 N) and the wattmeter reading during cutting indicates that 1.798 kW are being fed into the motor.

The input horsepower is

$$\text{hp}_{\text{in}} = \frac{1.798}{0.746} = 2.41 \text{ hp}$$

The horsepower consumed at the cutting edge can be determined from Eq. 4.69:

$$\text{hp} = \frac{\mathbf{F}_c \cdot \mathbf{V}_c}{33{,}000}$$

where

$$\mathbf{V}_c = \frac{\pi \times D}{12} \times \text{rpm}$$
$$= 150.5 \text{ ft/min } (45.87 \text{ m/min})$$

Substituting yields

$$\text{hp} = \frac{450 \times 150.5}{33{,}000} = 2.052 \text{ horsepower}$$

Solving for efficiency yields

$$\text{Eff} = \frac{\text{Output horsepower}}{\text{Input horsepower}} = \frac{2.052}{2.41}$$
$$= 0.851 \text{ or } 85.1\%$$

The 15% loss of energy in example 4.32 can readily be justified if an examination of the efficiency of individual machine elements is considered. If it is assumed that the input energy must be transmitted through the following system with the accompanying efficiencies,

Motor	0.97
Belt drive	0.95
One set of bearings	0.96
Second set of bearings	0.96

then the system efficiency can be written as

$$\begin{aligned} \text{Eff} &= \text{Eff(motor)} \times \text{Eff(belt)} \times \text{Eff(bearing 1)} \times \text{Eff(bearing 2)} \\ &= 0.97 \times 0.95 \times 0.96 \times 0.96 \\ &= 0.849 = 84.9\% \end{aligned}$$

The second factor of consideration in determining the power required to perform a specific metal cutting operation on a machine tool is that of *tare horsepower*. This is the amount of horsepower that is required to overcome the machine's resistance to motion when the machine is running and cutting is not taking place. This is often referred to as the *idling horsepower*. The tare horsepower can be significant in cases where machine tools have gear trains with a large number of gears in contact. Bearings with high frictional forces can also increase tare horsepower as can machine elements such as belt drives. Contrasting high-tare-horsepower machine tools are those that possess anti-friction bearings, small numbers of gears in the gear train, and direct motor drives. These have high efficiencies and may possess relatively low tare-horsepower demands, and as a result have relatively high efficiencies.

A means of calculating the required horsepower for a motor on a machine tool is to take into consideration three factors that influence the motor horsepower. These can be expressed analytically as

$$\text{hp}_m = \frac{\text{hp}_c}{\text{Eff}} + \text{hp}_t \tag{4.77}$$

where hp_m is the motor horsepower, hp_c is the horsepower required at cut, hp_t is the tare horsepower, and Eff is the machine efficiency. The following examples give applications of Eq. 4.77.

EXAMPLE 4.33. A machine tool with a power efficiency of 80% and a tare horsepower of 0.56 hp is used for a turning operation. The material being cut is found to have a unit horsepower of 1.3. The cutting velocity is set at 175 ft/min (53.34 m/min), with a feed of 0.010 in./rev (0.254 mm/rev) and a depth of cut of 0.375 in. (9.525 mm). Determine the motor horsepower required for this operation.

The volumetric rate of metal removal is equal to

$$\begin{aligned} R_v &= 12 \times f \times d \times \mathbf{V}_c = 12 \times 0.010 \times 0.375 \times 175 \\ &= 7.875 \text{ in.}^3\text{/min} \end{aligned}$$

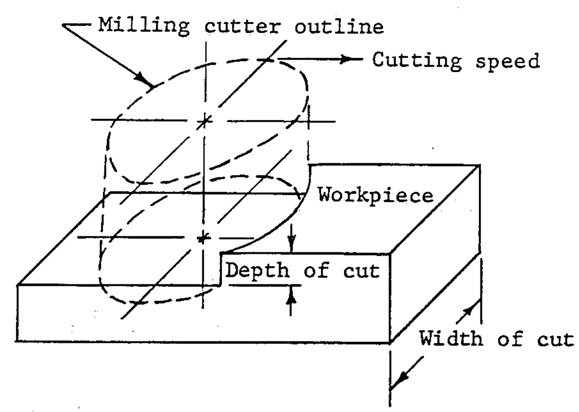
From the unit horsepower, the horsepower required at the cut is equal to

$$\begin{aligned} \text{hp}_c &= U_p \times R_v = 1.3 \times 7.875 \\ &= 10.24 \text{ horsepower} \end{aligned}$$

Substituting numerical values into Eq. 4.77 yields

$$\begin{aligned} \text{hp}_m &= \frac{\text{hp}_c}{\text{Eff}} + \text{hp}_t = \frac{10.24}{0.80} + 0.56 \\ &= 13.36 \text{ horsepower (9.966 kW)} \end{aligned}$$

EXAMPLE 4.34. The diagram shown below represents a 5-in.(127-mm)-diameter milling cutter machining a steel specimen that has a Bhn of 240 and a unit horsepower of 0.85. The milling cutter has eight teeth and the cutting velocity has been set at 200 ft/min (61 m/min). The depth of cut is 0.375 in. (9.525 mm) and the width of cut is 4 in. (101.5 mm). The feed rate is 0.004 in./tooth (0.106 mm/tooth). It is assumed that the milling takes place on a machine that has a power efficiency of 0.8 and a tare horsepower of 0.6.



From this information determine:

(a) The rpm of the cutter.
(b) The feed rate of the workpiece.
(c) The volumetric rate of metal removal.
(d) The horsepower required at the cut.
(e) The required motor horsepower.
(f) The torque generated by the cut.

The rpm of the cutter can be written as

$$\text{rpm} = \frac{12 \times V_c}{\pi \times D} = \frac{12 \times 200}{3.14 \times 5} = 152.9 \text{ rpm}$$

The feed rate of the workpiece is

$$\begin{aligned} f_m &= \text{feed/tooth} \times \text{number of teeth/rev} \times \text{rpm} \\ &= 0.004 \times 8 \times 152.9 \\ &= 4.89 \text{ in./min} \end{aligned}$$

The volumetric rate of metal removal is equal to

$$R_{vm} = f_m \times d \times w = 4.89 \times 0.375 \times 4$$
$$= 7.335 \text{ in.}^3/\text{min } (2.002 \text{ cm}^3/\text{sec})$$

Taking into account the unit horsepower, the horsepower required at the cut can be written as

$$\text{hp}_c = U_p \times R_v = 0.85 \times 7.335$$
$$= 6.23 \text{ horsepower } (4.651 \text{ kW})$$

The required motor horsepower can be expressed as

$$\text{hp}_m = \frac{\text{hp}_c}{\text{Eff}} + \text{hp}_t = \frac{6.23}{0.8} + 0.6$$
$$= 8.39 \text{ horsepower } (6.26 \text{ kW})$$

The torque exerted on the milling cutter can be extracted from Eq. 4.68.

$$T = \frac{\text{hp}_c \times 63{,}025}{\text{rpm}} = \frac{6.23 \times 63{,}025}{152.9}$$
$$= 2568 \text{ lb-in. } (289.7 \text{ N-m})$$

4.13 EFFECTIVE TOOL CLEARANCE ANGLES

The angle that appears on a tool as a clearance or relief angle is influenced in an actual cutting operation by the rate of feed. Figure 4.45 shows the effect of the lead angle with regard to reducing the

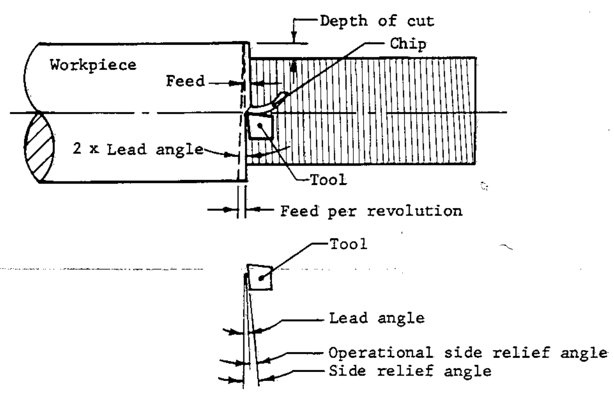


Fig. 4.45. Operational side relief angle.

side relief angle on a turning tool. An examination of Fig. 4.45 reveals that the lead angle also has an effect upon the so-called operational side relief angle. For this application, the operational side relief angle can be defined as the tool side relief angle minus the lead angle. This turns out to be the actual clearance angle between the tool and the workpiece during the cutting process.

Figure 4.46 shows the right triangle from which the lead angle can be extracted. The base of the triangle represents the lead, that is, the advance that the tool makes for each revolution of the workpiece. The lead is equivalent to the feed per revolution for a turning operation. The height of the triangle represents the circumference of the workpiece. The lead angle can be expressed as

$$\tan(\text{lead angle}) = \frac{\text{Lead}}{\text{Circumference}}$$

where lead = feed/rev = f, circumference = $\pi \times D$, and lead angle = L_a. As a result,

$$L_a = \tan^{-1}\left(\frac{f}{\pi \times D}\right) \tag{4.78}$$

The operational side relief angle can now be written as

$$O_a = S_a - L_a$$

where O_a is the operational relief angle, S_a is the side relief angle, and L_a is the lead angle, or

$$O_a = S_a - \tan^{-1}\left(\frac{f}{\pi \times D}\right) \tag{4.79}$$

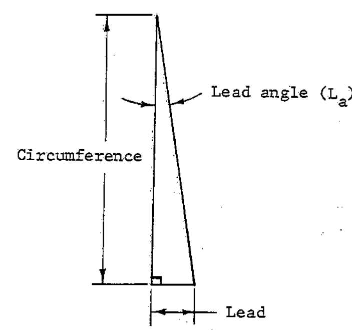


Fig. 4.46. Geometric model of lead angle.

The following example serves to illustrate how the lead angle reduces a side relief angle to an operational side relief angle.

EXAMPLE 4.35. A turning tool has ground into it a side relief angle of 6°. It is desired to determine the operational side relief angle for the machining of a 0.250-in. (6.35-mm) steel rod with a feed setting of 0.040 in./rev (1.016 mm/rev).

Substituting into Eq. 4.79 yields

$$O_a = 6° - \tan^{-1}\left(\frac{0.040}{3.14 \times 0.250}\right)$$
$$= 6° - 2.92°$$
$$= 3.08°$$

Of special interest in this case is the determination of the feed setting that reduces the operational side relief angle to zero. For this situation to exist, O_a must be equal to 0°. Under this circumstance, Eq. 4.79 can be written as

$$S_a = \tan^{-1}\left(\frac{f}{\pi \times D}\right)$$

from which

$$\tan S_a = \frac{f}{\pi \times D}$$

Isolating the feed yields

$$f = \pi \times D \times (\tan S_a) \qquad \textbf{(4.80)}$$

EXAMPLE 4.36. What is the value of the feed that will reduce the operational side relief angle to zero for the data in example 4.35?

Substituting into Eq. 4.80 yields

$$f = 3.14 \times 0.25 \times \tan 6°$$
$$= 0.0825 \text{ in./rev } (2.1 \text{ mm/rev})$$

As can be seen in example 4.36, any feed setting that is above 0.0825 in./rev (2.1 mm/rev) will result in a negative operational side relief angle, causing contact on the lower portion of the tool flank with no contact at the cutting edge. This means that rather than cutting with contact at the cutting edge, the turning tool will contact the workpiece at the lower portion of the tool flank, causing a rubbing action during the operation.

The diameter of the workpiece has a major influence on the operational side relief angle. This can be seen by examining Eq. 4.79. A

larger diameter with the same feed reduces the lead angle. Using the same feed and side relief angle as in example 4.35, but increasing the diameter from 0.25 in. (6.35 mm) to 2.00 in. (50.8 mm), reduces the lead angle to 0.364°. With the larger-diameter workpiece, the effect of the lead angle on reducing the operational side relief angle is minimal (0.364°), whereas with the smaller diameter it is significant, a 2.92° reduction.

The positioning of a turning tool relative to the center of a workpiece can have an influence on what may be considered the operational end relief angle. Figure 4.47 illustrates three examples of tool positioning relative to the horizontal centerline of the workpiece. In Fig. 4.47(a) the tool is set on center and the offset angle (ψ) is equal to zero. In this case, the operational end relief angle is the angle that has been ground into the tool. In Fig. 4.47(b), the offset angle (ψ) is set above the horizontal centerline, thus having an influence on reducing the end relief angle. Figure 4.47(c) shows the effect of setting the offset angle (ψ) below the horizontal centerline. In this case, the operational end relief angle is increased by the magnitude of the offset angle (ψ).

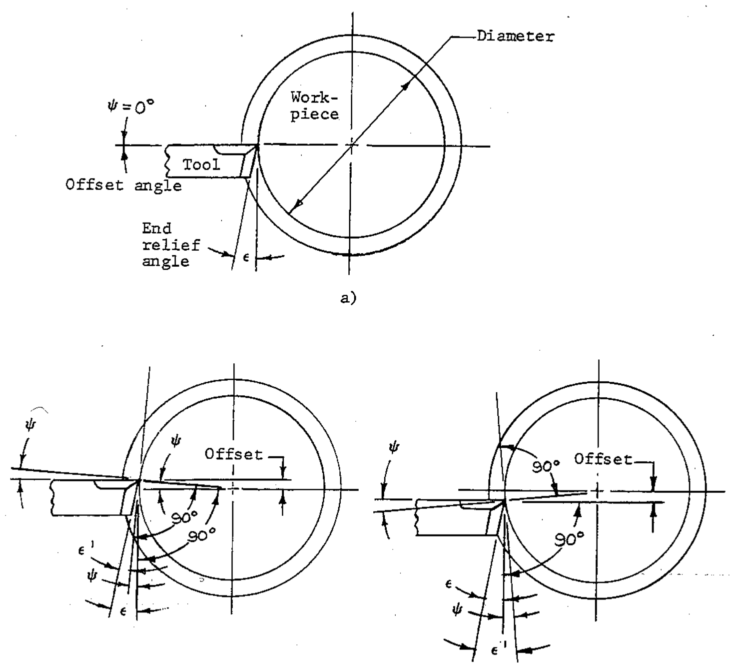


Fig. 4.47. Examples of effect of tool positioning on end relief angle.

Noting in Fig. 4.47 that the diameter represents the hypotenuse of the right triangle, the offset can be written as

$$\text{Off} = D \times \sin \psi \tag{4.81}$$

where Off is the vertical distance above or below the horizontal centerline, D is the diameter, and ψ is the offset angle. From Eq. 4.80, the offset angle can be isolated and expressed as

$$\psi = \sin^{-1}\left(\frac{\text{Off}}{D}\right) \tag{4.82}$$

An examination of Fig. 4.47(b) reveals that if the offset is above the horizontal centerline, then

$$\varepsilon' = \varepsilon - \psi \tag{4.83}$$

where ε' is the operational end relief angle, ε is the end relief angle, and ψ is the offset angle. For an offset below the horizontal centerline, the operational end relief angle is equal to

$$\varepsilon' = \varepsilon + \psi \tag{4.84}$$

To illustrate the importance of the offset position on the operational end relief angle, the following example is given.

EXAMPLE 4.37. It is desired to find the influence on the end relief angle of a 0.0625-in. (1.588-mm) turning tool offset, both above and below the horizontal centerline. The end relief angle ground into the tool is 7° and the diameter of the workpiece is 1 in. (25.4 mm).

The offset angle can be evaluated from Eq. 4.82 as

$$\psi = \sin^{-1}\left(\frac{\text{Off}}{D}\right) = \sin^{-1}\left(\frac{0.0625}{1.000}\right)$$
$$= 3.58°$$

With the offset above the horizontal centerline, the operational end relief angle is equal to

$$\varepsilon' = \varepsilon - \psi$$
$$= 7° - 3.58° = 3.42°$$

With the offset below the horizontal centerline, the operational end relief angle is equal to

$$\varepsilon' = \varepsilon + \psi = 7° + 3.58°$$
$$= 10.58°$$

As can be seen from the results of example 4.37, the offset has a significant influence on the operational end relief angle. Of interest also is that the offset influences the operational back rake angle. Figure 2.1 indicates tool angle designations. On a relative scale, as indicated in Fig. 4.47, the back rake angle is increased by the offset angle (ψ) if the offset (Off) is above the horizontal centerline and is decreased by the offset angle (ψ) if the offset (Off) is below the horizontal centerline.

There may be situations where the tool is set in a particular orientation, only to be offset by a deflection caused by the loading of the metal cutting process. A boring operation is a case of this type. Figure 4.48 illustrates the tool deflection on the boring bar caused by the cutting force. The deflection of the tool is designated as ΔY and is a response to the application of the cutting force ($\mathbf{F}_c$). If the tool was set at the horizontal centerline, then the deflection ΔY would be synonymous with the offset as discussed in example 4.37. In situations where the cutting force is high and the boring bar is slender, the deflection can be significant enough to appreciably alter the position of the tool. A means of evaluating the boring-bar deflection due to the action of the cutting force is through the following expression:

$$Y = \frac{\mathbf{F}_c \times L^3}{3 \times E \times I} \tag{4.85}$$

where $\mathbf{F}_c$ is the cutting force; E is the modulus of elasticity, which equals 30×10^6 psi (206.8×10^9 N/m^2) for steel; and I is the moment of inertia of cross-section area ($I = \pi \times D^4/64$ for a circular area). The following example illustrates the application of Eq. 4.85.

EXAMPLE 4.38. It is desired to determine the tool deflection on a boring bar caused by a cutting force loading. The steel boring bar has a 0.500-in. (12.7-mm) diameter and a length of 6 in. (152.4 mm). The cutting force has a value of 100 lb.

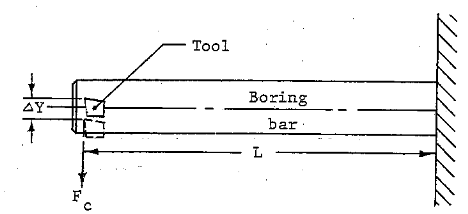


Fig. 4.48. Front view of boring bar.

Substituting into Eq. 4.85 yields

$$Y = \frac{F \times L^3}{3 \times E \times I} = \frac{100 \times 6^3 \times 64}{3 \times 30 \times 10^6 \times 3.14 \times (0.5)^4}$$

$$= 0.07827 \text{ in. } (1.988 \text{ mm})$$

As can be seen from the results of example 4.38, the deflection of a boring bar under a load can be significant.

4.14 THREE-DIMENSIONAL ANALYSIS OF RESULTANT CUTTING FORCE

Figure 4.49 illustrates the resolution of the resultant force that the tool exerts on the workpiece into three rectangular components, each parallel with its corresponding coordinate axis. The x component, labeled F_x, is the feed force and is a measure of the resistance the tool encounters during the cutting process in the direction of the feed. The y component, labeled F_y, is the cutting force and is a measure of the resistance the tool encounters in the direction of the cutting velocity. Lastly, the z component, labeled F_z, is the radial force and is a measure of the resistance the tool encounters in the direction of the radius of the workpiece.

Figure 4.50 shows the reactionary component forces to the forces that the tool exerts on the workpiece. Newton's third law of mechanics states that when two bodies exert forces on each other, these forces are equal in magnitude, opposite in direction, and act on the same line

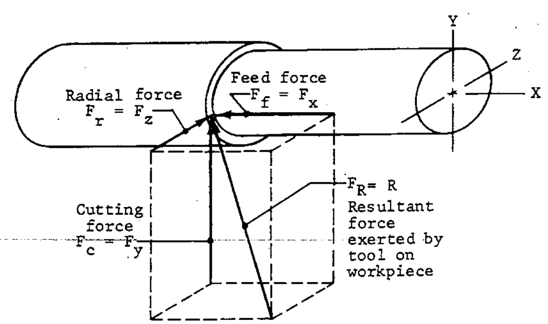


Fig. 4.49. Rectangular components of tool force acting on workpiece in turning operation.

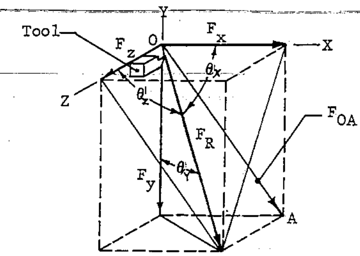


Fig. 4.50. Rectangular components of the workpiece forces acting on the tool.

of action. Figure 4.50 illustrates the component forces that the workpiece exerts on the cutting tool.

Of interest are the relationships of these component forces with the resultant cutting force. The magnitude of the rectangular components can be written as

$$F_x = F_R \cos\theta_x; \qquad F_y = F_R \cos\theta_y; \qquad F_z = F_R \cos\theta_z \tag{4.86}$$

By taking the ratio $1/F_R$ from each of the above expressions it can be written that

$$\frac{1}{F_R} = \frac{\cos\theta_x}{F_x} = \frac{\cos\theta_y}{F_y} = \frac{\cos\theta_z}{F_z} \tag{4.87}$$

The vectorial sum of F_x and F_y can be represented by a vector extending from O to A, as shown in Fig. 4.50. Its magnitude can be represented by applying the Pythagorean Theorem:

$$(\mathbf{F}_{OA})^2 = (F_x)^2 + (F_y)^2 \tag{4.88}$$

In turn, the resultant vectorial sum of $\mathbf{F}_{OA}$ and F_z must obviously be equal to F_R. Again, by applying the Pythagorean Theorem, it can be written that

$$(F_R)^2 = (\mathbf{F}_{OA})^2 + (F_z)^2$$

Substituting for $(\mathbf{F}_{OA})^2$ from Eq. 4.88 yields

$$(F_R)^2 = (F_x)^2 + (F_y)^2 + (F_z)^2$$

or

$$F_R = \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2} \tag{4.89}$$

By substituting values from Fig. 4.49, Eq. 4.89 can be written as

$$\mathbf{R} = \sqrt{(\mathbf{F}_f)^2 + (\mathbf{F}_c)^2 + (\mathbf{F}_r)^2} \tag{4.90}$$

By substituting in Eq. 4.89 the expressions for the component forces as listed in Eq. 4.87, the following identity emerges:

$$F_R = \sqrt{(F_R)^2 \cos^2\theta_x + (F_R)^2 \cos^2\theta_y + (F_R)^2 \cos^2\theta_z}$$

or

$$1 = \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z \tag{4.91}$$

The three angles θ_x, θ_y, and θ_z define the direction of the resultant cutting force F_R. The cosines of these angles are known as the *directional cosines of the cutting force*. The following example demonstrates the application of the three-dimensional resolution of the resultant cutting force.

EXAMPLE 4.39. Determine the magnitude of the resultant force (F_R) and the angular displacements of the resultant from the three space axes (x, y, z). Dynamometer measurements record values of the component forces as: $\mathbf{F}_f = 210$ lb (934 N); $\mathbf{F}_c = 320$ lb (1423 N); and $\mathbf{F}_r = 50$ lb (222 N).

For this application, substitution into Eq. 4.90 yields

$$\mathbf{R} = \sqrt{(\mathbf{F}_f)^2 + (\mathbf{F}_c)^2 + (\mathbf{F}_r)^2} = \sqrt{(320)^2 + (210)^2 + (50)^2}$$
$$= 386 \text{ lb } (1717 \text{ N})$$

Angular displacements can be evaluated from Eq. 4.86 as

$$\theta_x = \cos^{-1}\left(\frac{\mathbf{F}_f}{F_R}\right) = \cos^{-1}\left(\frac{210}{386}\right)$$
$$= 57°$$
$$\theta_y = \cos^{-1}\left(\frac{\mathbf{F}_c}{F_R}\right) = \cos^{-1}\left(\frac{320}{386}\right)$$
$$= 85.2°$$
$$\theta_z = \cos^{-1}\left(\frac{\mathbf{F}_r}{F_R}\right) = \cos^{-1}\left(\frac{50}{386}\right)$$
$$= 82.6°$$

Experiments have revealed that the cutting force $\mathbf{F}_c$, the feed force $\mathbf{F}_f$, and the radial force $\mathbf{F}_r$ are functions of the feed and depth of cut and can be expressed in the form

$$\text{Force} = \text{Constant} \times \text{Feed}^{(\text{exponent}_f)} \times \text{Depth of cut}^{(\text{exponent}_d)}$$

where the constant, the exponent$_f$, and the exponent$_d$ are fixed for a tool of a specified shape, but vary to different values as the shape of

MODELO empírico

the tool changes[3]. In other words, it can be written that for a tool of a particular shape,

$$\mathbf{F}_c = C_c \times f^{n_1} \times d^{n_2} \tag{4.92}$$
$$\mathbf{F}_f = C_f \times f^{n_3} \times d^{n_4} \tag{4.93}$$

and

$$\mathbf{F}_r = C_r \times f^{n_5} \times d^{n_6} \tag{4.94}$$

where $\mathbf{F}_c$ is the cutting force, $\mathbf{F}_f$ is the feed force, $\mathbf{F}_r$ is the radial force, C_c is a constant for cutting force, C_f is a constant for feed force, C_r is a constant for a radial force, n_1 is the exponent of feed for cutting force, n_2 is the exponent of depth of cut for cutting force, n_3 is the exponent of feed for feed force, n_4 is the exponent of depth of cut for feed force, n_5 is the exponent of feed for radial force, and n_6 is the exponent of depth of cut for radial force. Boston and Kraus found that for a turning operation on annealed low-carbon steel using a high-speed-steel tool with a signature (designation—see Fig. 2.1) of 8-14-6-6-6-0-3-3/16 R, running at approximately 80 ft/min (24.38 m/min), the equations for cutting force, feed force, and radial force were of the form

$$\mathbf{F}_c = 58{,}000 \times f^{0.68} \times d^{0.83} \tag{4.95}$$
$$\mathbf{F}_f = 31{,}800 \times f^{0.57} \times d^{1.31} \tag{4.96}$$
$$\mathbf{F}_r = 7250 \times f^{0.68} \times d^{0.47} \tag{4.97}$$

The determination of the resultant force from the individual component forces is demonstrated by the following example.

EXAMPLE 4.40. A turning operation running at 80 ft/min (24.38 m/min), with a feed of 0.015 in. (0.381 mm) and a depth of cut of 0.250 in. (6.35 mm), has equations for the component forces in the form of Eqs. 4.95–4.97. From these data, determine:

(a) The cutting force.
(b) The feed force.
(c) The radial force.
(d) The resultant force.
(e) The horsepower required of the cut.
(f) The unit horsepower.
(g) The motor horsepower required if the machine has an efficiency of 85% and a tare horsepower of 0.75.

[3] O. W. Boston and C. E. Kraus, "A Study of the Turning of Steel Employing a New-Type Three-Component Dynamometer," *Trans. ASME*, RP-58-1, 1936, pp. 47–53.

Substituting into Eq. 4.95 yields

$$\mathbf{F}_c = 58{,}000 \times (0.015)^{0.68} \times (0.25)^{0.83}$$
$$= 1053 \text{ lb } (4684 \text{ N})$$

From Eq. 4.96,

$$\mathbf{F}_f = 31{,}800 \times (0.015)^{0.57} \times (0.25)^{1.31}$$
$$= 472 \text{ lb } (2{,}100 \text{ N})$$

From Eq. 4.97,

$$\mathbf{F}_r = 7250 \times (0.015)^{0.68} \times (0.25)^{0.47}$$
$$= 217 \text{ lb } (966 \text{ N})$$

The resultant force can be determined by substituting into Eq. 4.90:

$$\mathbf{R} = \sqrt{(\mathbf{F}_f)^2 + (\mathbf{F}_c)^2 + (\mathbf{F}_r)^2}$$
$$= \sqrt{(472)^2 + (1052)^2 + (217)^2}$$
$$= 1173 \text{ lb } (5218 \text{ N})$$

From Eq. 4.69, the horsepower required of the cut is

$$\text{hp}_c = \frac{\mathbf{F}_c \cdot \mathbf{V}_c}{33{,}000} = \frac{1053 \times 80}{33{,}000}$$
$$= 2.55 \text{ horsepower } (1.902 \text{ kW})$$

Substituting into Eq. 4.72 yields the unit horsepower as being equal to

$$U_p = \frac{\text{hp}_c}{R_v} = \frac{2.55}{3.6}$$
$$= 0.708 \text{ hp/in.}^3\text{/min } (1.934 \text{ kW/cm}^3\text{/sec})$$

where

$$R_v = 12 \times f \times d \times \mathbf{V}_c$$
$$= 12 \times 0.015 \times 0.25 \times 80$$
$$= 3.6 \text{ in.}^3\text{/min } (0.9828 \text{ cm}^3\text{/sec})$$

Finally, from Eq. 4.77,

$$\text{hp}_m = \frac{\text{hp}_c}{E_{ff}} + \text{hp}_t = \frac{2.55}{0.85} + 0.75$$
$$= 3.75 \text{ horsepower } (2.8 \text{ kW})$$

Since the constants (c) and exponents (n) of the component force equations vary with the material being cut as well as with the shape of the tool, specifications for a particular operation needs to involve an experimental technique. The feed (f) and the depth of cut (d) are independent variables, that is, their values do not depend on each other. Therefore, by holding one of the independent variables constant, we can evaluate the effect of changes of the other independent variable on the component force. In other words, by holding the depth of cut constant, the influence of the feed on the cutting force can be expressed as

$$\mathbf{F}_c = C_c \times f^{n_1} \times d^{n_2}$$

or

$$\mathbf{F}_c = C_1 \times f^{n_1} \tag{4.98}$$

where

$$C_1 = C_c \times d^{n_2}$$

If experimental data are compiled between the cutting force and the feed, with the depth of cut fixed, then the constant (C_1) and the exponent (n_1) of the feed can be evaluated. To determine the exponent (n_2) of the depth of cut, a similar technique may be employed. This can be accomplished by keeping the feed constant and compiling experimental data between the cutting force and the depth of cut. In this case, the cutting force equation changes from

$$\mathbf{F}_c = C_c \times f^{n_1} \times d^{n_2}$$

to

$$\mathbf{F}_c = C_2 \times d^{n_2} \tag{4.99}$$

where

$$C_2 = C_c \times f^{n_1}$$

Assume that in an effort to determine the relationship of the cutting force as a function of the feed and depth of cut, the following experiment was conducted. Dynamometer readings of the cutting force were taken with the depth of cut fixed and with four different settings of the feed. Then additional dynamometer readings were taken, but this time with the feed fixed and with four different settings of the depth of cut. The experimental data are listed in Table 4.6. The value of the constant C_1 and the exponent n_1 in Eq. 4.98 can be evaluated by substituting experimental data from Table 4.6. From tests 1 and 2, it can be written that

$$\mathbf{F}_c = C_1 \times f^{n_1}$$
$$125.7 = C_1 \times (0.005)^{n_1}$$

Table 4.6. Cutting Force Experimental Data

Test	Feed (in./rev)	Feed (mm/rev)	Depth of Cut (in.)	Depth of Cut (mm)	Cutting Force (lb)	Cutting Force (N)
1	0.005	0.127	0.050	1.27	125.7	559
2	0.010	0.254	0.050	1.27	218.8	973
3	0.015	0.381	0.050	1.27	302.3	1344
4	0.020	0.508	0.050	1.27	380.7	1693
5	0.010	0.254	0.050	1.27	218.8	973
6	0.010	0.254	0.100	2.54	422.8	1881
7	0.010	0.254	0.150	3.81	621.4	2764
8	0.010	0.254	0.200	5.08	816.7	3633

and

$$218.8 = C_1 \times (0.010)^{n_1}$$

Isolating C_1 yields

$$C_1 = \frac{125.7}{(0.005)^{n_1}} = \frac{218.8}{(0.010)^{n_1}} \qquad (1)$$

or

$$\left(\frac{0.010}{0.005}\right)^{n_1} = \frac{218.8}{125.7} = 1.7407$$

Solving for n_1 yields

$$n_1 \log 2 = \log 1.7407$$

$$n_1 = \frac{\log 1.7407}{\log 2} = \frac{0.2407}{0.301} = 0.8$$

By substituting in Eq. 1 for the value of n_1, C_1 is found to be equal to

$$C_1 = \frac{125.7}{(0.005)^{0.8}} = \frac{125.7}{0.0144} = 8712.8$$

For the cutting operation described in Table 4.6, Eq. 4.98 can now be expressed as

$$F_c = 8712.8 \times f^{0.8}$$

The constant and exponent in Eq. 4.99 can also be evaluated by using data from Table 4.6. Taking the values from tests 6 and 7 yields

$$\mathbf{F}_c = C_2 \times d^{n_2}$$

$$422.8 = C_2 \times (0.100)^{n_2}$$

$$621.4 = C_2 \times (0.150)^{n_2}$$

from which

$$C_2 = \frac{422.8}{(0.100)^{n_2}} = \frac{621.4}{(0.150)^{n_2}} \qquad (2)$$

Isolating n_2, we obtain

$$\left(\frac{0.150}{0.100}\right)^{n_2} = \frac{621.4}{422.8}$$

$$(1.5)^{n_2} = 1.4697$$

$$n_2 = \frac{\log 1.4697}{\log 1.5} = \frac{0.1672}{0.17609} = 0.95$$

C_2 can now be evaluated from Eq. 2 as

$$C_2 = \frac{422.8}{(0.100)^{0.95}} = \frac{422.8}{0.1122} = 3768.2$$

For the cutting operation expressed by the data in Table 4.6, Eq. 4.99 can be written as

$$\mathbf{F}_c = 3768.2 \times d^{0.95}$$

With the availability of the value of the exponents in Eq. 4.92, the numerical value of the cutting constant can easily be determined for this particular cutting operation by substituting data from Table 4.6. We can illustrate by using data from test 8:

$$\mathbf{F}_c = C_c \times f^{n_1} \times d^{n_2}$$

$$816 = C_c \times (0.010)^{0.8} \times (0.200)^{0.95}$$

$$C_c = \frac{816.7}{0.02512 \times 0.21676} = 150{,}000$$

As a result, the cutting force for the operation described by the data of Table 4.6 can now be written as

$$\mathbf{F}_c = 150{,}000 \times f^{0.8} \times d^{0.95}$$

A point of interest is that the numerical values of the exponents n_1 and n_2 can also be determined graphically by plotting the experimental data. To illustrate, Eq. 4.98 can be written as

$$\mathbf{F}_c = C_1 \times f^{n_1}$$

or for convenience as

$$\log \mathbf{F}_c = n_1 \log f + \log C_1 \qquad (4.100)$$

which has the form of the equation of a straight line

$$y = mx + b$$

When plotted on log–log graph paper, Eq. 4.98 appears as a straight line, where the exponent n_1 represents the slope of the line. Figure 4.51 represents the plot. As can be seen,

$$\text{Slope} = n_1 = \tan\theta_1 = \frac{\log \mathbf{F}_{c4} - \log \mathbf{F}_{c1}}{\log f_4 - \log f_1} \tag{4.101}$$

Solving for the value of the exponent n_1 from the example listed in Table 4.6 by using data from tests 1 and 4 gives

$$n_1 = \frac{\log 380.7 - \log 125.7}{\log 0.020 - \log 0.005}$$

$$= \frac{2.5806 - 2.0993}{-1.699 - (-2.301)} = \frac{0.4813}{0.602}$$

$$= 0.8$$

This value could also be attained through a direct angular measurement from Fig. 4.51, where

$$n_1 = \tan\theta_1 = \tan 38.66° = 0.8$$

With a similar graphical approach, the value of the exponent n_2 can also be found. The experimental values represented by Eq. 4.99, plotted on log–log graph paper, appear in the form of a straight line.

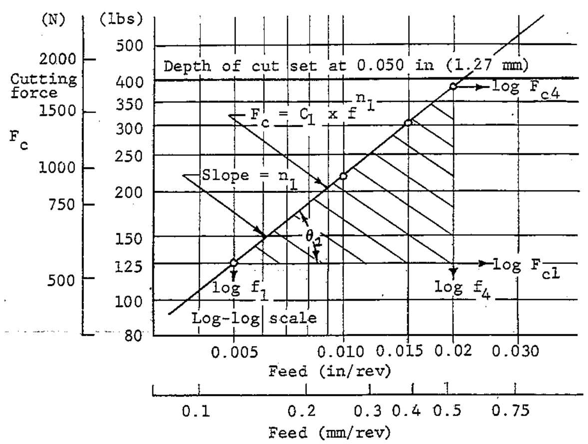


Fig. 4.51. Graphical representation of experimental data on log–log scale.

This is shown in Fig. 4.52. As can be seen,

$$\text{Slope} = n_2 = \tan\theta_2 = \frac{\log \mathbf{F}_{c8} - \log \mathbf{F}_{c5}}{\log d_8 - \log d_5} \tag{4.102}$$

Solving for the exponent n_2 by using data form tests 5 and 8 gives

$$n_2 = \frac{\log 816.7 - \log 218.8}{\log 0.2 - \log 0.05} = \frac{2.912 - 2.340}{-0.699 - (-1.301)}$$

$$= \frac{0.572}{0.602} = 0.95$$

The value of n_2 can also be extracted by a direct angular measurement from the graph. As a result,

$$n_2 = \tan\theta_2 = \tan 43.53° = 0.95$$

With the availability of the exponents n_1 and n_2, the value of the constant C_c in Eq. 4.92 can easily be evaluated by taking any set of values from the experimental data. As an example, using data from test 1 yields

$$\mathbf{F}_c = C_c \times f^{n_1} \times d^{n_2}$$

$$125.7 = C_c \times (0.005)^{0.8} \times (0.050)^{0.95}$$

$$C_c = \frac{125.7}{0.014427 \times 0.05808} = 150{,}000$$

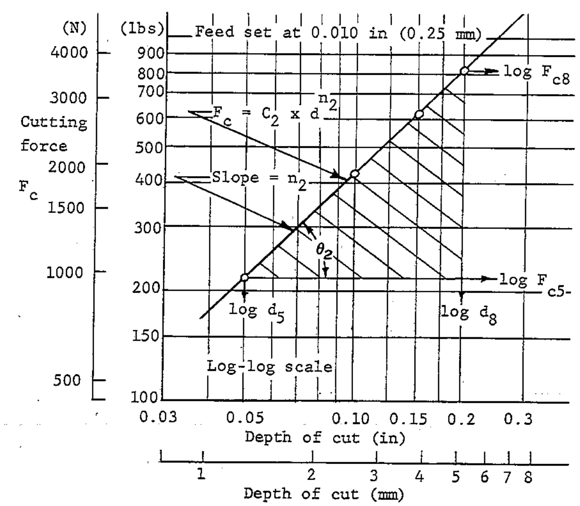


Fig. 4.52. Graphical representation of experimental data on log–log scale.

It is then confirmed that for the given experimental data, the cutting force equation can be written as

$$\mathbf{F}_c = 150{,}000 \times f^{0.8} \times d^{0.95}$$

The values of the constants and exponents for the feed force (Eq. 4.93) and the radial force (Eq. 4.94) can be determined from experimental data in a fashion similar to that employed for the cutting force (Eq. 4.92).

4.15 SUMMARY OF THE MECHANICS OF THE CUTTING PROCESS

The analysis and application of the forces acting between the work material and the tool constitutes the mechanics of the cutting process. Dynamometers, which are force-sensing instruments, provide a means of measuring cutting forces. Through a convenient design of a dynamometer, the resultant cutting force can be resolved into three space components. These components, in turn, enable the resolution of additional directional components that provide an insight into the interaction of the tool and workpiece.

The tool–chip interface force is a case in point. By determining the magnitude of the force acting parallel to the face of the tool, a measure of the tool–surface resistance to the sliding motion of the chip can be attained. This, in turn, enables us to determine the coefficient of friction, which reflects directly on the interaction of the tool and work-piece. In this way, a measure of the quality of the tool surface is determined.

Free-body diagrams of the tool, chip, and workpiece show how the resultant cutting force is transmitted during the cutting process. The free-body diagrams also provide a means of isolating component forces in directions covenient for analysis. Force magnitudes can also be extracted from free-body diagrams. A case in point is the evaluation of the magnitude of the force that acts in the direction of the chip shear plane. This force, distributed over the area of the shear plane, provides a measure of the plastic flow shear stress state that exists during the metal cutting operation.

Vector polygon graphical layouts of the component forces provide not only a visual display of the forces but also a means of expressing forces in vector summation equations. The graphical layouts are justified by the first and third laws of Newton's equilibrium state.

This analysis leads to the determination of the direction and magnitude of individual component forces that are represented in vector form.

The resolution of the resultant cutting force into three sets of perpendicular components extends the analysis of evaluating the effects of a component force in a particular direction. Force action along the shear plane, along the face of the tool, and along the cutting direction leads to quantification of cutting characteristics.

As an example, force action on the shear plane is a reflection of the shear stress of the work material and also influences the cutting force that acts on a plane perpendicular to the direction of the cutting force. The plane over which the cutting force is distributed is equal to the feed multiplied by the depth of cut, and the ratio of the cutting force divided by this area is called the *chip pressure*. It has been shown that the shear angle is a function of the tool side-rake angle and that it has a major influence on the chip pressure.

Force action on the face of the tool can be interpreted as a measure of the quality of the surface of the tool. To illustrate, a tool that provides a low friction force while the chip slides over it can be expected to use less energy and thus generate lower temperatures in the cutting process. The term *coefficient of friction* is a measure of the ratio of the frictional force to the force acting normal to the frictional surface. This term does not have a fixed value when considering a specific tool machining operation with a particular material. Rather, it has been shown experimentally that the coefficient of friction varies as a function of the cutting velocity, which, in turn, is related to the temperature acting on the tool and the workpiece.

Force action in the direction of the cutting velocity is of importance because it is related to the energy expended during the metal cutting operation. The product of the cutting force multiplied by the cutting velocity is expressed in units of work per unit time, which is a measure of power.

Velocity components other than the cutting velocity are also worth of consideration. As an example, the chip velocity is related to the cutting velocity through the shear angle and is affected by the chip thickness. Obviously, the relationship is inverse, that is, a thicker chip will produce a lower chip velocity.

An energy analysis of the major components of the resultant force, namely, the cutting force ($\mathbf{F}_c$), the feed force ($\mathbf{F}_f$), and the radial force ($\mathbf{F}_r$) reveals that the predominant amount of energy used in a metal cutting operation is consumed by the cutting force. As a result, for practical purposes of analysis, energy absorption by the feed and radial forces are negligible and need not be considered.

Rotary tools, such as drills and milling cutters, have similar component force designations as those for a single-point tool. The reason for this is that multiple-point tools can be reduced in the analysis of force components to a series of single-point tools spaced along the circumference of the tool. In analyzing rotary tools, the work performed is equal to the product of the torque and the angular displacement through which the torque acts. This is an important point insofar as confusion oftentimes develops with respect to the balance of the units. In this case, the radian is considered unitless and the work has assigned to it the same units as the torque.

The amount of work performed per unit time is the definition of *power*. It is the rate at which work (energy) is performed. The power per unit volume required for a metal cutting operation is the definition of *unit horse power*. This value is a measure of the power required to machine at a volumetric rate of unity. It has been found that the unit horsepower is related to the hardness of the work material. A harder material requires a higher unit horsepower during the metal cutting operation.

The amount of power required at the cut is not equal to the total power required to perform an operation on a machine tool. This is because (1) all machine tools require a certain amount of power to run in the idling mode and (2) all machine tools are less than 100% efficient. Therefore, in calculating the capacity of a machine in terms of motor horsepower, the machine efficiency as well as the tare (idling) horsepower must be taken into consideration.

Another point of interest is the effect that the feed and positioning of a tool have on the clearance angles that are ground into the tool. What may seem to be a desirable side relief angle in a turning operation can be eliminated completely if the lead angle of the feed is greater than the clearance angle. The result is a rubbing action rather than one of cutting. A similar situation can develop when the tool is set too high above center. This condition can eliminate the end relief angle in a turning operation. Deflections of tools, such as boring bars, under the forces of cutting can also influence the clearance angles. This takes place when the tool moves from its unloaded position as a result of absorbing the forces acting on it.

The relationship of the components of the resultant cutting force to cutting conditions, such as the feed and the depth of cut, can be extracted from experimental data. As an example, if one compiles data on the cutting force as a function of the feed, with the depth of cut fixed, then the relationship between cutting force and feed can be established. In a similar fashion, the relationship between cutting force and depth of cut can be established. This can be done if data are compiled where the cutting force is measured as a function of the depth

of cut, with the feed fixed. By combining the test results, expressions can be written relating the component cutting forces with cutting conditions.

PROBLEMS

PROBLEM 4.1. For a two-dimensional cutting force analysis similar to that shown in Fig. 4.3, determine the magnitude and direction of the resultant cutting force if dynamometer measurements are 1690 N for the cutting force ($\mathbf{F}_c$) and 556 N for the feed force ($\mathbf{F}_f$).

Answers: R = 1779 N
$\theta = 18.2°$

PROBLEM 4.2. Dynamometer readings register values of 450 lb for the cutting force ($\mathbf{F}_c$) and 156 lb for the feed force ($\mathbf{F}_f$). The cutting tool is found to have a rake angle of 10°. From this information, determine:

(a) The resultant force.
(b) The angle (θ) at which the resultant force acts.
(c) The friction force (**F**).
(d) The normal force (**N**).
(e) The coefficient of friction (μ).

Answers: **R** = 476.3 lb
$\theta = 19.12°$
F = 231.6 lb
N = 4.5.8 lb
$\mu = 0.557$

PROBLEM 4.3. A turning operation registers a cutting force of 1112 N and a feed force of 333.6 N. It is also found that the chip thickness is 1.143 mm for a corresponding feed of 0.381 mm/rev. The tool performing the cutting has a rake angle of 10°. From this information determine:

(a) The resultant force (**R**).
(b) The angle at which the resultant force acts (θ).
(c) The shear angle (α).
(d) The shear force ($\mathbf{F}_s$).
(e) The force normal to the shear plane ($\mathbf{F}_n$).

Answers: R = 1161 N
$\theta = 16.7°$
$\alpha = 19.02°$
$\mathbf{F}_s = 942.6$ N
$\mathbf{F}_n = 677.8$ N

PROBLEM 4.4. For a metal cutting operation, the shear stress is to be evaluated by using dynamometer readings. It is found through chip thickness measurements that the shear angle is 20°. Other data compiled include: $\mathbf{F}_c = 800$ lb; $\mathbf{F}_f = 275$ lb; $f = 0.020$ in./rev; and $d = 0.375$ in.

Answer: $S_s = 29{,}993$ psi

PROBLEM 4.5. A metal cutting operation is in the process of being evaluated for the magnitude of the three sets of perpendicular force components shown in Fig. 4.12, into which the resultant force can be resolved. Dynamometer readings give values of 1780 N for the cutting force ($\mathbf{F}_c$) and 670 N for the feed force ($\mathbf{F}_f$). The chip thickness ratio (r_a) is found to be equal to 0.27 and the tool rake angle is 5°. With the availability of these data, calculate:

(a) The shear angle (α).
(b) The resultant force (**R**).
(c) The angle of kinetic friction (ϕ).
(d) The frictional force (**F**).
(e) The normal force (**N**).
(f) The angle K as shown in Fig. 4.12.
(g) The shear force ($\mathbf{F}_s$).
(h) The force normal to the shear plane ($\mathbf{F}_n$).

Answers: $\alpha = 15.4°$
$\mathbf{R} = 1902$ N
$\phi = 25.63°$
$\mathbf{F} = 822.7$ N
$\mathbf{N} = 1714.9$ N
$K = 36.03°$
$\mathbf{F}_s = 1538.2$ N
$\mathbf{F}_n = 1118.8$ N

PROBLEM 4.6. Using the information provided in problem 4.4, calculate the chip pressure resistance and compare it with the shear stress.

Answers: $P_c = 106{,}666$ psi
$R_{sc} = 0.28$

PROBLEM 4.7. It is desired in a metal cutting operation to determine the value of the cutting force and the coefficient of friction by using the yield shear stress, which is found to be 220×10^6 Pa for the material being cut. For the operation, a 5°-side-rake-angle tool produces a

shear angle of 20°. The feed is set at 0.457 mm/rev and the depth of cut is 6.35 mm. Assume that $\mathbf{F}_n/\mathbf{F}_s = 1$.

Answers: $\mathbf{F}_c = 2391$ N
$\mu = 0.577$

PROBLEM 4.8. Calculate the expected cutting force for an operation where the shear stress is 32,000 psi and the shear angle is 18°. The feed is set at 0.010 in./rev and the depth of cut is 0.100 in. Assume that $\mathbf{F}_n/\mathbf{F}_s = 1$.

Answer: $\mathbf{F}_c = 130.6$ lb

PROBLEM 4.9. Determine the coefficient of friction and the angle of kinetic friction for a metal cutting operation using a 10°-rake-angle tool. Dynamometer readings are 1500 N for the cutting force ($\mathbf{F}_c$) and 800 N for the feed force ($\mathbf{F}_f$).

Answers: $\mu = 0.783$
$\phi = 38.07°$

PROBLEM 4.10. Calculate the chip velocity and the shear velocity for an operation being machined with a tool that has a side rake angle of 5°. The cutting velocity is 200 ft/min. Other machine settings include: chip thickness equal to 0.032 in. and feed equal to 0.014 in.

Answers: $\mathbf{V}_p = 87.47$ ft/min
$\mathbf{V}_s = 211.2$ ft/min

PROBLEM 4.11. Determine the amount of work performed by each of the three components of the resultant force for one revolution of a turning operation. The component forces are:

$$\mathbf{F}_c = 1000 \text{ N}, \quad \mathbf{F}_f = 350 \text{ N}, \quad \text{and} \quad \mathbf{F}_r = 100 \text{ N}$$

The diameter of the workpiece is 60 mm and is being machined at 50 m/min, with a feed of 0.4 mm/rev and a depth of cut of 6.0 mm.

Answers: $W_c = 188.4$ joules
$W_f = 0.14$ joules
$W_r = 0$ joules

PROBLEM 4.12. Using a graphical technique, confirm the answers of problem 4.10.

Answers: $\mathbf{V}_p = 87.47$ ft/min
$\mathbf{V}_s = 211.2$ ft/min

PROBLEM 4.13. A drill has acting on it a torque of 20 N-m and a thrust of 4000 N. The feed is set at 0.25 mm/rev. For one revolution of the drill, calculate the work done by the torque and by the thrust force.

Answers: $W_t = 125.66$ joules
$W_{st} = 1.0$ joules

PROBLEM 4.14. Solve for the torque and the thrust of a standard $\frac{3}{4}$-in. drill that is machining 200-Bhn steel with a feed setting of 0.010 in./rev.

Answers: $T = 368$ in.-lb
$T_{st} = 1455.7$ lb

PROBLEM 4.15. It is desired to determine the amount of power necessary to perform a given cut. The cutting velocity is 150 m/min, the feed is 0.5 mm/rev, the depth of cut is 5.0 mm, and the cutting force is 5000 N.

Answer: $P = 12.5$ kW

PROBLEM 4.16. A 3-in.-diameter milling cutter with eight cutting edges is programmed to rotate at 120 rpm. Calculate the cutting velocity and the feed per tooth for a workpiece feed of 2.5 in./min.

Answers: $\mathbf{V}_c = 94.2$ ft/min
$f_{\text{tooth}} = 0.0026$ in.

PROBLEM 4.17. A milling operation is set for a depth of cut of 12.7 mm, and a torque measurement indicates a value of 135.6 N-m. The rpm is 100 and the work feed is 44.45 mm/min. From these data, calculate the power required for the cutting operation.

Answer: $P = 1.42$ kW

PROBLEM 4.18. Determine the amount of horsepower required for a $\frac{3}{4}$-in.-diameter drilling operation that generates a torque of 310 in.-lb when running at 450 rpm.

Answer: hp = 2.21 horsepower

PROBLEM 4.19. A turning operation is observed to have a cutting force of 1500 N and a cutting velocity of 75 m/min. From these data, calculate the power required for the cutting operation.

Answer: $P = 1.875$ kw

PROBLEM 4.20. A 1-in.-diameter end mill is set to cut a 0.500-in.-deep by 1-in.-wide slot in a steel workpiece that has a Bhn of 180. The end mill is set to run at 80 ft/min with a feed of 0.0025 in./tooth. The end mill has six teeth (cutting edges). From these data, calculate:

(a) The feed set on the machine tool.
(b) The volumetric rate of metal removal.
(c) The horsepower consumed in the operation.
(d) The torque acting on the end mill by using a unit horsepower value from Table 4.5.
(e) The average cutting force acting on the end mill.

Answers: $f = 4.586$ in./min
$R_v = 2.293$ in.^{3}/min
$hp_c = 1.72$ horsepower
$T = 354.6$ in.-lb
$\mathbf{F}_c = 709.5$ lb

PROBLEM 4.21. A 12-mm-diameter high-speed standard drill is machining steel that possesses a unit power of 3.0 kW/cm^3/sec. The cutting velocity is set at 12 m/min and the feed rate is 40 mm/min. Neglecting the effects of the chisel edge, determine the power required for this operation as well as the torque acting on the drill.

Answers: $P = 0.264$ kW
$T = 7.93$ N-m

PROBLEM 4.22. Reference sources reveal that a medium carbon steel workpiece material has a unit horsepower of 0.8 hp/in.^{3}/min. For a turning operation, the cutting velocity is set at 500 ft/min with a corresponding feed of 0.022 in./rev and a depth of cut of 0.210 in. From the given data, calculate:

(a) The cutting force.
(b) The volumetric rate of machining.
(c) The horsepower consumed by the cut.

Answers: $\mathbf{F}_c = 1463$ lb
$R_v = 27.72$ in.^{3}/min
$hp_c = 22.176$ horsepower

PROBLEM 4.23. It is desired to determine the overall efficiency of a machine tool. A wattmeter is set up to measure the input power and a dynamometer is used to measure the cutting force. For a cutting velocity of 50 m/min, it is found that the wattmeter reading is 2 kW and the dynamometer reading is 2000 N.

Answer: $E_{ff} = 83.4\%$

PROBLEM 4.24. It is desired to determine the required motor horsepower of a turning operation. The machine being used has a power efficiency of 85% and a tare horsepower of 0.5. The cutting velocity is set at 200 ft/min, with a feed of 0.015 in./rev and a depth of cut of 0.250 in. The unit horsepower of the material being cut is found to be 1.2 hp/in.^{3}/min. With this information, determine:

(a) The volumetric rate of metal removal.
(b) The horsepower required at the cut.
(c) The motor horsepower.

Answers: $R_v = 9$ in.^{3}/min
$hp_c = 10.8$ horsepower
$hp_m = 14$ horsepower

PROBLEM 4.25. Determine the motor horsepower required for an operation where the required power to perform the cut is 5 kW, the machine efficiency is 80%, and the tare power is 0.3 kW.

Answer: $P = 6.55$ kW

PROBLEM 4.26. Determine the tool deflection on a steel boring bar that has a 0.75-in. diameter, a 5-in. length, and a 200-lb cutting force acting on it.

Answer: $Y = 0.018$ in.

PROBLEM 4.27. Dynamometer readings record the following values of component forces for a turning operation: $\mathbf{F}_f = 950$ N, $\mathbf{F}_c = 1500$ N, and $\mathbf{F}_r = 225$ N. From this information, calculate the resultant force (**R**).

Answer: $\mathbf{R} = 1789.7$ N

PROBLEM 4.28. A turning operation running at 80 ft/min, with a feed of 0.020 in./rev and a depth of cut of 0.375 in., has equations for component forces in the form of Eqs. 4.95–4.97. From these data, determine:

(a) The cutting force.
(b) The feed force.
(c) The radial force.

Answers: $\mathbf{F}_c = 1796$ lb
$\mathbf{F}_f = 946$ lb
$\mathbf{F}_r = 320$ lb

PROBLEM 4.29. Using the experimental data represented in Table 4.6, write the equation for the cutting force using SI units, where the feed is given in millimeters per revolution, the depth of cut is given in millimeters, and the cutting force is given in newtons..

Answer: $\mathbf{F}_c = 2321 \times f^{0.8} \times d^{0.95}$

PROBLEM 4.30. A metal cutting operation is described by the answer given in Problem 4.29.

(a) Calculate the cutting force for a feed of 0.127 mm/rev and a depth of cut of 1.27 mm.
(b) If the feed is increased to 0.254 mm/rev and the depth of cut is increased to 5.08 mm, determine the corresponding cutting force.

Answers: (a) $\mathbf{F}_c = 559$ N
(b) $\mathbf{F}_c = 3633$ N

BIBLIOGRAPHY

Baumeister, Theodore, *Marks' Standard Handbook for Mechanical Engineers*, McGraw-Hill, New York, 1978.

Beer, Ferdinand P., and Johnston, E. Russell, Jr., *Mechanics for Engineers—Statics*, McGraw-Hill, New York, 1976.

Boston, O. W., and Kraus, C. E., "A Study of the Turning of Steel Employing a New-Type Three-Component Dynamometer," *Trans. ASME*, RP-58-1, 1936.

Boston, Orlan William, *Metal Processing*, John Wiley & Sons, New York, 1951.

Cook, Nathan H., *Manufacturing Analysis*, Addison-Wesley, Reading, Massachusetts, 1966.

Costa, Leslie F., Jr., "A Measurement of the Coefficient of Friction as a Function of Cutting Speed," Senior Design Project, Southeastern Massachusetts University, North Dartmouth, Massachusetts, 1977.

Dallas, Daniel B., *Tool and Manufacturing Engineers Handbook*, Society of Manufacturing Engineers, Dearborn, Michigan, 1976.

Hoffman, Edward G., *Fundamentals of Tool Design*, Society of Manufacturing Engineers, Dearborn, Michigan, 1984.

Machining Data Handbook, Metcut Research Associates, Cincinnati, Ohio, 1972.

Manual on Cutting Metal, The American Society of Mechanical Engineers, New York, 1952.

Merchant, M. Eugene, "Basic Mechanics of the Metal-Cutting Process," *J. Appl. Mech.*, Vol. 15, September 1944.

Merchant, M. Eugene, "Mechanics of the Metal Cutting Process II, Plasticity Conditions in Orthogonal Cutting," *J. Appl. Mech.*, Vol. 16, June 1945.

Oberg, E., Jones, F. D., and Horton, H., *Machinery's Handbook*, 22nd ed. Industrial Press, New York, 1984.

Sen, Gopal Chandra, and Bhattacharyya, Amitabha, *Principles of Metal Cutting*, New Central Book Agency, Calcutta, India, 1969.

Shaw, M. C., and Oxford, C. J., "On the Drilling of Metals 2—The Torque and Thrust in Drilling," *Trans. ASME*, Vol. 79, 1957.

Shaw, Milton C., *Metal Cutting Principles*, MIT Press, Cambridge, Massachusetts, 1968.

Williams, R. A., "A Study of the Drilling Process," *J. Eng. Ind.*, 1973.